Arrow-Debreu Equilibrium

Econ 2100 Fall 2018

Lecture 22, November 14

Outline

1. Time and Uncertainty: dated events
2. Uncertainty in General Equilibrium: Arrow-Debreu Economies
   1. State-Contingent Commodities
   2. Preferences
   3. Production Sets
   4. Budget Sets
3. Arrow-Debreu Equilibrium
   1. Examples

General Equilibrium Under Uncertainty

Main Objective

Extend general equilibrium theory to account for time and uncertainty.

- We use the structure of the expected utility model, while time is added by thinking about a tree.
- Trade, at least for now, takes place at the very beginning, before anybody learns anything about the evolution of uncertainty.
- Individuals and firms trade state-contingent commodities; those are promises to deliver or receive different amounts of the goods as time and uncertainty evolve.

- Uncertainty is described by a finite set of states S with s as generic element.
- An observable subset of S is called an event. It is a collection of states.
  - S and $\varnothing$ are events by definition.
  - If A and B are events, then $A \cup B$, $A \cap B$, and $S \setminus A = \{s \in S : s \notin A\}$ are also events.
- If A is an event, $p(A) \in [0,1]$ is the probability of A.
- $p(A \cup B) = p(A) + p(B)$ when $A \cap B = \varnothing$.
  - The probability of an event is the sum of the probabilities of the individual states in the event, provided these states are themselves events.

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Example: Flipping a Coin

- Toss a coint twice:

$$S = \{(H,H),(H,T),(T,H),(T,T)\}$$

- The set of of all possible events is the set of all possible subsets of S.
- Suppose one observes the outcome of each toss over time; draw the corresponding tree
- The nodes in the figure correspond to what are called dated events.
  - The dated event $(0, \{(H,H),(H,T),(T,H),(T,T)\})$ is the situation at time 0 before anything has happened
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- One assignes probabilities to the states at time 2, and then use them to figure out the probabilities of all events.
  - If the coin is fair, then each state us equally likely, and we can attach numbers to the probability of each state and each event

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Example: Flipping a Coin with Partial Information

- Toss a coint twice:

$$S = \{(H, H), (H, T), (T, H), (T, T)\}$$

- The set of of all possible events is the set of all possible subsets of S.

- Suppose that at time 2 one only observes the number of H; these are the events

$$\{(T, T)\}, \ \{(T, H), (H, T)\}, \text{ and } \{(H, H)\}$$

- The information is different than before, and so shouyld be the tree.
  - The dated event $(0, \{(H, H), (H, T), (T, H), (T, T)\})$ is the situation at time 0 before anything has happened.
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  - Draw the tree.

- Since we only care about observable things, we only assign the following probabilites

$$p(S) = 1, \ p(\varnothing) = 0, \ p(T, T), \ p\{(T, H) \cup (H, T)\}, \text{ and } p(H, H)$$

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  - The dated events $(2,\{(H,H)\}),(2,\{(T,H),(H,T)\}),(2,\{(T,T)\})$ describe the situation at time 2 (after both tosses).
  - Draw the tree.
- Since we only care about observable things, we only assign the following probabilites
$$p(S)=1,\ p(\varnothing)=0,\ p(T,T),\ p\{(T,H)\cup(H,T)\},\text{ and } p(H,H)$$
  - If the coin is fair, then each state us equally likely, and we can attach numbers to the probability of each state and each event.

Example: Flipping a Coin with Partial Information

- Toss a coint twice:
$$S = \{(H, H), (H, T), (T, H), (T, T)\}$$
- The set of of all possible events is the set of all possible subsets of S.
- Suppose that at time 2 one only observes the number of H; these are the events
$$\{(T, T)\}, \ \{(T, H), (H, T)\}, \text{ and } \{(H, H)\}$$
- The information is different than before, and so shouyld be the tree.
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Partitions and Refinements

Definition

A partition of a set S is a set, $\mathcal{P}$, of nonempty subsets of S that are mutually disjoint and whose union is S:

$$\forall A, B \in \mathcal{P} \;\; A \cap B = \emptyset \text{ and } \bigcup_{A \in \mathcal{P}} A = \mathcal{S}$$

- This is sometimes called an information structure.

