

**Introduction to Abstract Algebraic Systems MATH-430-1070 (11365),  
Fall 2022**

**Exercise Set 4**

**1.** A group  $G$ , whose every subgroup has order equal to some power of a fixed prime number  $p$ , is called a *p-group*. Further, given a finite group  $G$  and a prime number  $p$ , a *Sylow p-subgroup* of  $G$  is such  $p$ -subgroup  $H$  for whom  $p \nmid [G : H]$ .

Find all Sylow subgroups with:

- (i)  $G = S_3, p = 2,$
- (ii)  $G = S_3, p = 3,$
- (iii)  $G = S_4, p = 3.$

**2.** A subgroup  $H$  of a group  $G$  is called a *characteristic subgroup*, if  $\phi(H) = H$  for every automorphism  $\phi : G \rightarrow G$ .

Show that if  $H$  is characteristic in  $G$  and  $G$  is a normal subgroup of some group  $K$ , then  $H$  is a normal subgroup of  $K$ . In particular,  $H$  is normal in  $G$ .

**3.** Artin textbook problem 11.8, chapter 2.

**4.** Artin textbook problem M9, chapter 2.

**5.** Artin textbook problem M10, chapter 2.