Introduction to Abstract Algebraic Systems MATH-430-1070 (11365), Fall 2022

Exercise Set 3

1. Find the explicit representation of the right and left cosets of the indicated subgroups H of $GL(n)$. In each case, determine whether the left and right cosets corresponding to the same $A \in GL(n)$ are equal.
(i) $H = SO(n)$,
(ii) $H = \{c\mathrm{Id};\ c \in \mathbb{R} \setminus \{0\}\}$,
(iii) H consists of $n \times n$ diagonal matrices with nonzero determinant.

2. For any two elements a, b of a finite group G, show that:

$$order(a) = order(a^{-1}) \quad \text{and} \quad order(ab) = order(ba)$$

3. Prove that a finite group G has exactly three subgroups if and only if G is cyclic and its order is a square of a prime number.

4. Show that the alternating group A_4 (namely, the subgroup of S_4 consisting of all even permutations) has order 12.

5. Show that A_4 has no subgroup of order 6. (This implies that the implication of the Lagrange theorem cannot be reversed.)