Arrow-Debreu Equilibrium

Econ 3030 Fall 2025

Lecture 25

Outline

1. Arrow-Debreu Equilibrium Recap
2. Arrow-Debreu Equilibrium With Only One Good
    1. Pareto Efficiency and Equilibrium
    2. Properties of Interior Equilibria

Contingent Claims

- There is a finite set of states S with s as generic element, and an observable subset of S is called an event.
- The set of dated events is $\Gamma = \{(t, A) : 0 \leq t \leq T, A \in \mathcal{P}_t, \text{ for all } t\}\}$, where the letter t is the date of occurrence of the event A.
  - The partition $\mathcal{P}_t$ is the set of events that occur up to time t.
    - Because information increases over time, $\mathcal{P}_{t+1}$ refines $\mathcal{P}_t$ for all t.
- A dated event contingent commodity vector

$$\mathbf{x} \in \mathbb{R}^{\Gamma \times L}$$

  is a title to the vector $\mathbf{x}_{(t,A)} \in \mathbb{R}^L$ if and only if dated event (t, A) occurs;
  - for each l, it specifies $x_{(t,A),l}$: a quantity of commodity l for each dated event (t, A).
  - This is a complete contingent plan that specifies what will happen at any future event.

Trade and Uncertainty

- Consumers' wealth may fluctuate across elements of Γ.
  - a consumer's endowment could be large in some dated events and small in others.
  - prices could change across dated events.
- Without trade, there is nothing consumers can do about these fluctuations.
- Trade, on the other hand, allows for insurance:
  - consumers can sell part of their initial endowment in some dated events and use the proceeds to buy consumption in other dated events;
  - firms can use more or less inputs or produce more or less outputs across different dated events, in order to maximize their profits.
- Markets for dated events contingent commodities make these trades possible.
- The central planner could also do all those things to achive Pareto efficiency.

Arrow-Debreu Economy With Complete Markets

- At date 0, there are $\Gamma \times L$ markets where agents can trade any amount of dated event contingent commodities.

Definition

$(\mathbf{x}^*, \mathbf{y}^*) \in \mathbb{R}_+^{\Gamma \times L \times I} \times \mathbb{R}^{\Gamma \times L \times J}$ and $\mathbf{p}^* \in \mathbb{R}_+^{\Gamma \times L}$ are an Arrow-Debreu equilibrium if

1. For every j, $\mathbf{y}_j^* \in \mathbb{R}^{\Gamma \times L}$ maximizes profits: $\mathbf{p}^* \cdot \mathbf{y}_j^* \geq \mathbf{p}^* \cdot \mathbf{y}_j$ for all $\mathbf{y}_j \in Y_j \subset \mathbb{R}^{\Gamma \times L}$
2. For every i, $\mathbf{x}_i^* \in \mathbb{R}_+^{\Gamma \times L}$ is maximal in the budget set:

$$\mathbf{x}_i^* \succsim \mathbf{x}_i \qquad \text{for all } \mathbf{x}_i \in \left\{ \mathbf{x}_i \in X_i \subset \mathbb{R}_+^{\Gamma \times L} : \mathbf{p}^* \cdot \mathbf{x}_i \leq \mathbf{p}^* \cdot \boldsymbol{\omega}_i + \sum_{j=1}^{J} \theta_{ji} \left(\mathbf{p}^* \cdot \mathbf{y}_j^* \right) \right\}$$

3. Markets clear: $\sum_{i=1i}^{I} \mathbf{x}_i^* \leq \sum_{i=1}^{I} \boldsymbol{\omega}_i + \sum_{j=1}^{J} \mathbf{y}_j^*$ and $p_{l(t,A)} = 0$ if the inequality is strict

