Portfolio management
with small transaction costs

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April 1 - 6, 2013

In these series of talks, I consider the classical Merton problem of lifetime consumption, portfolio optimization problem in a market with small transaction costs. The problem of investment and consumption in a market with transaction costs was first studied by Magill & Constantinides in 1976 and later by Constantinides in 1986. Since then, starting with the classical paper of Davis & Norman (1990) an impressive understanding of this problem has been achieved. The problem of proportional transaction costs is a special case of a singular stochastic control problem in which the state process can have controlled discontinuities. The related partial differential equation for this class of optimal control problems is a quasi-variational inequality which contains a gradient constraint. Technically, the multi-dimensional setting presents intriguing free boundary problems.

It is well known that in practice the proportional transaction costs are small and in the limiting case of zero costs, one recovers the classical problem of Merton. Then, a natural approach to simplify the problem is to obtain an asymptotic expansion in terms of the small transaction costs. The first proof in this direction was obtained in the appendix of my joint paper with Shreve in 1994. Later several rigorous results and formal asymptotic results have been obtained. The rigorous results have been restricted to one space dimensions with the exception of the recent paper by Bichuch & Shreve.

In these talks, I first describe the dynamic programming approach. Then, the asymptotic problem will be studied using the techniques of homogenization and the formal matched asymptotics. The rigorous proofs will be based on techniques from the theory of viscosity solutions and the perturbed test

functions of Evans. General utilities, multi-dimensional problems, fixed and
and proportional costs will be treated.