Uniqueness, Stability, and Gross Substitutes

Econ 2100 Fall 2018

Lecture 21, November 12

Outline

1. Uniquenness (in pictures)
2. Stability
3. Gross Substitute Property

Uniqueness and Stability

We have dealt with efficiency and existence of equilibria, but there are many unanswered questions.

1. Is a competitive equilibrium unique? If not, how many Walrasian equilibria are there?
2. Is there a procedure that leads the prices to end up in equilibrium if one starts at a situation in which demand does not equal supply? In other words, is there a natuaral price adjustment process that converges to an equilibrium?
3. Does Walrasian equilibrium impose meaningful restrictions on observable data? In particular, what can we say about how a change in endowments will change equilibrium prices?

- The first two questions have somewhat negative answers.
  - There can be a lot of Walrasian equilibria for a given specification of preferences and endowments (but typically not an infinite number).
  - There is no reason to believe that dynamic adjustment processes will converge to a Walrasian equilibrium outcome.
- The third question has a positive answer.
  - If we observe data on endowments and prices for a fixed set of agents trading at equilibrium prices, and then are asked to predict equilibrium prices and quantities for these same agents after a change in endowments, we will generally be able to say something (though maybe not all that much) about what the new equilibrium prices and quantities will be.

Uniqueness and Stability

We have dealt with efficiency and existence of equilibria, but there are many unanswered questions.

1. Is a competitive equilibrium unique? If not, how many Walrasian equilibria are there?
2. Is there a procedure that leads the prices to end up in equilibrium if one starts at a situation in which demand does not equal supply? In other words, is there a natuaral price adjustment process that converges to an equilibrium?
3. Does Walrasian equilibrium impose meaningful restrictions on observable data? In particular, what can we say about how a change in endowments will change equilibrium prices?

- The first two questions have somewhat negative answers.
  - There can be a lot of Walrasian equilibria for a given specification of preferences and endowments (but typically not an infinite number).
  - There is no reason to believe that dynamic adjustment processes will converge to a Walrasian equilibrium outcome.
- The third question has a positive answer.
  - If we observe data on endowments and prices for a fixed set of agents trading at equilibrium prices, and then are asked to predict equilibrium prices and quantities for these same agents after a change in endowments, we will generally be able to say something (though maybe not all that much) about what the new equilibrium prices and quantities will be.

Uniqueness and Stability

We have dealt with efficiency and existence of equilibria, but there are many unanswered questions.

1. Is a competitive equilibrium unique? If not, how many Walrasian equilibria are there?
2. Is there a procedure that leads the prices to end up in equilibrium if one starts at a situation in which demand does not equal supply? In other words, is there a natuaral price adjustment process that converges to an equilibrium?
3. Does Walrasian equilibrium impose meaningful restrictions on observable data? In particular, what can we say about how a change in endowments will change equilibrium prices?

- The first two questions have somewhat negative answers.
  - There can be a lot of Walrasian equilibria for a given specification of preferences and endowments (but typically not an infinite number).
  - There is no reason to believe that dynamic adjustment processes will converge to a Walrasian equilibrium outcome.
- The third question has a positive answer.

Uniqueness and Stability

We have dealt with efficiency and existence of equilibria, but there are many unanswered questions.

1. Is a competitive equilibrium unique? If not, how many Walrasian equilibria are there?
2. Is there a procedure that leads the prices to end up in equilibrium if one starts at a situation in which demand does not equal supply? In other words, is there a natuaral price adjustment process that converges to an equilibrium?
3. Does Walrasian equilibrium impose meaningful restrictions on observable data? In particular, what can we say about how a change in endowments will change equilibrium prices?

- The first two questions have somewhat negative answers.
  - There can be a lot of Walrasian equilibria for a given specification of preferences and endowments (but typically not an infinite number).
  - There is no reason to believe that dynamic adjustment processes will converge to a Walrasian equilibrium outcome.
- The third question has a positive answer.
  - If we observe data on endowments and prices for a fixed set of agents trading at equilibrium prices, and then are asked to predict equilibrium prices and quantities for these same agents after a change in endowments, we will generally be able to say something (though maybe not all that much) about what the new equilibrium prices and quantities will be.

Uniqueness and Stability

We have dealt with efficiency and existence of equilibria, but there are many unanswered questions.

1. Is a competitive equilibrium unique? If not, how many Walrasian equilibria are there?
2. Is there a procedure that leads the prices to end up in equilibrium if one starts at a situation in which demand does not equal supply? In other words, is there a natuaral price adjustment process that converges to an equilibrium?
3. Does Walrasian equilibrium impose meaningful restrictions on observable data? In particular, what can we say about how a change in endowments will change equilibrium prices?

- The first two questions have somewhat negative answers.
  - There can be a lot of Walrasian equilibria for a given specification of preferences and endowments (but typically not an infinite number).
  - There is no reason to believe that dynamic adjustment processes will converge to a Walrasian equilibrium outcome.
- The third question has a positive answer.
  - If we observe data on endowments and prices for a fixed set of agents trading at equilibrium prices, and then are asked to predict equilibrium prices and quantities for these same agents after a change in endowments, we will generally be able to say something (though maybe not all that much) about what the new equilibrium prices and quantities will be.

Uniqueness and Stability

We have dealt with efficiency and existence of equilibria, but there are many unanswered questions.

1. Is a competitive equilibrium unique? If not, how many Walrasian equilibria are there?
2. Is there a procedure that leads the prices to end up in equilibrium if one starts at a situation in which demand does not equal supply? In other words, is there a natuaral price adjustment process that converges to an equilibrium?
3. Does Walrasian equilibrium impose meaningful restrictions on observable data? In particular, what can we say about how a change in endowments will change equilibrium prices?

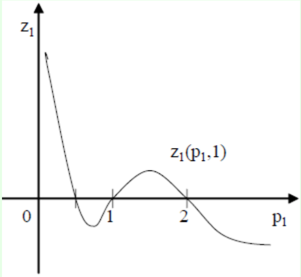
- The first two questions have somewhat negative answers.
  - There can be a lot of Walrasian equilibria for a given specification of preferences and endowments (but typically not an infinite number).
  - There is no reason to believe that dynamic adjustment processes will converge to a Walrasian equilibrium outcome.
- The third question has a positive answer.
  - If we observe data on endowments and prices for a fixed set of agents trading at equilibrium prices, and then are asked to predict equilibrium prices and quantities for these same agents after a change in endowments, we will generally be able to say something (though maybe not all that much) about what the new equilibrium prices and quantities will be.

Lack of Uniqueness

Multiple Equilibria

Two goods and two consumers: $u_1(x_{11}, x_{21}) = x_{11} - \frac{1}{8}(x_{21})^{-8}$ and $u_2(x_{12}, x_{22}) = -\frac{1}{8}(x_{12})^{-8} + x_{22}$; $\omega_1 = (2, r)$ and $\omega_2 = (r, 2)$; $r = 2^{8/9} - 2^{1/9}$.

- $z_1(p_1, 1)$:



- Small changes in preferences or endowments do not change aggregate excess demand much: multiplicity of equilibria is robust.

