Singular Perturbations of Partial Differential Equations and Games

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In these lectures we will examine singular perturbations of Partial Differential Equations (PDEs) and their interpretation/motivation in the context of optimal control and differential games. We will consider a variety of results, including work due to E. N. Barron, L. C. Evans, A. Friedman, P. L. Lions, and P. E. Souganidis. Starting with a short review of the "value" of games, we will devote special attention to the dynamic programming principle and finding the PDEs (a.k.a. Hamilton-Jacobi-Bellman equations) satisfied by a "value" of these games. As we now know, a natural tool for studying solutions of these PDEs is the Crandall-Lions theory of viscosity solutions. Consequently, we also present a brief introduction to the topic, including basic definitions and results. Beginning our study of singular perturbations, we consider a simple example of differential games with Lipschitz controls, and the singular perturbation of its associated Hamilton-Jacobi-Bellman equation. Moving to deeper applications, we consider existence and $C^{1,1}$ regularity of solutions of convex, fully nonlinear, uniformly elliptic PDEs via games. Concluding our tour, we consider applications in homogenization and the approximation of second order parabolic equations by coupled systems of first order Hamilton-Jacobi equations.