ECON 2001

Exam 1

10 August 2012

This is a closed book exam. Please read the entire exam before starting.
You have 60 minutes to answer all FOUR questions.
Start each question on a new page please.

Question 1

Consider $a, b \in \mathbb{R}$ and prove the following: if for all $\varepsilon > 0$, $a \leq \varepsilon$, then $a \leq 0$.

Question 2

Let A and B be non-empty subsets of $\mathbb{R}$. Prove that $\sup(A \cup B) = \sup\{\sup(A), \sup(B)\}$.

HINT: One can show that $x = y$ by proving that $x \leq y$ and $x \geq y$.

Question 3

Show that the following function $\rho : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}_+$ is a metric on $\mathbf{R}^n$ (suppose that $n \geq 3$).

$$\rho(x, y) = \min\{d(x, y), 1\},$$

where $d(\cdot) : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$ is a valid metric on $\mathbf{R}^n$.

HINT: A metric is a function $d : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}_+$ that satisfies three properties. What are they?

Question 4

Given a function $u : \mathbf{R} \to \mathbf{R}$ and two real numbers w and l such that $u(w) \neq u(w - l)$, let c be defined implicitly by the following equality

$$u(c) = pu(w - l) + (1 - p)u(w).$$

where $p \in (0, 1)$. Show that if u is continuous the c defined above always exist and that if, in addition, u is strictly increasing then this c is unique (a function $f : \mathbf{R} \to \mathbf{R}$ is *strictly increasing* if $y > x \Rightarrow f(y) > f(x)$).