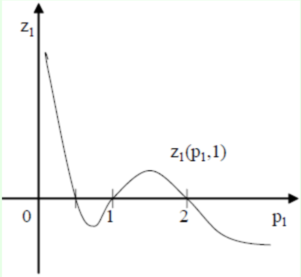
- Since $z_1(\cdot, 1)$ starts above zero and finishes below zero (why would that be?), the picture also suggests that the number of equilibria, if finite, is odd.
  - This is a general result that follows from the Index Theorem.

Lack of Uniqueness

Multiple Equilibria

Two goods and two consumers: $u_1(x_{11}, x_{21}) = x_{11} - \frac{1}{8}(x_{21})^{-8}$ and $u_2(x_{12}, x_{22}) = -\frac{1}{8}(x_{12})^{-8} + x_{22}$; $\omega_1 = (2, r)$ and $\omega_2 = (r, 2)$; $r = 2^{8/9} - 2^{1/9}$.

- $z_1(p_1, 1)$:



- Small changes in preferences or endowments do not change aggregate excess demand much: multiplicity of equilibria is robust.

- Since $z_1(\cdot, 1)$ starts above zero and finishes below zero (why would that be?), the picture also suggests that the number of equilibria, if finite, is odd.
  - This is a general result that follows from the Index Theorem.

Local Uniqueness

- Even if there are multiple Walrasian equilibria, the previous example suggests that each of these equilibria could be locally unique.
- What does that mean? Equilibria are 'isolated points'.

- Local uniqueness: there is no other Walrasian equilibrium price vector 'close' to the orginal equilibrium price vector.
- Formally, an equilibrium is not locally unique if its price vector p^* is the limit of a sequence of other equilibrium prices.
  - In the example on the previous slide, there were three Walrasian equilibria and each of them is locally unique.

Results

- In many economies, competitive equilibria are locally unique.
- Therefore, the set of Walrasian equilibria is finite.
- The formal statement says something like: the set of initial endowment such that equilibria are locally unique has measure 1.
- Equilibria are "generically" unique.

Local Uniqueness

- Even if there are multiple Walrasian equilibria, the previous example suggests that each of these equilibria could be locally unique.
- What does that mean? Equilibria are 'isolated points'.
- Local uniqueness: there is no other Walrasian equilibrium price vector 'close' to the orginal equilibrium price vector.
- Formally, an equilibrium is not locally unique if its price vector p^* is the limit of a sequence of other equilibrium prices.
  - In the example on the previous slide, there were three Walrasian equilibria and each of them is locally unique.

Results

- In many economies, competitive equilibria are locally unique.
- Therefore, the set of Walrasian equilibria is finite.
- The formal statement says something like 'the set of initial endowment such that equilibria are locally unique has measure 1'.
- Equilibria are "generically" unique.

Local Uniqueness

- Even if there are multiple Walrasian equilibria, the previous example suggests that each of these equilibria could be locally unique.
- What does that mean? Equilibria are ‘isolated points’.

- Local uniqueness: there is no other Walrasian equilibrium price vector ‘close’ to the orginal equilibrium price vector.
- Formally, an equilibrium is not locally unique if its price vector p^* is the limit of a sequence of other equilibrium prices.
    - In the example on the previous slide, there were three Walrasian equilibria and each of them is locally unique.

Results

- In many economies, competitive equilibria are locally unique.
- Therefore, the set of Walrasian equilibria is finite.
- The formal statement says something like: the set of initial endowment such that equilibria are locally unique has measure 1.
- Equilibria are “generically” unique.

Local Uniqueness

- Even if there are multiple Walrasian equilibria, the previous example suggests that each of these equilibria could be locally unique.
- What does that mean? Equilibria are ‘isolated points’.

- Local uniqueness: there is no other Walrasian equilibrium price vector ‘close’ to the orginal equilibrium price vector.
- Formally, an equilibrium is not locally unique if its price vector p^* is the limit of a sequence of other equilibrium prices.
  - In the example on the previous slide, there were three Walrasian equilibria and each of them is locally unique.

Results

- In many economies, competitive equilibria are locally unique.
- Therefore, the set of Walrasian equilibria is finite.
- The formal statement says something like: the set of initial endowment such that equilibria are locally unique has measure 1.
- Equilibria are "generically" unique.

Local Uniqueness

- Even if there are multiple Walrasian equilibria, the previous example suggests that each of these equilibria could be locally unique.
- What does that mean? Equilibria are 'isolated points'.

- Local uniqueness: there is no other Walrasian equilibrium price vector 'close' to the orginal equilibrium price vector.
- Formally, an equilibrium is not locally unique if its price vector p^* is the limit of a sequence of other equilibrium prices.
  - In the example on the previous slide, there were three Walrasian equilibria and each of them is locally unique.

Results

- In many economies, competitive equilibria are locally unique.
- Therefore, the set of Walrasian equilibria is finite.
- The formal statement says something like: the set of initial endowment such that equilibria are locally unique has measure 1.
- Equilbria are "generically" unique
  - What does generically mean?

Local Uniqueness

- Even if there are multiple Walrasian equilibria, the previous example suggests that each of these equilibria could be locally unique.
- What does that mean? Equilibria are 'isolated points'.

- Local uniqueness: there is no other Walrasian equilibrium price vector 'close' to the orginal equilibrium price vector.
- Formally, an equilibrium is not locally unique if its price vector p^* is the limit of a sequence of other equilibrium prices.
  - In the example on the previous slide, there were three Walrasian equilibria and each of them is locally unique.

Results

- In many economies, competitive equilibria are locally unique.
- Therefore, the set of Walrasian equilibria is finite.
- The formal statement says something like: the set of initial endowment such that equilibria are locally unique has measure 1.
- Equilbria are "generically" unique
  - What does generically mean?

Local Uniqueness

- Even if there are multiple Walrasian equilibria, the previous example suggests that each of these equilibria could be locally unique.
- What does that mean? Equilibria are ‘isolated points’.

- Local uniqueness: there is no other Walrasian equilibrium price vector ‘close’ to the orginal equilibrium price vector.
- Formally, an equilibrium is not locally unique if its price vector p^* is the limit of a sequence of other equilibrium prices.
  - In the example on the previous slide, there were three Walrasian equilibria and each of them is locally unique.

Results

- In many economies, competitive equilibria are locally unique.
- Therefore, the set of Walrasian equilibria is finite.
- The formal statement says something like: the set of initial endowment such that equilibria are locally unique has measure 1.
- Equilbria are “generically” unique
  - What does generically mean?

Local Uniqueness

- Even if there are multiple Walrasian equilibria, the previous example suggests that each of these equilibria could be locally unique.
- What does that mean? Equilibria are ‘isolated points’.

- Local uniqueness: there is no other Walrasian equilibrium price vector ‘close’ to the orginal equilibrium price vector.
- Formally, an equilibrium is not locally unique if its price vector p^* is the limit of a sequence of other equilibrium prices.
  - In the example on the previous slide, there were three Walrasian equilibria and each of them is locally unique.

Results

- In many economies, competitive equilibria are locally unique.
- Therefore, the set of Walrasian equilibria is finite.
- The formal statement says something like: the set of initial endowment such that equilibria are locally unique has measure 1.
- Equilbria are “generically” unique
  - What does generically mean?

Local Uniqueness

- Even if there are multiple Walrasian equilibria, the previous example suggests that each of these equilibria could be locally unique.
- What does that mean? Equilibria are ‘isolated points’.