The One Good Timeless Exchange Economy

- Only one commodity ($L = 1$), and no firms.
- Only one future time period ($T = 1$) and no consumption at time $t = 0$,
  - dated events are $(1, \{s\})$, so we label everything just by the realized state $s \in S$.
- A consumption bundle is an S-dimensional vector $\mathbf{x}_i \in X_i \subset \mathbb{R}^S_+$:
$$\mathbf{x}_i \succsim \mathbf{y}_i \quad \text{if and only if} \quad U_i(\mathbf{x}_i) \geq U_i(\mathbf{y}_i)$$
  where preferences are subjective expected utility
$$U_i(\mathbf{x}_i) = \sum_{s=1}^{S} \pi_{si} u_i(x_{si})$$
  and π_{si} is the probability consumer i assigns to state s
  - Assume $u_i(\cdot)$ is differentiable, strictly increasing, and weakly concave.
    - $u_i(\cdot)$ could be state-dependent (just more notation).
- Note that
$$\frac{\partial \sum_{s=1}^{S} \pi_{si} u_i(x_{si})}{\partial x_{si}} = \pi_{si} u_i'(x_{si})$$
  - The marginal utility of consumption in state s depends on the derivative of the utility function as well as the probability of that state.

General Equilibirum Is Almost Finance

- A consumption bundle $\mathbf{x}_i = (x_{1i}, .., x_{Si})$ is the promise to receive quantities of the only good across different states.
  - We think of this good as money, and consumption bundles are similar to financial instruments
    - In financial markets, agents trade particular contracts that are denominated in money and have yields also denominated in money.
- Prices measure how much agents have to pay for the delivery of one unit of the only good (money) in state s;
  - these are sometimes called state prices since they measure the value of money in each state.
- Next we use the first and second welfare theorem for the differentiable economy to characterize Pareto optimal allocations and equilibria.

Arrow-Debreu Equilibrium With One Good

- At date 0, there are S markets where agents can trade any amount of state contingent commodities.
- Prices are $\mathbf{p} \in \mathbb{R}_+^S$, where $p_s \geq 0$ is the price of one unit of the good to be delivered in state s.
- For each i, preferences are over $\mathbb{R}_+^S$ and initial endowments are $\boldsymbol{\omega}_i \in \mathbb{R}_+^S$.

Definition

$\mathbf{x}^* \in \mathbb{R}_+^{S \times I}$ and $\mathbf{p}^* \in \mathbb{R}_+^S$ are an Arrow-Debreu equilibrium if

1. For every i, $\mathbf{x}_i^* \in \mathbb{R}_+^S$ solves:

$$\max_{\mathbf{x}_i \in \mathbb{R}_+^S} \sum_{s=1}^{S} \pi_{si} u_i(x_{si}) \qquad \text{subject to} \qquad \sum_{s=1}^{S} p_s x_{si} \leq \sum_{i=1}^{I} p_s \omega_{si}$$

2. Markets clear:

$$\sum_{i=1}^{I} x_{si}^* \leq \sum_{i=1}^{I} \omega_{si} \qquad \text{for } s = 1, \ldots, S$$

Pareto Optimality

- A Pareto optimal allocation solves the planner's problem $\max \sum_{i=1}^{I} \lambda_i U_i(\mathbf{x}_i)$

Pareto optimality in the Arrow-Debreu economy

An allocation is Pareto optimal if and only if it solves:

$$\max_{\mathbf{x}_i \in \mathbb{R}_+^S} \sum_{i=1}^{I} \lambda_i \left[\sum_{s=1}^{S} \pi_{si} u_i(x_{si}) \right] \quad \text{subject to} \quad \sum_{i=1}^{I} x_{si} = \sum_{i=1}^{I} \omega_{si} \quad \text{for } s = 1, \ldots, S$$

- Using the results from a few classes back.

Necessary (and sufficient) conditions for Pareto optimality

At an interior solution $\hat{x}_{si}$ consumption is strictly positive in each state, and we have the following first order condition:

$$\lambda_i \pi_{si} u_i'(\hat{x}_{si}) = \mu_s \quad \text{for } s = 1, \ldots, S \quad \text{for } i = 1, \ldots, I.$$

where μ_s is the Lagrange multiplier given by the feasibility condition in state s.