Definition

If $\mathcal{F}$ and $\mathcal{P}$ are partitions of S, $\mathcal{F}$ refines $\mathcal{P}$ if every A in $\mathcal{P}$ is a union of sets in $\mathcal{F}$. That is, for every A in $\mathcal{P}$, the sets in $\mathcal{F}$ that are subsets of A form a partition of A.

- Partitions can be used to express the revelation of information over time.

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Information Over Time and Dated Events

- Information is revealed over periods $t = 0, 1, ..., T$ and that S is the set of possible states of the world.

- The amount of information available at time t is represented by a partition, $\mathcal{P}_t$, of S.
- As information increases over time, then $\mathcal{P}_{t+1}$ refines $\mathcal{P}_t$, for all t.
- The partition $\mathcal{P}_t$ is the set of events that occur up to time t.
- Suppose that such a sequence of partitions, $\mathcal{P}_t$, is given, for $t = 0, 1, ..., T$.

Definition

The set of dated events equals $\Gamma = \{(t, A) : 0 \leq t \leq T,\ A \in \mathcal{P}_t, \text{ for all } t\}$, where in a pair (t, A) the letter t is the date of occurrence of the event A.

- Use Γ to also denote the cardinality of this set.
- The special case in which $T = 1$ corresponds to the "timeless" model of uncertainty we discussed when talking about individual decision making.

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Contingent Claims and Trade

- All decisions are made at time -1: individuals exchange promises to deliver or receive amounts of the commodities in pre-specified dated events.

Definition

A contingent claim is an agreement to deliver or receive an amount of a specified commodity in a specified dated event.

- The set of all contingent claims is the set of all vectors of quantities of commodities in all dated events:

$$\mathbb{R}^{\Gamma \times L} = \{x : x \text{ is a function from } \Gamma \text{ to } \mathbb{R}^L\}$$

  - An element of this set specifies a quantity of some commodity ℓ for each dated event: $x_{(t,A),\ell}$, where (t, A) is a dated event in Γ and ℓ is one of the L commodities.

- Trade in the contingent claims occurs at time -1; trades are made against a single unit of account (money), and no trade occurs after the initial moment.
- Agents exchange promises for future delivery or receipt; as time unfolds, only deliveries and receipts corresponding to realized events will take place.
  - In periods $0, 1, \ldots, T$, deliveries are made and taken according to the contingent claims (contracts) purchased and sold at the beginning.

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General Equilibrium Under Uncertainty

Definitions

- For each commodity $l = 1, .., L$, and each dated event in $(t, A) \in \Gamma$, one unit of dated event contingent commodity l (t, A) is a title to receive one unit of good l if and only if event A at time t occurs.
- A dated event contingent commodity vector

$$x \in \mathbb{R}^{\Gamma \times L}$$

is a title to receive the commodity vector $x_{(t,A)} \in \mathbb{R}^L$ if and only if dated event (t, A) occurs.

- These are binding promises to obtain certain quantities of each of the goods in a given state, after that state is realized:
  - state-contingent commodities are "contracts".
- A state-contingent commodity vector is a random variable: it assigns a vector in $\mathbb{R}^L$ to each dated event.

REMARK: All State Contingent Commodities Are Traded

- We assume that there is a market for each of these state contingent commodities.

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Preferences Over State Contingent Goods

Expected Utility Preferences Over State Contingent Commodities

- Agent i's consumption set $X_i \subset \mathbb{R}_+^{\Gamma \times L}$ contains state-contingent commodities.
- Agent i's preferences $\succsim_i$ are over X_i.
  - Preferences take into account the passage of time time as well as uncertainty. Typically, later consumption is less "valuable" than earlier consumption, and less likely events are not as "valuable" as more likely events.

- These are ex-ante preferences; they describe what i thinks at time -1 about what she would like to consume in the future.
  - i makes trade-offs between random variables; one more unit of good l in dated event (t, A) versus one less unit of good l' in dated event (t', A').
- Preferences reflect consumers probability distributions across dated events.
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- Preferences reflect consumers probability distributions across dated events.
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Preferences Over State Contingent Goods

Expected Utility Preferences Over State Contingent Commodities

- Agent i's consumption set $X_i \subset \mathbb{R}_+^{\Gamma \times L}$ contains state-contingent commodities.
- Agent i's preferences $\succsim_i$ are over X_i.
  - Preferences take into account the passage of time time as well as uncertainty.
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Initial Endowment

Definition

The endowment of agent i is a state-contingent commodity bundle

$$\omega_i \in \mathbb{R}_+^{\Gamma \times L}$$

When dated event (t, A) occurs, agent i has endowment $\omega_{(t,A)i} \in \mathbb{R}_+^L$.

