

GIOVANNI PRODI SEMINAR :
VARIATIONAL MULTISCALE METHODS IN MATERIAL SCIENCE

COORDINATED BY: DR. MARTA LEWICKA AND DR. ANJA SCHLÖMERKEMPER

INSTITUT FÜR MATHEMATIK
JULIUS-MAXIMILIANS-UNIVERSITÄT WÜRZBURG

TUESDAYS 14:15-15:45, ROOM SE 40
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Problems from Materials Science provide a rich source of challenges in research directions traditionally associated with pure mathematics, as: Mathematical Analysis, Differential Equations, Geometry, Calculus of Variations. These problems often have a multiscale character; for example, the atomic structure of a material influences its macroscopic properties. Purely continuum models also raise multiscale issues, involving the formation and evolution of patterns as a consequence of loading or phase transformation.

The purpose of this seminar is to study new results in the application of Variational Multiscale Methods to systems related to Materials Science. Different approaches to coupling of the two extreme scales: the microscopic and the continuum will be at the center of the attention. Other recent advances, such as: emergence of wrinkling patterns driven by loading or growth, prediction of dislocations and fracture, dimension reduction, microstructures and thin films of nematic liquid crystal, will be considered, with an eye on open problems and possible extensions.

As a starting point, we plan to discuss the following topics:

- (1). The geometric rigidity estimate, which paved the way for rigorous derivation of various plate theories through methods of Calculus of Variations.
 - G. Friesecke, R. James and S. Müller, *A theorem on geometric rigidity and the derivation of nonlinear plate theory from three dimensional elasticity*, Comm. Pure. Appl. Math., **55** (2002), 1461–1506.
- (2). The method of convex integration, with application to the Born-Infeld equations.
 - Stefan Müller, Mariapia Palombaro, *On a differential inclusion related to the Born-Infeld equations* (2012)
- (3). Analysis of the elastic ground states of a nematic glass in the membrane approximation, inducing three-dimensional shapes from flat sheets of material, at the nanoscale all the way to macroscopic objects, including nondevelopable surfaces.
 - Carl D. Modes, Kaushik Bhattacharya and Mark Warner *Disclination-mediated thermo-optical response in nematic glass sheets* (2010)
- (4). Consider the deformation of a thin elastic film bonded to a thick compliant substrate. Taking a variational viewpoint which views the buckling of the film as a problem of energy-driven pattern formation, one can identify the scaling law of the

minimum energy with respect to the physical parameters of the problem, and we prove that a herringbone pattern achieves the optimal scaling.

- Robert V. Kohn and Hoai-Minh Nguyen, *Analysis of a compressed thin film bonded to a compliant substrate: the energy scaling law* (2012)

The seminar will be self-contained. Knowledge of Lebesgue measure and integration, Lebesgue spaces L^p and basic knowledge of Sobolev spaces $W^{1,p}$, as well as the standard linear algebra material and the very basic differential geometry in \mathbb{R}^3 will be assumed.