Properties of Pareto Optimal Consumption Bundles

Necessary (and sufficient) conditions for Pareto optimality

At an interior solution $\hat{x}_{si}$ consumption is strictly positive in each state, and we have the following first order condition:

$$\lambda_i \pi_{si} u_i'(\hat{x}_{si}) = \mu_s \qquad \text{for } s = 1, \dots, S \qquad \text{for } i = 1, \dots, I.$$

where μ_s is the Lagrange multiplier given by the feasibility condition in state s.

- Summing over states, one gets

$$\lambda_i \sum_{t=1}^{S} \pi_{ti} u_i'(\hat{x}_{ti}) = \sum_{t=1}^{S} \mu_t \qquad \text{for } i = 1, \dots, I.$$

- Dividing the first order condition above by this expression we obtain:

$$\frac{\lambda_i \pi_{si} u_i'(\hat{x}_{si})}{\lambda_i \sum_{t=1}^{S} \pi_{ti} u_i'(\hat{x}_{ti})} = \frac{\pi_{si} u_i'(\hat{x}_{si})}{\sum_{t=1}^{S} \pi_{ti} u_i'(\hat{x}_{ti})} = \frac{\mu_s}{\sum_{t=1}^{S} \mu_t}$$

  for $s = 1, \dots, S$ and for $i = 1, \dots, I$.

- The right hand side does not depend on i... comparison with equilibirum shortly.

Properties of Interior Pareto Optimal Allocations

An interior Pareto optimal allocation $\hat{x}$ must satisfy:

$$\frac{\pi_{si} u_i'(\hat{x}_{si})}{\sum_{t=1}^{S} \pi_{ti} u_i'(\hat{x}_{ti})} = \frac{\mu_s}{\sum_{t=1}^{S} \mu_t} \qquad \text{for } s = 1, ..., S \quad \text{and } i = 1, ..., I$$

- Thus:

$$\begin{pmatrix} \frac{\pi_{1i} u_i'(\hat{x}_{1i})}{\sum_{t=1}^{S} \pi_{ti} u_i'(\hat{x}_{ti})} \\ \cdots \\ \frac{\pi_{si} u_i'(\hat{x}_{si})}{\sum_{t=1}^{S} \pi_{ti} u_i'(\hat{x}_{ti})} \\ \cdots \\ \frac{\pi_{Si} u_i'(\hat{x}_{Si})}{\sum_{t=1}^{S} \pi_{ti} u_i'(\hat{x}_{ti})} \end{pmatrix} = \begin{pmatrix} \frac{\mu_1}{\sum_{t=1}^{S} \mu_t} \\ \cdots \\ \frac{\mu_s}{\sum_{t=1}^{S} \mu_t} \\ \cdots \\ \frac{\mu_S}{\sum_{t=1}^{S} \mu_t} \end{pmatrix} = \begin{pmatrix} \frac{\pi_{1j} u_j'(\hat{x}_{1j})}{\sum_{t=1}^{S} \pi_{tj} u_j'(\hat{x}_{tj})} \\ \cdots \\ \frac{\pi_{sj} u_j'(\hat{x}_{sj})}{\sum_{t=1}^{S} \pi_{tj} u_j'(\hat{x}_{tj})} \\ \cdots \\ \frac{\pi_{Sj} u_j'(\hat{x}_{Sj})}{\sum_{t=1}^{S} \pi_{tj} u_j'(\hat{x}_{tj})} \end{pmatrix}$$

  for any i, j (the Lagrange multipliers do not depend on the identity of the consumer).
- Better-than sets have a common support.