- Local uniqueness: there is no other Walrasian equilibrium price vector ‘close’ to the orginal equilibrium price vector.
- Formally, an equilibrium is not locally unique if its price vector p^* is the limit of a sequence of other equilibrium prices.
  - In the example on the previous slide, there were three Walrasian equilibria and each of them is locally unique.

Results

- In many economies, competitive equilibria are locally unique.
- Therefore, the set of Walrasian equilibria is finite.
- The formal statement says something like: the set of initial endowment such that equilibria are locally unique has measure 1.
- Equilbria are “generically” unique
  - What does generically mean?

Local Uniqueness

- Even if there are multiple Walrasian equilibria, the previous example suggests that each of these equilibria could be locally unique.
- What does that mean? Equilibria are ‘isolated points’.

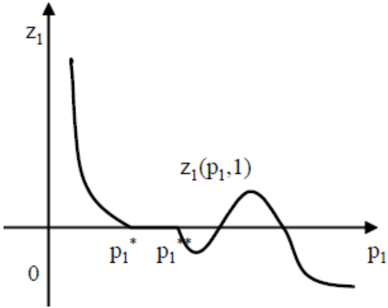
- Local uniqueness: there is no other Walrasian equilibrium price vector ‘close’ to the orginal equilibrium price vector.
- Formally, an equilibrium is not locally unique if its price vector p^* is the limit of a sequence of other equilibrium prices.
  - In the example on the previous slide, there were three Walrasian equilibria and each of them is locally unique.

Results

- In many economies, competitive equilibria are locally unique.
- Therefore, the set of Walrasian equilibria is finite.
- The formal statement says something like: the set of initial endowment such that equilibria are locally unique has measure 1.
- Equilbria are “generically” unique
  - What does generically mean?

Local Uniqueness: Example

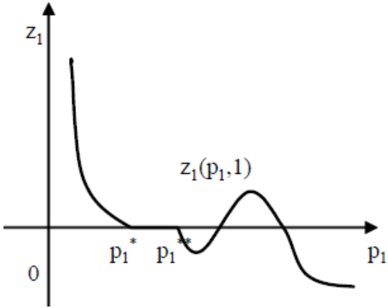
- We have local non-uniqueness if $z_1(p_1, 1)$ is equal to zero over some interval of prices $[p_1^*, p_1^{**}]$.



- The figure illustrates how this is extremely special.
  - Any small perturbation of $z_1(\cdot, 1)$, for example because of a small change in endowments, will move it a little and yield a finite number of locally unique equilibria.

Local Uniqueness: Example

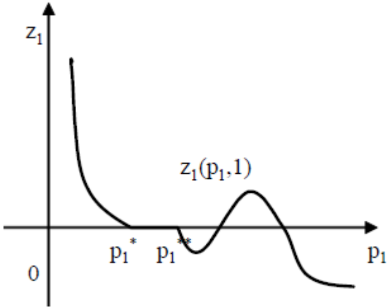
- We have local non-uniqueness if $z_1(p_1, 1)$ is equal to zero over some interval of prices $[p_1^*, p_1^{**}]$.



- The figure illustrates how this is extremely special.
  - Any small perturbation of $z_1(\cdot, 1)$, for example because of a small change in endowments, will move it a little and yield a finite number of locally unique equilibria.

Local Uniqueness: Example

- We have local non-uniqueness if $z_1(p_1, 1)$ is equal to zero over some interval of prices $[p_1^*, p_1^{**}]$.



- The figure illustrates how this is extremely special.
  - Any small perturbation of $z_1(\cdot, 1)$, for example because of a small change in endowments, will move it a little and yield a finite number of locally unique equilibria.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge 'naturally' to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called "tâtonnement" (french for "trial and error").

Tâtonnement

- Agents meet in a public square and a "Walrasian" auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge ‘naturally’ to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called “tâtonnement” (french for “trial and error”).

Tâtonnement

- Agents meet in a public square and a “Walrasian” auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge 'naturally' to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called "tâtonnement" (french for "trial and error").

Tâtonnement

- Agents meet in a public square and a "Walrasian" auctioneer posts some prices
- Afterwards, agents call out their demands at those prices
- The auctioneer sees demands and then posts new (adjusted) prices
- This process continues until prices are posted so that demand equals supply
- If that is the case, the auctioneer stops, announces prices, and trade occurs

- Clearly, this is a very abstract way to think about price convergence.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge 'naturally' to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called "tâtonnement" (french for "trial and error").

Tâtonnement

- Agents meet in a public square and a "Walrasian" auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.
  - In the stock market, trade actually takes place right after prices are announced.
  - Second, consumers might not want to announce their true demands.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge 'naturally' to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called "tâtonnement" (french for "trial and error").

Tâtonnement

- Agents meet in a public square and a "Walrasian" auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge 'naturally' to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called "tâtonnement" (french for "trial and error").

Tâtonnement

- Agents meet in a public square and a "Walrasian" auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.
  - In the stock market, trade actually takes place right after prices are announced.
  - Second, consumers might not want to announce their true demands.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge 'naturally' to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called "tâtonnement" (french for "trial and error").

Tâtonnement

- Agents meet in a public square and a "Walrasian" auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.
  - In the stock market, trade actually takes place right after prices are announced.
  - Second, consumers might not want to announce their true demands.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge ‘naturally’ to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called “tâtonnement” (french for “trial and error”).

Tâtonnement

- Agents meet in a public square and a “Walrasian” auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge 'naturally' to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called "tâtonnement" (french for "trial and error").

Tâtonnement

- Agents meet in a public square and a "Walrasian" auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge 'naturally' to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called "tâtonnement" (french for "trial and error").

Tâtonnement

- Agents meet in a public square and a "Walrasian" auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.
  - In the stock market, trade actually takes place right after prices are announced.
  - Second, consumers might not want to announce their true demands.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge 'naturally' to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called "tâtonnement" (french for "trial and error").

Tâtonnement

- Agents meet in a public square and a "Walrasian" auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.
  - In the stock market, trade actually takes place right after prices are announced.
  - Second, consumers might not want to announce their true demands.

Tâtonnement

- How do equilbrium prices come to be? Is there a way for prices to converge 'naturally' to their equilibrium value even if one starts with the wrong prices?

- As much as we would like a simple yes answer, things are much more tricky.
- Walras suggested the following price adjustment process called "tâtonnement" (french for "trial and error").

Tâtonnement

- Agents meet in a public square and a "Walrasian" auctioneer posts some prices.
- Afterwards, agents call out their demands at those prices.
- The auctioneer sees demands and then posts new (adjusted) prices.
- This process continues until prices are posted so that demand equals supply.
- If that is the case, the auctioneer stops, announces prices, and trade occurs.

- Clearly, this is a very abstract way to think about price convergence.
    - In the stock market, trade actually takes place right after prices are announced.
    - Second, consumers might not want to announce their true demands.

Tâtonnement Stability

- A possible tâtonnement price adjustment rule is:
$$p(t+1) = p(t) + \alpha z(p(t)) \text{ for small } \alpha > 0.$$
    - prices go up when excess demand is positive, and down when it is negative.