- The consumer knows she will be rich in some dated events poor in others.
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Trade Under Uncertainty

- Consumers know that their wealth may fluctuate.
- This is because consumer i's initial endowment $\omega_{(t,A)}$ changes with the (t, A).
  - maybe i's endowment is large in one dated event and small in another; or
  - maybe i's endowment of desirable goods is large in some dated events and small in others.
- Without ex-ante (time -1) trade, consumers cannot insure themselves against these income and consumpion fluctuations.
- Trade allows for insurance:
  - consumers can sell so of their initial endowment in some dated events, and use the proceeds to buy consumption in other states; or
  - they can sell some of their initial endowment of particular goods in some dated events in which they will have a lot of those, and use the proceeds to buy more of other goods in those states;
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- Ex-ante markets for state-contingent commodities make these trades possible.

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Production Plans for State Contingent Goods

- Like consumers, firms make decisions at time -1 while actual production takes place from time 0 onward.
- They decide what output vector to produce in each dated event.
- Production plans are also state-contingent vectors.

Definition

The production possibility set is represented by a set $Y_j \subset \mathbb{R}^{\Gamma \times L}$ for each firm j.

- Firms 'produce' state-contingent commodities.
- A $y \in \mathbb{R}^{\Gamma \times L}$ is a binding promise to buy the corresponding inputs and sell the corresponding outputs in each dated event.
- The firm sells future production and makes a profit that is distributed to consumers.
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Markets and Prices in an Arrow-Debreu Economy

- In the Arrow-Debreu model each state-contingent commodity is traded.

Arrow-Debreu Markets

- There are $\Gamma \times S$ markets open at date -1.
- In these markets, a trade specifies various amounts of goods to be delivered in various states:
  - there is a price for each good in each state.
- Agents trade promises to receive, or give, amounts of good l if and when event A occurs at time t.
  - In finance lingo, these are "forward" markets where "futures" are traded.

- Trades of these 'contracts' are agreed upon now by all parties involved.
- Even though decisions are made now, the 'physical' exchange of goods only happens after uncertainty is resolved.
- This system of payments and deliveries is well defined only if everyone knows which state occurred: symmetric information.
  - We prevent situations where i says "This is event A" and j says "No, that is event A'"; if that happens, state-contingent markets may not function properly.

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Budget Constraint

- A consumer's income is given by the current value of (i) the state-contingent commodities she owns and (ii) her share of state-contingent profits:
  - She sells her future endowment.
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Budget Set

$$B_i(p, \omega_i) = \left\{ x_i \in X_i : \begin{array}{c} \sum_{(t,A)\in\Gamma} \sum_{l=1}^{L} p_{l(t,A)} x_{l(t,A)i} \\ \leq \\ \sum_{(t,A)\in\Gamma} \sum_{l=1}^{L} p_{l(t,A)} \omega_{l(t,A)i} + \sum_{j=1}^{J} \theta_{ji} \sum_{(t,A)\in\Gamma} \sum_{l=1}^{L} p_{ls} y_{l(t,A)j} \end{array} \right\}$$

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Budget Constraint Notation

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Consumer i cannot spend more than she has:

$$\left\{ \overbrace{\sum_{(t,A)\in\Gamma} \overbrace{\sum_{l=1}^{L} p_{l(t,A)} x_{l(t,A)i}}^{\text{expenditure in } (t,A)}}^{\text{overall expenditure}} \leq \overbrace{\sum_{(t,A)\in\Gamma} \overbrace{\sum_{l=1}^{L} p_{l(t,A)} \omega_{l(t,A)i}}^{\text{revenue from endowment in } (t,A)}}^{\text{overall revenue}} + \sum_{j=1}^{J} \theta_{ji} \overbrace{\sum_{(t,A)\in\Gamma} \overbrace{\sum_{l=1}^{L} p_{l(t,A)} y_{l(t,A)j}}^{j \text{ profits in } (t,A)}}^{j \text{ profits}} \right\}$$

