ECON 2001

Exam 2

17 August 2011

This is a closed book exam. Please read the entire exam before starting.
You have 60 minutes to answer all FOUR questions.
Start each question on a new page.

Question 1

Consider the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$$

1. Find A's eigenvalues.
2. Find A's eigenvectors.
3. Argue that A is diagonalizable and provide the matrices that accomplish its diagonalization.

Hint for the last part: A is diagonalizable if we can find matrices P and D, with D diagonal, such that $P^{-1}AP = D$.

Question 2

Show that if A and B are similar matrices then they have the same eingenvalues.

Question 3

Prove that any metric space X that contains a finite numbr of elements is sequentially compact.

Question 4

Let T be a linear transformation $T : V \to V$. For any integer $i > 1$, define $T^i : V \to V$ as

$$T^i = T^{i-1} \circ T$$

so that $T^2 = T \circ T = T(T(v))$, $T^3 = T^2 \circ T = T \circ T \circ T = T(T(T(v)))$ and so on. Assume that there is an $x \in V$ such that $T^n(x) = 0$ but $T^{n-1}(x) \neq 0$ for some $n > 0$. Prove that $x, T(x), T^2(x), ..., T^{n-1}(x)$ are linearly independent.
Hints: by contradiction; remember that $T(0) = 0$ for any linear transformation.