- The only stationary points of this process are price vectors p^* at which $z(p^*) = 0$, i.e. Walrasian equilibrium prices.
- An equilibrium price vector is locally stable if the price adjustment converges to it from any nearby starting prices.
    - If an equilibrium price vector is stable a small perturbation away from it moves the economy away from equilibrium only for a short time as the auctioneer finds another equilibrium quickly.
- An equilibrium price vector is globally stable if the price adjustment rule converges to it from any initial prices.
- Unfortunately, Walrasian tâtonnement can cycle without converging; Scarf (1960) provides examples where both global and local stability fail.
- The "convergence process" research agenda has been pretty much abandoned.

Tâtonnement Stability

- A possible tâtonnement price adjustment rule is:
$$p(t+1) = p(t) + \alpha z(p(t)) \text{ for small } \alpha > 0.$$
    - prices go up when excess demand is positive, and down when it is negative.
- The only stationary points of this process are price vectors p^* at which $z(p^*) = 0$, i.e. Walrasian equilibrium prices.
- An equilibrium price vector is locally stable if the price adjustment converges to it from any nearby starting prices.
    - If an equilibrium price vector is stable a small perturbation away from it moves the economy away from equilibrium only for a short time as the auctioneer finds another equilibrium quickly.
- An equilibrium price vector is globally stable if the price adjustment rule converges to it from any initial prices.
- Unfortunately, Walrasian tâtonnement can cycle without converging; Scarf (1960) provides examples where both global and local stability fail.
- The "convergence process" research agenda has been pretty much abandoned.

Tâtonnement Stability

- A possible tâtonnement price adjustment rule is:
  $$p(t+1) = p(t) + \alpha z(p(t)) \text{ for small } \alpha > 0.$$
  - prices go up when excess demand is positive, and down when it is negative.
- The only stationary points of this process are price vectors p^* at which $z(p^*) = 0$, i.e. Walrasian equilibrium prices.
- An equilibrium price vector is locally stable if the price adjustment converges to it from any nearby starting prices.
  - If an equilibrium price vector is stable a small perturbation away from it moves the economy away from equilibrium only for a short time as the auctioneer finds another equilibrium quickly.
- An equilibrium price vector is globally stable if the price adjustment rule converges to it from any initial prices.
- Unfortunately, Walrasian tâtonnement can cycle without converging; Scarf (1960) provides examples where both global and local stability fail.
- The "convergence process" research agenda has been pretty much abandoned.

Tâtonnement Stability

- A possible tâtonnement price adjustment rule is:

$$p(t+1) = p(t) + \alpha z(p(t)) \text{ for small } \alpha > 0.$$

  - prices go up when excess demand is positive, and down when it is negative.

- The only stationary points of this process are price vectors p^* at which $z(p^*) = 0$, i.e. Walrasian equilibrium prices.
- An equilibrium price vector is locally stable if the price adjustment converges to it from any nearby starting prices.
  - If an equilibrium price vector is stable a small perturbation away from it moves the economy away from equilibrium only for a short time as the auctioneer finds another equilibrium quickly.
- An equilibrium price vector is globally stable if the price adjustment rule converges to it from any initial prices.
- Unfortunately, Walrasian tâtonnement can cycle without converging; Scarf (1960) provides examples where both global and local stability fail.
- The "convergence process" research agenda has been pretty much abandoned.

Tâtonnement Stability

- A possible tâtonnement price adjustment rule is:
$$p(t+1) = p(t) + \alpha z(p(t)) \text{ for small } \alpha > 0.$$
  - prices go up when excess demand is positive, and down when it is negative.

- The only stationary points of this process are price vectors p^* at which $z(p^*) = 0$, i.e. Walrasian equilibrium prices.
- An equilibrium price vector is locally stable if the price adjustment converges to it from any nearby starting prices.
  - If an equilibrium price vector is stable a small perturbation away from it moves the economy away from equilibrium only for a short time as the auctioneer finds another equilibrium quickly.
- An equilibrium price vector is globally stable if the price adjustment rule converges to it from any initial prices.
- Unfortunately, Walrasian tâtonnement can cycle without converging; Scarf (1960) provides examples where both global and local stability fail.
- The "convergence process" research agenda has been pretty much abandoned.

Tâtonnement Stability

- A possible tâtonnement price adjustment rule is:

$$p(t+1) = p(t) + \alpha z(p(t)) \text{ for small } \alpha > 0.$$

  - prices go up when excess demand is positive, and down when it is negative.

- The only stationary points of this process are price vectors p^* at which $z(p^*) = 0$, i.e. Walrasian equilibrium prices.
- An equilibrium price vector is locally stable if the price adjustment converges to it from any nearby starting prices.
  - If an equilibrium price vector is stable a small perturbation away from it moves the economy away from equilibrium only for a short time as the auctioneer finds another equilibrium quickly.
- An equilibrium price vector is globally stable if the price adjustment rule converges to it from any initial prices.
- Unfortunately, Walrasian tâtonnement can cycle without converging; Scarf (1960) provides examples where both global and local stability fail.
- The "convergence process" research agenda has been pretty much abandoned.

Tâtonnement Stability

- A possible tâtonnement price adjustment rule is:

$$p(t+1) = p(t) + \alpha z(p(t)) \text{ for small } \alpha > 0.$$

  - prices go up when excess demand is positive, and down when it is negative.

- The only stationary points of this process are price vectors p^* at which $z(p^*) = 0$, i.e. Walrasian equilibrium prices.
- An equilibrium price vector is locally stable if the price adjustment converges to it from any nearby starting prices.
  - If an equilibrium price vector is stable a small perturbation away from it moves the economy away from equilibrium only for a short time as the auctioneer finds another equilibrium quickly.
- An equilibrium price vector is globally stable if the price adjustment rule converges to it from any initial prices.
- Unfortunately, Walrasian tâtonnement can cycle without converging; Scarf (1960) provides examples where both global and local stability fail.
- The “convergence process” research agenda has been pretty much abandoned.

Sonnenschein-Debreu-Mantel Theorem

- We have proved that (in an exchange economy) aggregate excess demand is continuous, homogeneous of dregree zero, satisfies Walras' law, and has certain boundary properties (excess demand goes to infinity as prices go to zero).
- Are there any other restrictions on aggregate excess demand that can be derived from utility maximization?
- Not really.

Theorem

Suppose we have an open and bounded subset $B \subset \mathbb{R}^L_{++}$ and a continuous function $f : B \to \mathbb{R}^L_{++}$ satisfying homogeneity of degree zero and Walras' Law. Then, there exists an economy with aggregate excess demand function $z(p)$ satisfying $f(p) = z(p)$ on B.

- How to interpret this result? The traditional interpretation is that anything goes: there are no extra properties of aggregate excess demand that can be derived from individual's optimal behavior.
- This implies aggregate excess demand need not satisfy properties that hold for individual excess demand (think revealed preferences).
- An implication is that utility maximization imposes no extra conditions on equilibrium prices.

Sonnenschein-Debreu-Mantel Theorem

- We have proved that (in an exchange economy) aggregate excess demand is continuous, homogeneous of dregree zero, satisfies Walras' law, and has certain boundary properties (excess demand goes to infinity as prices go to zero).
- Are there any other restrictions on aggregate excess demand that can be derived from utility maximization?
- Not really.

