ECON 2001

Exam 2

16 August 2013

This is a closed book exam. Please read the entire exam before starting.
You have 60 minutes to answer all FOUR questions.
Start each question on a new page.

Question 1

Let X be a compact metric space and let $\{U_\lambda : \lambda \in \Lambda\}$ be an open cover of X. Show that there exists some real number $\varepsilon > 0$ such that any closed ball in X of radius ε is entirely contained in at least one set U_λ.
Hints: by contradiction (be careful how to state this); use a sequence of balls of radii $1, 1/2, 1/3, ...$ and the fact that X is compact.

Question 2

Let X and Y be two non-empty sets and $\Psi : X \to Y$ a correspondence. We say that Ψ is

$$\text{injective if } \Psi(x) \cap \Psi(x') = \emptyset \text{ for any distinct } x, x' \in X$$

$$\text{surjective if } \Psi(X) = Y$$

where the image of a set is defined by $\Psi(S) = \cup\{\Psi(x) : x \in S\}$. Prove that Ψ is injective and surjective if and only if $\Psi = f^{-1}$ for some function $f : Y \to X$.

Question 3

Prove that if A is a symmetric matrix such that $A^3 = I$ then $A = I$.

Question 4

Let T and S be a linear transformations from V to itself, where V is a finite dimensional vector space. Prove that $T \circ S$ is invertible if and only if both T and S are invertible.
HINT: remember that a linear transformation R is invertible if and only if $\ker R = \{\mathbf{0}\}$.