ECON 2001
Exam 3
22 August 2014

This is a closed book exam. Please read the entire exam before starting.
You have 60 minutes to answer all FOUR questions.
Start each question on a new page.

Question 1

Suppose $f : \mathbf{R} \to \mathbf{R}$ is differentiable in the interval (a, b); let $a < c < d < b$ and assume $f'(c) < 0 < f'(d)$ (where f' denotes the first derivative of f). Prove that the restriction of f to $[c, d]$ does not achieve a global minimum at c or d.
Hint: use the definition of derivative.

Question 2

Let $f : \mathbf{R} \times \mathbf{R}^2 \to \mathbf{R}$ be defined by

$$f(x, y, z) = x^2 + y^2 + z^2 - 4$$

and let $f(x, y, z) = 0$ be an identity that always holds.

1. Consider the point $(x, y, z) = (2, 0, 1)$. Can you use the implicit function theorem to derive z as a function of x and y around this point? If so, call the function $g(x, y)$ and find its derivative at the given point.
2. Consider the point $(x, y, z) = (0, 2, 0)$. Can you use the implicit function theorem to derive z as a function of x and y around this point? If so, call the function $g(x, y)$ and find its derivative at the given point.
3. Consider the point $(x, y, z) = (1, 1, \sqrt{2})$. Can you use the implicit function theorem to derive z as a function of x and y around this point? If so, call the function $g(x, y)$ and find its derivative at the given point.

Question 3

Let $f : \mathbf{R} \times \mathbf{R}^2 \to \mathbf{R}$ be defined by:

$$f(x; a, b) = x^a - bx$$

where $0 < a < 1$.

1. Is this function concave or convex as a function of x?
2. Find
$$x^*(a, b) \equiv \arg\max_x f(x; a, b)$$
3. Define $f^*(a, b) \equiv f(x^*(a, b), a, b)$, and find $\frac{\partial f^*(a,b)}{\partial a}$.

Question 4

Prove that $f : \mathbb{R}^n \to \mathbb{R}$ is quasiconvex if and only if

$$f(\lambda x + (1 - \lambda)y) \leq \max\{f(x), f(y)\}$$

for all $x, y \in \mathbb{R}^n$ and $\lambda \in [0, 1]$.
HINT: $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is quasiconvex if and only if the set $\{z \in \mathbb{R}^n : f(z) \leq a\}$ is convex for all $a \in \mathbb{R}$.