Properties of Interior Pareto Optimal Allocations

An interior Pareto optimal allocation $\hat{x}$ must satisfy:

$$\frac{\pi_{si} u_i'(\hat{x}_{si})}{\sum_{t=1}^{S} \pi_{ti} u_i'(\hat{x}_{ti})} = \frac{\mu_s}{\sum_{t=1}^{S} \mu_t} \qquad \text{for } s = 1, ..., S \quad \text{and } i = 1, ..., I$$

- For any two states s and t this yields

$$\frac{\pi_{si} u_i'(\hat{x}_{si})}{\sum_{s=1}^{S} \pi_{si} u_i'(\hat{x}_{si})} = \frac{\pi_{sj} u_j'(\hat{x}_{sj})}{\sum_{s=1}^{S} \pi_{sj} u_j'(\hat{x}_{sj})} \qquad \text{for any } i \text{ and } j$$

and

$$\frac{\pi_{ti} u_i'(x_{ti}^*)}{\sum_{s=1}^{S} \pi_{si} u_i'(\hat{x}_{si})} = \frac{\pi_{tj} u_j'(x_{tj}^*)}{\sum_{s=1}^{S} \pi_{sj} u_j'(\hat{x}_{sj})} \qquad \text{for any } i \text{ and } j$$

- Therefore:

$$\frac{\pi_{si} u'_{si}(\hat{x}_{si})}{\pi_{ti} u'_{ti}(\hat{x}_{ti})} = \frac{\pi_{sj} u'_{sj}(\hat{x}_{sj})}{\pi_{tj} u'_{tj}(\hat{x}_{tj})} \qquad \text{for any } i \text{ and } j \text{ and } s \text{ and } t$$

Properties of Interior Pareto Optimal Allocations

Necessary conditions for Pareto optimality

An interior Pareto optimal allocation $\hat{\mathbf{x}}$ must satisfy

$$\frac{\pi_{si} u_i'(\hat{x}_{si})}{\pi_{ti} u_i'(\hat{x}_{ti})} = \frac{\pi_{sj} u_j'(\hat{x}_{sj})}{\pi_{tj} u_j'(\hat{x}_{tj})} \qquad \text{for any } i \text{ and } j \text{ and } s \text{ and } t$$

- If $\pi_{si} = \pi_{sj}$ for all s, then probabilities drop out and we are left with the marginal utility ratio for states s and t
- Then we can make conclusions about Pareto optima: when $\pi_{si} = \pi_{sj}$ for all s

$$u_i'(\hat{x}_{si}) \geq u_i'(\hat{x}_{ti}) \qquad \Leftrightarrow \qquad u_j'(\hat{x}_{sj}) \geq u_j'(\hat{x}_{tj})$$

  - Since we have $u'' \leq 0$ then

$$\hat{x}_{si} - \hat{x}_{ti} \leq 0 \qquad \Leftrightarrow \qquad \hat{x}_{sj} - \hat{x}_{tj} \leq 0$$

- An interior Pareto optimal consumption bundle $\hat{\mathbf{x}}$ is comonotonic across agents:

$$(x_{si} - x_{ti})(x_{sj} - x_{tj}) \geq 0$$

Properties of Interior Competitive Equilibria

Equilibrium in the Arrow-Debreu economy

At an equilibrium $\mathbf{x}^*, \mathbf{p}^*$ each consumer i solves

$$\max_{\mathbf{x}_i \in \mathbb{R}_+^S} \sum_{s=1}^{S} \pi_{si} u_i (x_{si})$$

subject to:

$$\sum_{s=1}^{S} p_s x_{si} \leq \sum_{i=1}^{I} p_s \omega_{si}$$

Necessary (and sufficient) conditions for Arrow-Debreu Equilibrium

The first order conditions yield

$$\pi_{si} u_i' (x_{si}^*) = \eta_i p_s^* \qquad \text{for } s = 1, ..., S \qquad \text{for } i = 1, ..., I.$$

where η_i is the Lagrange multiplier connected to consumer i budget constraint.

Properties of Interior Competitive Equilibria

Necessary (and sufficient) conditions for Arrow-Debreu Equilibrium

The first order conditions yield

$$\pi_{si} u_i'(x_{si}^*) = \eta_i p_s^* \qquad \text{for } s = 1, \ldots, S \qquad \text{for } i = 1, \ldots, I.$$

where η_i is the Lagrange multiplier connected to consumer i budget constraint.

