

COLLOQUIUM

Hermitian eigenvalue problem and its generalization to any semisimple group: A survey

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The classical Hermitian eigenvalue problem addresses the following question: What are the possible eigenvalues of the sum $A+B$ of two Hermitian matrices A and B , provided we fix the eigenvalues of A and B . A systematic study of this problem was initiated by H. Weyl (1912). By virtue of contributions from a long list of mathematicians, notably Weyl (1912), Horn (1962), Klyachko (1998) and Knutson-Tao (1999), the problem is finally settled. The solution asserts that the eigenvalues of $A+B$ are given in terms of certain system of linear inequalities in the eigenvalues of A and B . These inequalities are given explicitly in terms of certain triples of Schubert classes in the singular cohomology of Grassmannians and the standard cup product. Belkale (2001) gave an optimal set of inequalities for the problem in this case. The Hermitian eigenvalue problem has been extended by Berenstein-Sjamaar (2000) and Kapovich-Leeb-Millson (2005) for any semisimple complex algebraic group G . Their solution is again in terms of a system of linear inequalities obtained from certain triples of Schubert classes in the singular cohomology of the partial flag varieties G/P (P being a maximal parabolic subgroup) and the standard cup product. However, their solution is far from being optimal. In a joint work with P. Belkale, we define a deformation of the cup product in the cohomology of G/P and use this new product to generate our system of inequalities which solves the problem for any G optimally (as shown by Ressayre).

I together with Belkale also solve the corresponding ‘multiplicative eigenvalue problem’.

The lecture will take place in Thackeray 704 at 3:30pm.
Refreshments will start at 3:00pm.