$$= \left\{ \sum_{(t,A)\in\Gamma} p_{(t,A)} \cdot x_{(t,A)i} \leq \sum_{(t,A)\in\Gamma} p_{(t,A)} \cdot \omega_{(t,A)i} + \sum_{j=1}^{J} \theta_{ji} \sum_{(t,A)\in\Gamma} p_{(t,A)} \cdot y_{(t,A)j} \right\}$$

$$= \left\{ p \cdot x_i \leq p \cdot \omega_i + \sum_{j=1}^{J} \theta_{ji} (p \cdot y_j) \right\}$$

where $p_{(t,A)} \in \mathbb{R}_+^L$ for each s, and $p \in \mathbb{R}_+^{\Gamma \times L}$.

Summary

The Economy

- An Arrow-Debreu economy is described by a set Γ of states of the world and:
  - for each agent i: a consumption set $X_i \subset \mathbb{R}_+^{\Gamma \times L}$, a preference relation $\succsim_i$ on X_i, an endowment vector $\omega_i \in \mathbb{R}_+^{\Gamma \times L}$, and shares $\theta_{ji} \geq 0$ denoting her ownership of each firm;
  - for each firm j: a production possibility set $Y_j \subset \mathbb{R}^{\Gamma \times L}$.

- Individual characteristics (endowment, preferences, and production) can depend on the realized date event.

Remark

Relative to the original general equilibirum model, this is nothing but a change of labels.
There are L "physical" commodities, but $\Gamma \times L$ commodities are traded. This model, then, corresponds to the original GE setup with L replaced by $\Gamma \times L$.

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- For all $j = 1, ..., J$, firm j production set is $Y_j \subset \mathbb{R}^{\Gamma \times N}$.
- For all $i = 1, ..., I$, consumer i's preferences are defined over $\mathbb{R}_+^{\Gamma \times N}$
- For all $i = 1, ..., I$, consumer i's initial endowment is $\omega_i \in \mathbb{R}_+^{\Gamma \times N}$.

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- In an Arrow-Debreu (complete markets) economy trading is possible in all contingent claims.

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With this description of an Economy, all that is different from before is the dimensionality of consumption and production spaces. The definitions of competitive equilibrium and Pareto optimality stay the same.

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Robinson With Dated Events

- There are two periods, and two equally likely states in the second period.
  - Thus $S = \{s, s'\}$, and the events are $\{s\}$ and $\{s'\}$.
- Hence the dated events thus are $(0, \{s, s'\})$, $(1, \{s\})$, and $(1, \{s'\})$.
- The consumer is endowed with one unit of the only good in period 0, and none in period 1:

$$\omega = \left(\omega_{(0,S)}, \omega_{(1,s)}, \omega_{(1,s')}\right) = (1, 0, 0)$$

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- The firm's output is $\sqrt{-y_{(0,S)}}$ in dated event $(1, s)$, and $-y(0, S)$ in dated event $(1, s')$, where $-y(0, S)$ is the firm's input of the good in period 0:

$$Y\left\{y_{(0,S)}, y_{(1,s)}, y_{(1,s')} : y_{(0,S)} \leq 0,\ y_{(1,s)} \leq \sqrt{-y_{(0,S)}},\ y_{(1,s')} \leq -y_{(0,S)}\right\}$$

  The time zero good can be used to produce goods in the future.

Robinson With Dated Events

- There are two periods, and two equally likely states in the second period.
  - Thus $S = \{s, s'\}$, and the events are $\{s\}$ and $\{s'\}$.
- Hence the dated events thus are $(0, \{s, s'\})$, $(1, \{s\})$, and $(1, \{s'\})$.
- The consumer is endowed with one unit of the only good in period 0, and none in period 1:

$$\omega = \left(\omega_{(0,S)}, \omega_{(1,s)}, \omega_{(1,s')}\right) = (1, 0, 0)$$

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  The time zero good can be used to produce goods in the future.

