

Isometric immersions: rigidity, regularity and aspects from nonlinear elasticity

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September 21, 2012

The study of isometric immersions of Riemannian manifolds was one of the first topics studied at the dawn of modern Differential Geometry. Among the main problems already discussed in the 19th century are the problems of existence and rigidity of given classes of isometric immersions. While these problems and many more were settled for C^1 isometric immersions by the pioneering work of Nash in the fifth decade of the last century, the answers have remained only partial in case a higher regularity (for example C^2) is assumed or required. A natural analytical question would be to explore the properties of isometric immersions lying somewhat in between these two categories. Sobolev spaces of isometries are the natural candidate in this regard.

In the meantime, the question of rigidity of elastic structures is a long-standing problem in engineering, solid mechanics and mathematical theory of elasticity. Recent developments in the theory of elasticity has lead to new nonlinear theories of rods, plates and shells involving isometries and infinitesimal isometries. The variational approach to elasticity leads us to consider isometric or infinitesimal isometries of Sobolev regularity. The main problems are rigidity, regularity, approximation by smooth isometries and matching properties. Also, recent studies of the pre-strained materials have revived the interest in, among others, the problems regarding existence of isometric immersions.

After reviewing the classical theory, we will discuss some recent results regarding Sobolev isometric immersions and Sobolev solutions to Monge-Ampère equations. Some applications in nonlinear elasticity will be discussed. We will then move forward to consider major open problems and possible venues for future research.

The lecture will take place in Thackeray 704 at 4:00pm.
Refreshments will start at 3:30pm.