Theorem

Suppose we have an open and bounded subset $B \subset \mathbb{R}^L_{++}$ and a continuous function $f : B \rightarrow \mathbb{R}^L_{++}$ satisfying homogeneity of degree zero and Walras' Law. Then, there exists an economy with aggregate excess demand function $z(p)$ satisfying $f(p) = z(p)$ on B.

- How to interpret this result? The traditional interpretation is that anything goes: there are no extra properties of aggregate excess demand that can be derived from individual's optimal behavior.
- This implies aggregate excess demand need not satisfy properties that hold for individual excess demand (think revealed preferences).
- An implication is that utility maximization imposes no extra conditions on equilibrium prices.

Sonnenschein-Debreu-Mantel Theorem

- We have proved that (in an exchange economy) aggregate excess demand is continuous, homogeneous of dregree zero, satisfies Walras' law, and has certain boundary properties (excess demand goes to infinity as prices go to zero).
- Are there any other restrictions on aggregate excess demand that can be derived from utility maximization?
- Not really.

Theorem

Suppose we have an open and bounded subset $B \subset \mathbb{R}^L_{++}$ and a continuous function $f : B \rightarrow \mathbb{R}^L_{++}$ satisfying homogeneity of degree zero and Walras' Law. Then, there exists an economy with aggregate excess demand function $z(p)$ satisfying $f(p) = z(p)$ on B.

- How to interpret this result? The traditional interpretation is that anything goes: there are no extra properties of aggregate excess demand that can be derived from individual's optimal behavior.
- This implies aggregate excess demand need not satisfy properties that hold for individual excess demand (think revealed preferences).
- An implication is that utility maximization imposes no extra conditions on equilibrium prices.

Sonnenschein-Debreu-Mantel Theorem

- We have proved that (in an exchange economy) aggregate excess demand is continuous, homogeneous of dregree zero, satisfies Walras' law, and has certain boundary properties (excess demand goes to infinity as prices go to zero).
- Are there any other restrictions on aggregate excess demand that can be derived from utility maximization?
- Not really.

Theorem

Suppose we have an open and bounded subset $B \subset \mathbb{R}^L_{++}$ and a continuous function $f : B \to \mathbb{R}^L_{++}$ satisfying homogeneity of degree zero and Walras' Law. Then, there exists an economy with aggregate excess demand function $z(p)$ satisfying $f(p) = z(p)$ on B.

- How to interpret this result? The traditional interpretation is that anything goes: there are no extra properties of aggregate excess demand that can be derived from individual's optimal behavior.
- This implies aggregate excess demand need not satisfy properties that hold for individual excess demand (think revealed preferences).
- An implication is that utility maximization imposes no extra conditions on equilibrium prices.

Sonnenschein-Debreu-Mantel Theorem

- We have proved that (in an exchange economy) aggregate excess demand is continuous, homogeneous of dregree zero, satisfies Walras' law, and has certain boundary properties (excess demand goes to infinity as prices go to zero).
- Are there any other restrictions on aggregate excess demand that can be derived from utility maximization?
- Not really.

Theorem

Suppose we have an open and bounded subset $B \subset \mathbb{R}^L_{++}$ and a continuous function $f : B \rightarrow \mathbb{R}^L_{++}$ satisfying homogeneity of degree zero and Walras' Law. Then, there exists an economy with aggregate excess demand function $z(p)$ satisfying $f(p) = z(p)$ on B.

- How to interpret this result? The traditional interpretation is that anything goes: there are no extra properties of aggregate excess demand that can be derived from individual's optimal behavior.
- This implies aggregate excess demand need not satisfy properties that hold for individual excess demand (think revealed preferences).
- An implication is that utility maximization imposes no extra conditions on equilibrium prices.

Brown-Matzkin Theorem

- Suppose one observes prices and individual endowments corresponding to the equilibrium of an economy.
- Now change individual endowments, but not preferences.
- Can any new equilibrium price vector obtain?

Theorem

There exist prices and endowments $(p, \{\omega_i\}_{i=1}^N)$ and $(\hat{p}, \{\hat{\omega}_i\}_{i=1}^N)$ such that there are no strictly monotone preferences $\{\succsim_i\}_{i=1}^N$ with the property that p is a Walrasian equilibrium price vector for the economy characterized by $\{\omega_i, \succsim_i\}_{i=1}^N$ and $\hat{p}$ is a Walrasian equilibrium price vector for the economy characterized by $\{\hat{\omega}_i \succsim_i\}_{i=1}^N$.

- This says that not all price vectors are possible in equilibrium, if preferences are strictly monotone.
- The result can also be connected to Afriat's theorem and the weak axiom.
- The following 'proof' is adapted from Jon Levin's notes.

Brown-Matzkin Theorem

- Suppose one observes prices and individual endowments corresponding to the equilibrium of an economy.
- Now change individual endowments, but not preferences.
- Can any new equilibrium price vector obtain?

Theorem

There exist prices and endowments $(p, \{\omega_i\}_{i=1}^N)$ and $(\hat{p}, \{\hat{\omega}_i\}_{i=1}^N)$ such that there are no strictly monotone preferences $\{\succsim_i\}_{i=1}^N$ with the property that p is a Walrasian equilibrium price vector for the economy characterized by $\{\omega_i, \succsim_i\}_{i=1}^N$ and $\hat{p}$ is a Walrasian equilibrium price vector for the economy characterized by $\{\hat{\omega}_i \succsim_i\}_{i=1}^N$.

- This says that not all price vectors are possible in equilibrium, if preferences are strictly monotone.
- The result can also be connected to Afriat's theorem and the weak axiom.
- The following 'proof' is adapted from Jon Levin's notes.

Brown-Matzkin Theorem

- Suppose one observes prices and individual endowments corresponding to the equilibrium of an economy.
- Now change individual endowments, but not preferences.
- Can any new equilibrium price vector obtain?

Theorem

There exist prices and endowments $(p, \{\omega_i\}_{i=1}^N)$ and $(\hat{p}, \{\hat{\omega}_i\}_{i=1}^N)$ such that there are no strictly monotone preferences $\{\succsim_i\}_{i=1}^N$ with the property that p is a Walrasian equilibrium price vector for the economy characterized by $\{\omega_i, \succsim_i\}_{i=1}^N$ and $\hat{p}$ is a Walrasian equilibrium price vector for the economy characterized by $\{\hat{\omega}_i \succsim_i\}_{i=1}^N$.

- This says that not all price vectors are possible in equilibrium, if preferences are strictly monotone.
- The result can also be connected to Afriat's theorem and the weak axiom.
- The following 'proof' is adapted from Jon Levin's notes.