- Summing over states, one gets

$$\sum_{s=1}^{S} \pi_{si} u_i'(x_{si}^*) = \eta_i \sum_{s=1}^{S} p_s^* \qquad \text{for } i = 1, \ldots, I.$$

- Dividing the first order condition above by this expression we obtain:

$$\frac{\pi_{si} u_i'(x_{si}^*)}{\sum_{t=1}^{S} \pi_{ti} u_i'(x_{ti}^*)} = \frac{\eta_i p_s^*}{\eta_i \sum_{t=1}^{S} p_t^*} = \frac{p_s^*}{\sum_{s=1}^{S} p_s^*}$$

  for $s = 1, \ldots, S$ and for $i = 1, \ldots, I$.

- The right hand side does not depend on i: prices are established in the market..

Properties of Interior Competitive Equilibria

An interior Arrow-Debreu equilibrium $\mathbf{x}^*$ must satisfy:

$$\frac{\pi_{si} u_i'(x_{si}^*)}{\sum_{t=1}^S \pi_{ti} u_i'(x_{ti}^*)} = \frac{p_s^*}{\sum_{s=1}^S p_s^*} \qquad \text{for } s = 1, ..., S \quad \text{and } i = 1, ..., I$$

- Therefore, for any two consumers i and j:

$$\begin{pmatrix} \frac{\pi_{1i} u_i'(x_{1i}^*)}{\sum_{t=1}^S \pi_{ti} u_i'(x_{ti}^*)} \\ \cdots \\ \frac{\pi_{si} u_i'(x_{si}^*)}{\sum_{t=1}^S \pi_{ti} u_i'(x_{ti}^*)} \\ \cdots \\ \frac{\pi_{Si} u_i'(x_{Si}^*)}{\sum_{t=1}^S \pi_{ti} u_i'(x_{ti}^*)} \end{pmatrix} = \begin{pmatrix} \frac{p_1^*}{\sum_{t=1}^S p_t^*} \\ \cdots \\ \frac{p_s^*}{\sum_{t=1}^S p_t^*} \\ \cdots \\ \frac{p_S^*}{\sum_{t=1}^S p_t^*} \end{pmatrix} = \begin{pmatrix} \frac{\pi_{1j} u_j'(x_{1j}^*)}{\sum_{t=1}^S \pi_{tj} u_j'(x_{tj}^*)} \\ \cdots \\ \frac{\pi_{sj} u_j'(x_{sj}^*)}{\sum_{t=1}^S \pi_{tj} u_j'(x_{tj}^*)} \\ \cdots \\ \frac{\pi_{Sj} u_j'(x_{Sj}^*)}{\sum_{t=1}^S \pi_{tj} u_j'(x_{tj}^*)} \end{pmatrix}$$

- Better-than sets have a common support given by the (normalized) price vector.

Equilibrium and Pareto Optimality

Pareto optimality necessary conditions

$$\lambda_i \pi_{si} u_i'(\hat{x}_{si}) = \mu_s \qquad \text{for } s = 1, \ldots, S \quad \text{and } i = 1, \ldots, I.$$

Arrow-Debreu equilibibirum necessary conditions

$$\pi_{si} u_i'(x_{si}^*) = \eta_i p_s^* \qquad \text{for } s = 1, \ldots, S \quad \text{and } i = 1, \ldots, I.$$

Welfare Theorems

Since the welfare theorems hold, $\hat{x}_{si} = x_{si}^*$ for all s and i, and therefore:

$$\eta_i = \frac{1}{\lambda_i} \quad \text{and} \quad \mu_s = p_s^* \qquad \text{for } s = 1, \ldots, S \quad \text{and } i = 1, \ldots, I.$$

- Thus

$$\frac{\pi_{si} u_i'(x_{si}^*)}{\sum_{t=1}^{S} \pi_{ti} u_i'(x_{ti}^*)} = \frac{\mu_s}{\sum_{t=1}^{S} \mu_t} = \frac{p_s^*}{\sum_{t=1}^{S} p_t^*} \qquad \text{for } s = 1, \ldots, S \quad \text{and } i = 1, \ldots, I.$$

