Econ 2001 Summer 2015

Problem Set 14

1. Which of the following functions are concave or convex (on $\mathbf{R}^n$)?

   (a) $f(x) = 3e^x + 5x^4 - \ln x$,

   (b) $f(x, y) = -3x^2 + 2xy - y^2 + 3x - 4y + 1$,

   (c) $f(x, y, z) = 3e^x + 5y^4 - \ln z$, and

   (d) $f(x, y, z) = Ax^a y^b z^c$, for $a, b, c > 0$.

2. For each of the following functions defined on $\mathbf{R}^2$, find the critical points and classify them as local maxima, local minima, or else. Also determine if any of the local maxima/minima are global maxima/minima.

   (a) $xy^2 + x^3y - xy$,

   (b) $x^2 - 6xy + 2y^2 + 10x + 2y - 5$,

   (c) $x^4 + x^2 - 6xy + 3y^2$, and

   (d) $3x^4 + 3x^2y - y^3$.

3. Consider the function $f(x, y) = e^{ax^{\frac{1}{2}} - y}$, where $x \geq 0$, and $y \in \mathbf{R}$. Determine, for each value of the parameter a whether f is quasiconcave, quasiconvex, or both.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be any increasing function. Then f is both quasiconcave and quasiconvex.

5. Prove the following:

   (a) f is a (strictly) convex function if and only if $-f$ is a (strictly) concave function.

   (b) f is a (strictly) quasi-convex function if and only if $-f$ is a (strictly) quasi-concave function.

6. Suppose $f : X \longrightarrow \mathbb{R}$ attains a maximum on $X \subset \mathbb{R}^n$. Prove the following.

   (a) If f is quasi-concave, then the set of maximizers is convex.

   (b) If f is strictly quasi-concave, then the maximizer of f is unique.

7. Do all the proofs missing from today's slides.