Brown-Matzkin Theorem

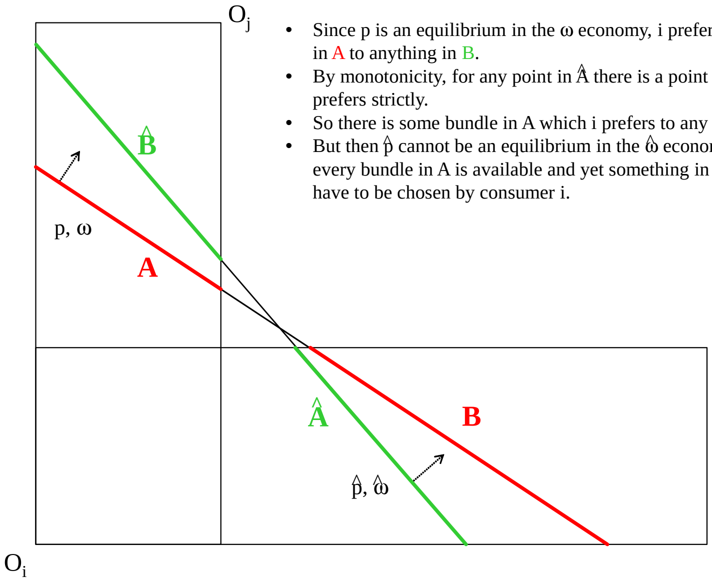
- Suppose one observes prices and individual endowments corresponding to the equilibrium of an economy.
- Now change individual endowments, but not preferences.
- Can any new equilibrium price vector obtain?

Theorem

There exist prices and endowments $(p, \{\omega_i\}_{i=1}^N)$ *and* $(\hat{p}, \{\hat{\omega}_i\}_{i=1}^N)$ *such that there are no strictly monotone preferences* $\{\succsim_i\}_{i=1}^N$ *with the property that* p *is a Walrasian equilibrium price vector for the economy characterized by* $\{\omega_i, \succsim_i\}_{i=1}^N$ *and* $\hat{p}$ *is a Walrasian equilibrium price vector for the economy characterized by* $\{\hat{\omega}_i \succsim_i\}_{i=1}^N$.

- This says that not all price vectors are possible in equilibrium, if preferences are strictly monotone.
- The result can also be connected to Afriat's theorem and the weak axiom.
- The following 'proof' is adapted from Jon Levin's notes.

Brown-Matzkin Theorem



- Since p is an equilibrium in the ω economy, i prefers something in A to anything in B.
- By monotonicity, for any point in $\hat{A}$ there is a point in B that i prefers strictly.
- So there is some bundle in A which i prefers to any bundle in $\hat{A}$.
- But then $\hat{p}$ cannot be an equilibrium in the $\hat{\omega}$ economy because every bundle in A is available and yet something in $\hat{A}$ would have to be chosen by consumer i.

Gross Substitutes

- We now consider a particular class of economies in which we can get affirmative answers to the uniqueness and stability questions.
- Two goods are gross substitutes if an increase in the price of one good increases the demand for the other.
- A demand function satisfies the gross substitutes property if an increase in the price of one good increases the demand for all other goods.

Definition

A Walrasian demand function $x^*(p)$ satisfies the gross substitutes property if, whenever p and p' are such that $p'_k > p_k$ and $p'_l = p_l$ for all $l \neq k$, then $x^*_l(p') > x^*_l(p)$ for all $l \neq k$.

Remark

If Walrasian demand satisfyies gross substitutes for all consumers, then individual and aggregate excess demand functions also satisfy the gross substitutes property.

Proof.

Homework □

Gross Substitutes

- We now consider a particular class of economies in which we can get affirmative answers to the uniqueness and stability questions.

- Two goods are gross substitutes if an increase in the price of one good increases the demand for the other.
- A demand function satisfies the gross substitutes property if an increase in the price of one good increases the demand for all other goods.

Definition

A Walrasian demand function $x^*(p)$ satisfies the gross substitutes property if, whenever p and p' are such that $p'_k > p_k$ and $p'_l = p_l$ for all $l \neq k$, then $x^*_l(p') > x^*_l(p)$ for all $l \neq k$.

Remark

If Walrasian demand satisfyies gross substitutes for all consumers, then individual and aggregate excess demand functions also satisfy the gross substitutes property.

Proof.

Homework □

Gross Substitutes

- We now consider a particular class of economies in which we can get affirmative answers to the uniqueness and stability questions.

- Two goods are gross substitutes if an increase in the price of one good increases the demand for the other.
- A demand function satisfies the gross substitutes property if an increase in the price of one good increases the demand for all other goods.

Definition

A Walrasian demand function $x^*(p)$ satisfies the gross substitutes property if, whenever p and p' are such that $p'_k > p_k$ and $p'_l = p_l$ for all $l \neq k$, then $x^*_l(p') > x^*_l(p)$ for all $l \neq k$.

Remark

If Walrasian demand satisfyies gross substitutes for all consumers, then individual and aggregate excess demand functions also satisfy the gross substitutes property.

Proof.

Homework ∎

Gross Substitutes

- We now consider a particular class of economies in which we can get affirmative answers to the uniqueness and stability questions.
- Two goods are gross substitutes if an increase in the price of one good increases the demand for the other.
- A demand function satisfies the gross substitutes property if an increase in the price of one good increases the demand for all other goods.

Definition

A Walrasian demand function $x^*(p)$ satisfies the gross substitutes property if, whenever p and p' are such that $p'_k > p_k$ and $p'_l = p_l$ for all $l \neq k$, then $x^*_l(p') > x^*_l(p)$ for all $l \neq k$.

Remark

If Walrasian demand satisfyies gross substitutes for all consumers, then individual and aggregate excess demand functions also satisfy the gross substitutes property.

Proof.

Homework □

Gross Substitutes

- We now consider a particular class of economies in which we can get affirmative answers to the uniqueness and stability questions.

- Two goods are gross substitutes if an increase in the price of one good increases the demand for the other.
- A demand function satisfies the gross substitutes property if an increase in the price of one good increases the demand for all other goods.

Definition

A Walrasian demand function $x^*(p)$ satisfies the gross substitutes property if, whenever p and p' are such that $p'_k > p_k$ and $p'_l = p_l$ for all $l \neq k$, then $x^*_l(p') > x^*_l(p)$ for all $l \neq k$.

Remark

If Walrasian demand satisfyies gross substitutes for all consumers, then individual and aggregate excess demand functions also satisfy the gross substitutes property.

Proof.

Homework □

Gross Substitutes and Uniqueness

Theorem

If the aggregate excess demand function satisfies gross substitutes, the economy has at most one Walrasian equilibrium

Proof.

We need to show that $z(p) = 0$ has at most one (normalized) solution.

- By contradiction: p and p' are not linearly dependent and $z(p) = z(p') = 0$.
- By homogeneity of degree zero, normalize the price vectors so that $p_l \geq p'_l$ for all $l = 1, .., L$ and $p_k = p'_k$ for some k.[a]
- Move from p' to p in $n-1$ steps, increasing the prices of each good $l \neq k$ in turn.
- At each step where a dimension of price increases strictly (and there must be at least one such step), the aggregate demand for good k must strictly increase, so that $z_k(p) > z_k(p') = 0$, yielding a contradiction.

$\square$

[a] Set $\lambda_l \equiv \frac{p'_l}{p_l}$, let $\tilde{l} \equiv \arg\max_l \lambda_l$, and then define $\tilde{p}_l \equiv \lambda_{\tilde{l}} p$. Then $\tilde{p}_{\tilde{l}} = p'_{\tilde{l}}$ and $\tilde{p}_l = \lambda_{\tilde{l}} p_l \geq \lambda_l p_l = p'_l$.

Gross Substitutes and Uniqueness

Theorem

If the aggregate excess demand function satisfies gross substitutes, the economy has at most one Walrasian equilibrium

Proof.

We need to show that $z(p) = 0$ has at most one (normalized) solution.