Equilibrium and Pareto Optimality

- In both Pareto optimality and equilibrium we have:

$$\begin{pmatrix} \frac{\pi_{1i} u_i'(x_{1i}^*)}{\sum_{t=1}^{S} \pi_{ti} u_i'(x_{ti}^*)} \\ \cdots \\ \frac{\pi_{si} u_i'(x_{si}^*)}{\sum_{t=1}^{S} \pi_{ti} u_i'(x_{ti}^*)} \\ \cdots \\ \frac{\pi_{Si} u_i'(x_{Si}^*)}{\sum_{t=1}^{S} \pi_{ti} u_i'(x_{ti}^*)} \end{pmatrix} = \begin{pmatrix} \frac{\pi_{1j} u_j'(x_{1j}^*)}{\sum_{t=1}^{S} \pi_{tj} u_j'(x_{tj}^*)} \\ \cdots \\ \frac{\pi_{sj} u_j'(x_{sj}^*)}{\sum_{t=1}^{S} \pi_{tj} u_j'(x_{tj}^*)} \\ \cdots \\ \frac{\pi_{Sj} u_j'(x_{Sj}^*)}{\sum_{t=1}^{S} \pi_{tj} u_j'(x_{tj}^*)} \end{pmatrix} = \begin{pmatrix} \frac{p_1^*}{\sum_{t=1}^{S} p_t^*} \\ \cdots \\ \frac{p_s^*}{\sum_{t=1}^{S} p_t^*} \\ \cdots \\ \frac{p_S^*}{\sum_{t=1}^{S} p_t^*} \end{pmatrix}$$

- Better-than sets have a common support given by the (normalized) price vector.

Equilibrium, Pareto Optimality, and Probabilities

- The support conditions define a probability distribution over the state space.
- Why? Each element of the vector

$$\left(\frac{\pi_{1i}u_i'\left(x_{1i}^*\right)}{\sum_{t=1}^S \pi_{ti}u_i'\left(x_{ti}^*\right)},\ldots,\frac{\pi_{si}u_i'\left(x_{si}^*\right)}{\sum_{t=1}^S \pi_{ti}u_i'\left(x_{ti}^*\right)},\ldots,\frac{\pi_{Si}u_i'\left(x_{Si}^*\right)}{\sum_{t=1}^S \pi_{ti}u_i'\left(x_{ti}^*\right)}\right)$$

  is a number between 0 and 1, and their sum is

$$\sum_{s=1}^S \frac{\pi_{si}u_i'\left(x_{si}^*\right)}{\sum_{t=1}^S \pi_{ti}u_i'\left(x_{ti}^*\right)} = 1$$

- These are sometimes called "risk-neutral probabilities". Why?
  - If the vNM utility index is linear, then $u_i'\left(x_{si}^*\right) = constant$

$$\frac{\pi_{si}u_i'\left(x_{si}^*\right)}{\sum_{t=1}^S \pi_{ti}u_i'\left(x_{ti}^*\right)} = \frac{\pi_{si} \times constant}{\sum_{t=1}^S \pi_{ti} \times constant} = \frac{\pi_{si}}{\sum_{t=1}^S \pi_{ti}}$$

- In equilibrium the state prices equal a common probability distribution.
  - think of it as the probability distribution over the state space assigned by an hypothetical risk-neutral agent (the market).

An Example: Exchange Economy

Two individuals, one good, and two states

We have: $I = 2$, $L = 1$, $S = 2$, $T = 1$, and there is no consumption at time 0.

