COLLOQUIUM

Weyl's asymptotic law in the setting of Lèvy processes

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In October 1910, the Dutch physicist Hendrik Antoon Lorentz, delivered a series of six lectures (the Paul Wolfskehl lectures) to the faculty of the University of Göttingen titled "old and new problems in physics". During the fourth lecture, with David Hilbert and his student Hermann Weyl present in the audience, he conjectured that the number of eigenvalues for the Laplacian for a region D in three space not exceeding the positive number λ is proportional to the volume of D times $\lambda^{3/2}$, when λ is large. (The problem had been raised a month earlier by Arnold Sommerfeld at a lecture in Könisberg.) Hilbert predicted that the conjecture would not be proved in his lifetime. He was wrong by several years. The conjecture was proved by Weyl in 1912.

Weyl's celebrated theorem, commonly referred to as Weyl's Law, has been extended and refined in many directions with connections to many areas of mathematics and physics. In this talk we first give an overview of some of the classical results in the field and discuss the elegant connections to Brownian motion first explored by Mark Kac in the 50's and 60's. We will then discuss problems that arise when the Brownian motion, which "goes" with the classical Laplacian, is replaced by other Lèvy processes. Such processes share many important properties with Brownian motion. We will look at a class of interesting examples that have been widely studied recently, the rotationally invariant stable processes that "go" with fractional powers of the Laplacian.

The lecture will take place in Thackeray 704 at 3:30pm. Refreshments will start at 3:00pm.