- By contradiction: p and p' are not linearly dependent and $z(p) = z(p') = 0$.
- By homogeneity of degree zero, normalize the price vectors so that $p_l \geq p'_l$ for all $l = 1, .., L$ and $p_k = p'_k$ for some k.[a]
- Move from p' to p in $n - 1$ steps, increasing the prices of each good $l \neq k$ in turn.
- At each step where a dimension of price increases strictly (and there must be at least one such step), the aggregate demand for good k must strictly increase, so that $z_k(p) > z_k(p') = 0$, yielding a contradiction.

□

[a] Set $\lambda_l \equiv \frac{p'_l}{p_l}$, let $\tilde{l} \equiv \arg\max_l \lambda_l$, and then define $\tilde{p}_l \equiv \lambda_{\tilde{l}} p$. Then $\tilde{p}_{\tilde{l}} = p'_{\tilde{l}}$ and $\tilde{p}_l = \lambda_{\tilde{l}} p_l \geq \lambda_l p_l = p'_l$.

Gross Substitutes and Uniqueness

Theorem

If the aggregate excess demand function satisfies gross substitutes, the economy has at most one Walrasian equilibrium

Proof.

We need to show that $z(p) = 0$ has at most one (normalized) solution.

- By contradiction: p and p' are not linearly dependent and $z(p) = z(p') = 0$.
- By homogeneity of degree zero, normalize the price vectors so that $p_l \geq p'_l$ for all $l = 1, .., L$ and $p_k = p'_k$ for some k.[a]
- Move from p' to p in $n-1$ steps, increasing the prices of each good $l \neq k$ in turn.
- At each step where a dimension of price increases strictly (and there must be at least one such step), the aggregate demand for good k must strictly increase, so that $z_k(p) > z_k(p') = 0$, yielding a contradiction.

□

[a] Set $\lambda_l \equiv \frac{p'_l}{p_l}$, let $\tilde{l} \equiv \arg\max_l \lambda_l$, and then define $\tilde{p}_l \equiv \lambda_{\tilde{l}} p$. Then $\tilde{p}_{\tilde{l}} = p'_{\tilde{l}}$ and $\tilde{p}_l = \lambda_{\tilde{l}} p_l \geq \lambda_l p_l = p'_l$.

Gross Substitutes and Uniqueness

Theorem

If the aggregate excess demand function satisfies gross substitutes, the economy has at most one Walrasian equilibrium

Proof.

We need to show that $z(p) = 0$ has at most one (normalized) solution.

- By contradiction: p and p' are not linearly dependent and $z(p) = z(p') = 0$.
- By homogeneity of degree zero, normalize the price vectors so that $p_l \geq p'_l$ for all $l = 1, .., L$ and $p_k = p'_k$ for some k.[a]
- Move from p' to p in $n-1$ steps, increasing the prices of each good $l \neq k$ in turn.
- At each step where a dimension of price increases strictly (and there must be at least one such step), the aggregate demand for good k must strictly increase, so that $z_k(p) > z_k(p') = 0$, yielding a contradiction.

□

[a] Set $\lambda_l \equiv \frac{p'_l}{p_l}$, let $\tilde{l} \equiv \arg\max_l \lambda_l$, and then define $\tilde{p}_l \equiv \lambda_{\tilde{l}} p$. Then $\tilde{p}_{\tilde{l}} = p'_{\tilde{l}}$ and $\tilde{p}_l = \lambda_{\tilde{l}} p_l \geq \lambda_l p_l = p'_l$.

Gross Substitutes and Uniqueness

Theorem

If the aggregate excess demand function satisfies gross substitutes, the economy has at most one Walrasian equilibrium

Proof.

We need to show that $z(p) = 0$ has at most one (normalized) solution.

- By contradiction: p and p' are not linearly dependent and $z(p) = z(p') = 0$.
- By homogeneity of degree zero, normalize the price vectors so that $p_l \geq p'_l$ for all $l = 1, .., L$ and $p_k = p'_k$ for some k.[a]
- Move from p' to p in $n - 1$ steps, increasing the prices of each good $l \neq k$ in turn.
- At each step where a dimension of price increases strictly (and there must be at least one such step), the aggregate demand for good k must strictly increase, so that $z_k(p) > z_k(p') = 0$, yielding a contradiction.

$\square$

[a] Set $\lambda_l \equiv \frac{p'_l}{p_l}$, let $\tilde{l} \equiv \arg\max_l \lambda_l$, and then define $\tilde{p}_l \equiv \lambda_{\tilde{l}} p$. Then $\tilde{p}_{\tilde{l}} = p'_{\tilde{l}}$ and $\tilde{p}_l = \lambda_{\tilde{l}} p_l \geq \lambda_l p_l = p'_l$.

Gross Substitutes and Stability

- When the gross substitutes property holds, Walrasian tatonnement converges to the unique equilibrium.

Proposition

Suppose that the aggregate excess demand function $z(p)$ satisfies gross substitutes and that $z(p^*) = 0$. Then for any p not collinear with p^*, $p^* \cdot z(p) > 0$.

Proof.

Homework (assume $L = 2$ for simplicity). □

- This Lemma is related to the weak axiom of revealed preference. Gross substitutes implies that the weak axiom holds if one compares p^*, the unique equilibrium price vector, to any other price vector p.

Theorem

Suppose that the aggregate excess demand function $z(p)$ satisfies gross substitutes, and that p^* is the Walrasian equilibrium price vector. Then the tatonnement adjustment process with $\frac{dp}{dy} = \alpha z(p(t))$, with $\alpha > 0$, converges to p^* as $t \to \infty$ for any initial condition $p(0)$.

Gross Substitutes and Stability

- When the gross substitutes property holds, Walrasian tatonnement converges to the unique equilibrium.

Proposition

Suppose that the aggregate excess demand function $z(p)$ satisfies gross substitutes and that $z(p^*) = 0$. Then for any p not collinear with p^*, $p^* \cdot z(p) > 0$.

Proof.

Homework (assume $L = 2$ for simplicity). □

- This Lemma is related to the weak axiom of revealed preference. Gross substitutes implies that the weak axiom holds if one compares p^*, the unique equilibrium price vector, to any other price vector p.

Theorem

Suppose that the aggregate excess demand function $z(p)$ satisfies gross substitutes, and that p^* is the Walrasian equilibrium price vector. Then the tatonnement adjustment process with $\frac{dp}{dy} = \alpha z(p(t))$, with $\alpha > 0$, converges to p^* as $t \to \infty$ for any initial condition $p(0)$.

Gross Substitutes and Stability

- When the gross substitutes property holds, Walrasian tatonnement converges to the unique equilibrium.

Proposition

Suppose that the aggregate excess demand function $z(p)$ satisfies gross substitutes and that $z(p^*) = 0$. Then for any p not collinear with p^*, $p^* \cdot z(p) > 0$.

Proof.

Homework (assume $L = 2$ for simplicity). □

- This Lemma is related to the weak axiom of revealed preference. Gross substitutes implies that the weak axiom holds if one compares p^*, the unique equilibrium price vector, to any other price vector p.

Theorem

Suppose that the aggregate excess demand function $z(p)$ satisfies gross substitutes, and that p^* is the Walrasian equilibrium price vector. Then the tatonnement adjustment process with $\frac{dp}{dy} = \alpha z(p(t))$, with $\alpha > 0$, converges to p^* as $t \to \infty$ for any initial condition $p(0)$.