- Each consumer's utility function is differentiable

$$\pi_{1i} u_i (x_{1i}) + \pi_{2i} u_i (x_{2i}) \qquad \text{with } i = A, B$$

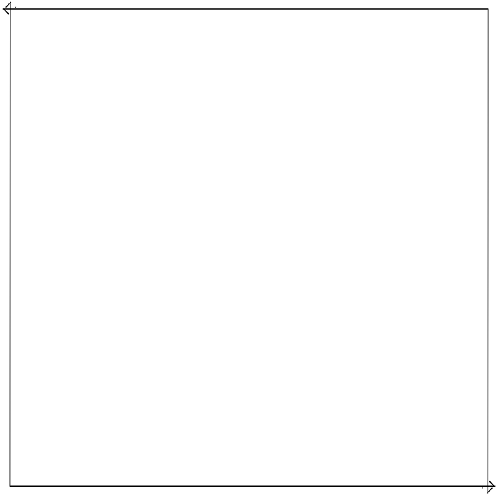
- Suppose the initial endowments are

$$\boldsymbol{\omega}_A = (1, 0) \qquad \text{and} \qquad \boldsymbol{\omega}_B = (0, 1)$$

  - There is no aggregate uncertainty: the aggregate endowment is the same in both states:
$$\boldsymbol{\omega} = \boldsymbol{\omega}_A + \boldsymbol{\omega}_B = (1, 1);$$
    - The only uncertainty is about who is the rich consumer.

An Example: Exchange Economy

$$\omega = \omega_A + \omega_B = (1, 1)$$



- In an Edgeworth box, the certainty lines (what is that?) overlap.
- What is special about allocations on the certainty line? They provide Full Insurance.

An Example: Exchange Economy

Pareto Optimality with two individuals, one good, and two states

- Marginal rates of substitution must be equal: $\frac{\pi_{1A} u_A'(x_{1A})}{\pi_{2A} u_A'(x_{2A})} = \frac{\pi_{1B} u_B'(x_{1B})}{\pi_{2B} u_B'(x_{2B})}$
  - If consumers have the same probabilities, $\pi_{1A} = \pi_{1B}$ and $\frac{\pi_{1A}}{\pi_{2A}} = \frac{\pi_{1B}}{\pi_{2B}}$.
    - The MRS condition becomes $\frac{u_A'(x_{1A})}{u_A'(x_{2A})} = \frac{u_B'(x_{1B})}{u_B'(x_{2B})}$.
    - Pareto optimal allocations do not depend on probabilities.

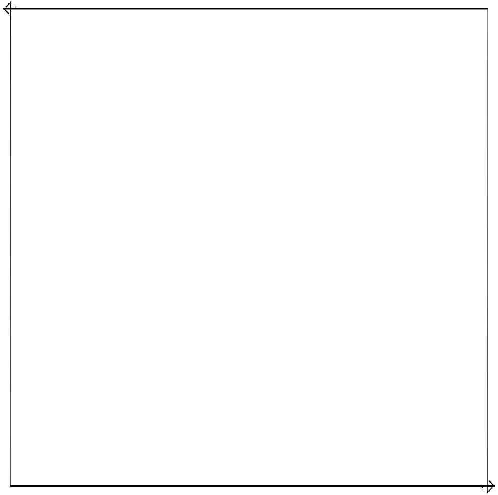
Equilibrium with two individuals, one good, and two states

- Utility maximization requires: $\frac{\pi_{1A} u_A'(x_{1A})}{\pi_{2A} u_A'(x_{2A})} = \frac{p_1}{p_2}$ and $\frac{\pi_{1B} u_B'(x_{1B})}{\pi_{2B} u_B'(x_{2B})} = \frac{p_1}{p_2}$
  - If $\pi_{1A} = \pi_{1B}$ the probabilities, again, drop out: $\frac{\pi_{1A} u_A'(x_{1A})}{\pi_{2A} u_A'(x_{2A})} = \frac{p_1}{p_2} = \frac{\pi_{1B} u_B'(x_{1B})}{\pi_{2B} u_B'(x_{2B})}$, and thus $\frac{u_A'(x_{1A})}{u_A'(x_{2A})} = \frac{u_B'(x_{1B})}{u_B'(x_{2B})}$.
    - Equilibrium allocations do not depend on probabilities either,
    - but the equilibrium prices do depend on probabilities.

- In both cases individuals' consumption must be equal in the two states (why?).

An Example: Exchange Economy

$$\omega = \omega_A + \omega_B = (1, 1)$$



- Pareto optimal and equilibrium allocations provide Full Insurance.
- What can we say about equilibrium prices?

