VORTEX STRETCHING AND CRITICALITY FOR THE 3D NSE

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The purpose of this lecture is to present a mathematical evidence – in a statistically significant sense – of a geometric scenario leading to criticality of the problem of (possible) singularity formation in the 3D Navier-Stokes equations [Dascaliuc and Grujić; 2012].

It will be shown that – in a suitably defined region of intense vorticity, comprised of vortex filaments – the scale of local linear sparseness needed to prevent the formation of singularities (this is based on a very intimate interplay between the diffusion in the model – quantified by the local-in-time analytic smoothing in L^∞ – and the geometric properties of the harmonic measure [Grujić; 2001, 2012]) coincides with a scale associated with the diameters of vortex filaments (this is based on an a priori sup-in-time L^1-estimate on the vorticity [Constantin; 1990] and the method of detecting multiscale sign fluctuations across physical scales introduced in [Dascaliuc and Grujić; 2011], this time with an eye on detecting the range of scales of positivity of the vortex stretching term, i.e., identifying the range of longitudinal scales associated with the creation and persistence of vortex filaments.