Proof.

We show that the distance between $p(t)$ and p^* (denoted $D(p)$) decreases monotonically with t.

$$D(p) = \frac{1}{2} \sum_{l=1}^{L} (p_l - p_l^*)^2$$

- Then

$$\begin{aligned} \frac{dD(p(t))}{dt} &= \sum_{l=1}^{L} (p_l - p_l^*) \frac{dp_l}{dt} = \alpha \sum_{l=1}^{L} (p_l - p_l^*) z_l(p(t)) \\ &\underbrace{=}_{\text{by Walras' Law}} -\alpha p^* \cdot z(p) \leq 0 \end{aligned}$$

  - The last inequality is strict unless p is proportional to p^*.

- Since $D(p(t))$ is decreasing monotonically over time then $\frac{dp}{dy}$ must converge, either to zero or to some positive number.
- In the former case, $p(t) \to p^*$, and we are done.
- In the latter case, $p(t)$ becomes nearly proportional to p^* as $t \to \infty$; this means the relative prices of $p(t)$ converge to those of p^*. □

Proof.

We show that the distance between $p(t)$ and p^* (denoted $D(p)$) decreases monotonically with t.

$$D(p) = \frac{1}{2} \sum_{l=1}^{L} (p_l - p_l^*)^2$$

- Then

$$\begin{aligned} \frac{dD(p(t))}{dt} &= \sum_{l=1}^{L} (p_l - p_l^*) \frac{dp_l}{dt} = \alpha \sum_{l=1}^{L} (p_l - p_l^*) z_l(p(t)) \\ &\underbrace{=}_{\text{by Walras' Law}} -\alpha p^* \cdot z(p) \leq 0 \end{aligned}$$

  - The last inequality is strict unless p is proportional to p^*.
- Since $D(p(t))$ is decreasing monotonically over time then $\frac{dp}{dy}$ must converge, either to zero or to some positive number.
- In the former case, $p(t) \to p^*$, and we are done.
- In the latter case, $p(t)$ becomes nearly proportional to p^* as $t \to \infty$; this means the relative prices of $p(t)$ converge to those of p^*. □

Proof.

We show that the distance between $p(t)$ and p^* (denoted $D(p)$) decreases monotonically with t.

$$D(p) = \frac{1}{2}\sum_{l=1}^{L}(p_l - p_l^*)^2$$

- Then

$$\frac{dD(p(t))}{dt} = \sum_{l=1}^{L}(p_l - p_l^*)\frac{dp_l}{dt} = \alpha\sum_{l=1}^{L}(p_l - p_l^*)\, z_l(p(t))$$
$$\underbrace{=}_{\text{by Walras' Law}} -\alpha p^* \cdot z(p) \leq 0$$

  - The last inequality is strict unless p is proportional to p^*.

- Since $D(p(t))$ is decreasing monotonically over time then $\frac{dp}{dy}$ must converge, either to zero or to some positive number.
- In the former case, $p(t) \to p^*$, and we are done.
- In the latter case, $p(t)$ becomes nearly proportional to p^* as $t \to \infty$; this means the relative prices of $p(t)$ converge to those of p^*. □

Proof.

We show that the distance between $p(t)$ and p^* (denoted $D(p)$) decreases monotonically with t.

$$D(p) = \frac{1}{2} \sum_{l=1}^{L} (p_l - p_l^*)^2$$

- Then

$$\begin{aligned} \frac{dD(p(t))}{dt} &= \sum_{l=1}^{L} (p_l - p_l^*) \frac{dp_l}{dt} = \alpha \sum_{l=1}^{L} (p_l - p_l^*) z_l(p(t)) \\ &\underbrace{=}_{\text{by Walras' Law}} -\alpha p^* \cdot z(p) \leq 0 \end{aligned}$$

    - The last inequality is strict unless p is proportional to p^*.

- Since $D(p(t))$ is decreasing monotonically over time then $\frac{dp}{dy}$ must converge, either to zero or to some positive number.
- In the former case, $p(t) \to p^*$, and we are done.
- In the latter case, $p(t)$ becomes nearly proportional to p^* as $t \to \infty$; this means the relative prices of $p(t)$ converge to those of p^*.

Proof.

We show that the distance between $p(t)$ and p^* (denoted $D(p)$) decreases monotonically with t.

$$D(p) = \frac{1}{2}\sum_{l=1}^{L}\left(p_l - p_l^*\right)^2$$

- Then

$$\begin{aligned} \frac{dD(p(t))}{dt} &= \sum_{l=1}^{L}\left(p_l - p_l^*\right)\frac{dp_l}{dt} = \alpha\sum_{l=1}^{L}\left(p_l - p_l^*\right) z_l(p(t)) \\ &\underbrace{=}_{\text{by Walras' Law}} -\alpha p^* \cdot z(p) \leq 0 \end{aligned}$$

  - The last inequality is strict unless p is proportional to p^*.

- Since $D(p(t))$ is decreasing monotonically over time then $\frac{dp}{dy}$ must converge, either to zero or to some positive number.
- In the former case, $p(t) \longrightarrow p^*$, and we are done.
- In the latter case, $p(t)$ becomes nearly proportional to p^* as $t \longrightarrow \infty$; this means the relative prices of $p(t)$ converge to those of p^*.

Proof.

We show that the distance between $p(t)$ and p^* (denoted $D(p)$) decreases monotonically with t.

$$D(p) = \frac{1}{2}\sum_{l=1}^{L}(p_l - p_l^*)^2$$

- Then

$$\begin{aligned}\frac{dD(p(t))}{dt} &= \sum_{l=1}^{L}(p_l - p_l^*)\frac{dp_l}{dt} = \alpha\sum_{l=1}^{L}(p_l - p_l^*)\,z_l(p(t)) \\ &\underbrace{=}_{\text{by Walras' Law}} -\alpha p^*\cdot z(p) \le 0\end{aligned}$$

  - The last inequality is strict unless p is proportional to p^*.

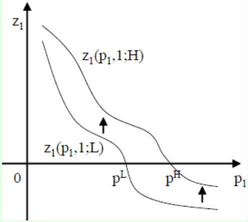
- Since $D(p(t))$ is decreasing monotonically over time then $\frac{dp}{dy}$ must converge, either to zero or to some positive number.
- In the former case, $p(t) \longrightarrow p^*$, and we are done.
- In the latter case, $p(t)$ becomes nearly proportional to p^* as $t \rightarrow \infty$; this means the relative prices of $p(t)$ converge to those of p^*. □

Gross Substitutes and Comparative Statics

A change that increases good k's excess demand increases its equilibrium price.

Two Goods Comparative Statics

- Set $p_2 = 1$, and assume good 1 is a normal good for all consumers.
- Increase the endowment of good 2. For any p_1, this increases aggregate demand for good 1 (why?), and threfore increases aggregate excess demand.
- The original curve is $z_1(p_1, 1; L)$, and the new one is $z_1(p_1, 1; H)$.



- Because $z_1(p_1, 1; L)$ is continuous and crosses zero only once (remember that equilibrium is unique), the new equilibrium must have a higher price for good 1.

- This example is easy to formalize, and generalizes to more goods.

Next Class

- Uncertainty and time.