

Topics in the Calculus of Variations
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This fifteen-hour compact course will provide a rapid introduction to a variety of modern techniques and results in the fields of Calculus of Variations and Analysis, with the eye on applications centered around the Mathematical Theory of Elasticity and Phase Transitions.

The course will be self-contained. A tentative outlay is as follows:

Lectures 1 - 4:

- The notion of Γ -convergence and its fundamental properties.
- Examples of Γ -convergence in the linearized elasticity.
- The fundamental role of Korn's inequality.
- Nonlinear elasticity of shells with arbitrary geometry: derivation of the von Kármán energy as the Γ -limit of the nonlinear energies.
- The fundamental role of the geometric rigidity inequality.

Lectures 5 - 8:

- A proof of Korn's inequality.
- A proof of Friesecke-James-Müller's geometric rigidity inequality.

Lectures 9 - 10:

- The hierarchy of the limiting theories for elastic plates and shells.
- Energy scaling regimes and the matching properties.
- A proof of the matching property for convex shells.

Lectures 10 - 15:

- The Aviles-Giga functional. The heuristic similarity with the Föppl-von Kármán functional in the high energy regimes.
- A compactness result in the gradient theory of phase transitions.
- The Div-Curl lemma (a self-contained proof due to Polisevskii).
- The Young measures and their basic properties.
- The proof of the Desimone-Müller-Kohn-Otto compactness result.

Prerequisites: Knowledge of Lebesgue measure and integration, Lebesgue spaces L^p and basic knowledge of Sobolev spaces $W^{1,p}$, as well as the standard linear algebra material and the basic differential geometry in \mathbb{R}^3 will be assumed.

Dates:

Monday 17-th from 14:00, Tuesday 18-th from 14:00, Wednesday 19-th from 14:00, Thursday 20-th from 14:00 PLUS Exam in January