Another Exchange Economy Example

Another economy with two individuals, one good, and two states

- Same economy as before, except for the initial endowments
- Initial endowments are

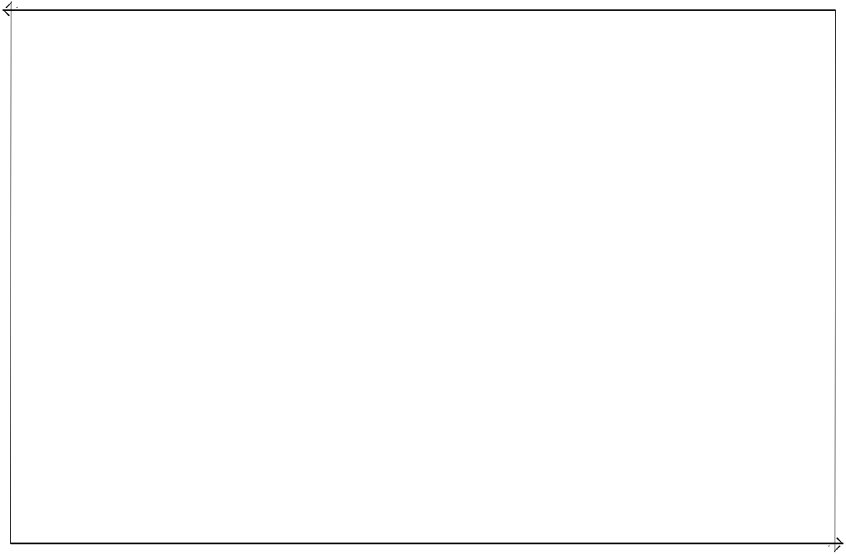
$$\omega_A = (2,0) \qquad \text{and} \qquad \omega_B = (0,1)$$

  - There is aggregate uncertainty: the aggregate endowment differ in the two states; the economy is richer in state 1.
    - In an Edgeworth box, the certainty lines no longer overlap.
  - Can we have full insurance of both consumers?
- The conditions for an allocation to be Pareto optimal (and an equilibrium) are the same as before.

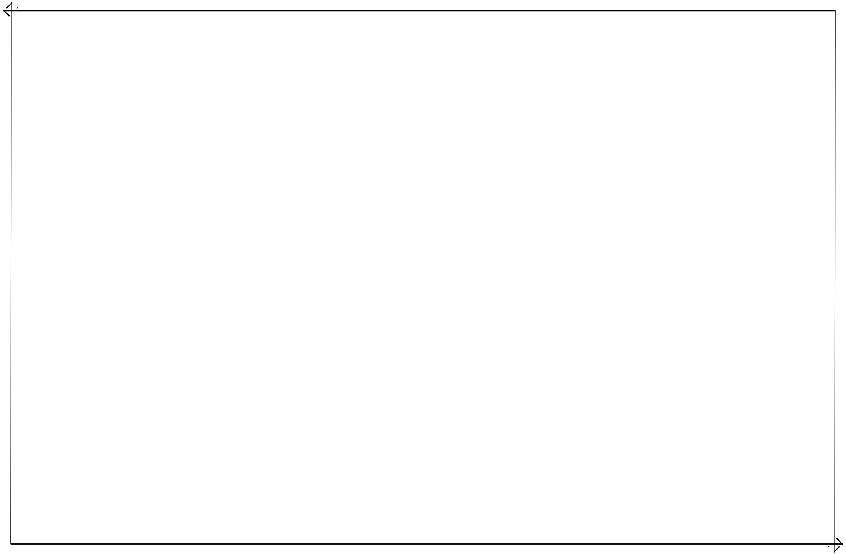
- Should Pareto optimal allocations be different from the previous example?
- How about the competitive equilibirum allocations and the prices?

An Example: Exchange Economy

$$\omega = \omega_A + \omega_B = (2, 1)$$



An Example: Comonotonic Allocations



Next Class

- Timing of Trades
- Radner Equilibrium
- Securities and Asset Prices