Robinson With Dated Events: Pareto Optimum

- A Pareto optimal allocation solves the problem

$$\max_{x_{(0,S)}, x_{(1,s)}, x_{(1,s')}} \left[\log x_{(0,S)} + \frac{1}{2} \log x_{(1,s)} + \frac{1}{2} \log x_{(1,s')} \right] \text{ subject to}$$

$$\begin{aligned} y_{(0,S)} &\leq 0,\ 0 \leq x_{(0,S)} \leq 1 + y_{(0,S)} \\ 0 &\leq x_{(1,s)} \leq \sqrt{-y_{(0,S)}},\ 0 \leq x_{(1,s')} \leq -y_{(0,S)} \end{aligned}$$

- A little work (that you should replicate at home) shows that the solution is

$$x_{(0,S)} = \frac{4}{7}, x_{(1,s)} = y_{(1,s')} = \sqrt{\frac{3}{7}}, x_{(1,s)} = y_{(1,s')} = \frac{3}{7}, y_{(0,S)} = -\frac{3}{7}$$

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- The competitive equilibrium (with $p_{(0,S)} = 1$) can be found using the first order conditions of the consumer's utility maximization problem.
  - For $x_{(0,S)}$: $\frac{\partial u(x_{(0,S)},x_{(1,s)},x_{(1,s')})}{\partial x_{(0,S)}} = \lambda p_{(0,S)} \quad \Rightarrow \quad \frac{1}{x_{(0,S)}} = \lambda p_{(0,S)} \quad \Rightarrow \quad \lambda = \frac{7}{4}$
  - For $x_{(1,a)}$:
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General Equilibrium Under Uncertainty Alone

- Suppose there is no time and there are S states.
- The set of all vectors of contingent claims is $\mathbb{R}^{S \times L}$
- One can think of each commodity bundle as an S-dimensional random variable (a random vector).
- If there is only one commodity, $L = 1$, one typically thinks of it as money.
- Rather than labeling dated events as $(0, \{s\})$ for each $s \in S$ we may as well drop the 0 and just label each event by the realized state s.

Edgeworth Box Examples

Notation

- Two consumer, two equally likely states,

$$u_A\left(x_{sA}, x_{s'A}\right) = \frac{1}{2}\log x_{sA} + \frac{1}{2}\log x_{s'A} \text{ and } \omega_A = (2, 0)$$

$$u_B\left(x_{sB}, x_{s'B}\right) = \frac{1}{2}\log x_{sB} + \frac{1}{2}\log x_{s'B} \text{ and } \omega_B = (0, 2)$$

- What is the CE where the price of the good (money) in state s equals one?

- Since the utility functions are Cobb-Douglas, the consumers spends half of their income on each good:

$$x^*_{sA} = 1,\ x^*_{s'A} = \frac{1}{p_{s'}}, \text{ and } x^*_{sB} = p_{s'}, x^*_{s'B} = 1$$

- From the supply equal demand equation for the good in state s we then get $1 + p_{s'} = 2$ which yields $p^*_{s'} = 1$.

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- Similar reasning gives that consumer A spends $\frac{1}{4}$ of her income on the good in state s, so

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- From the supply equal demand equation for the good in state s we then get $\frac{1}{2} + p_{s'} = 2$ which yields $p^*_{s'} = \frac{3}{2}$.
- As one consumer gives more weight to state s', the equilibrium allocation and prices change.
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- As one consumer gives more weight to state s', the equilibrium allocation and prices change.
  - the good in state s' is more valuable.

Edgeworth Box Examples

Notation

- Suppose we have the same economy but

$$u_A(\cdot) = \frac{1}{4}\log x_{sA} + \frac{3}{4}\log x_{s'A} \text{ and } u_B(\cdot) = \frac{1}{2}\log x_{sB} + \frac{1}{2}\log x_{s'B}$$

- Similar reasning gives that consumer A spends $\frac{1}{4}$ of her income on the good in state s, so

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- From the supply equal demand equation for good s we then get $\frac{1}{2} + \frac{1}{2}p_{s'} = 2$ which yields $p_{s'}^* = 3$.
- When both consumers agree one state is three times more likely than the other, the price of money in that state is three times the price of money in the other state.

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Next Week

- Focus on the model in which there is no time.
- Introduce financial markets.