Noise

- Definition: Noise is unwanted signal energy in the passband of a communications device
- Types
  - White Noise (Thermal)
  - Shot Noise
    - Due to Active Devices
    - Electrons Moving Across Boundaries
  - Impluse Noise
  - Crosstalk
- Effect is Signal Corruption

Theoretical Basis

- Random Signal is Added to Desired Signal
- The Characteristics of this Random Signal Depends on the Noise Type
  - Thermal Noise has a Gaussian Amplitude Distribution
  - Shot and Impulse Noise has a Poisson Arrival Distribution
- The Noise Signal has an Average Frequency Characteristic
  - Power Spectral Density, $P(f)$, which is the Fourier Transform of the Autocorrelation function
  - Noise can be filtered just like any other signal

Thermal Noise

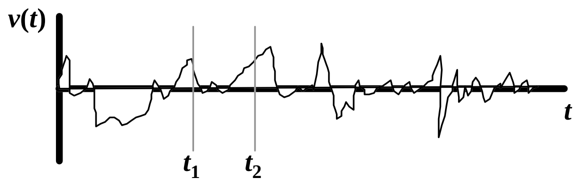
- Due to Random Transitions of Electrons in Materials
- Proportional to Temperature
- White Noise
  - Noise that is Not Dependent on Frequency
  - Assume Power Spectral Density is Constant

Martin B.H. Weiss

University of Pittsburgh

Characterization of Thermal Noise

- Let $n(t)$ be the Noise Signal in the Time Domain
  - The Peak Voltage at Any Given Time is Random
    - Assume that the *Distribution* of Peak Voltage is Gaussian
    - Assume the Mean Voltage is Zero
  - The Noise Voltage at Time t_1 is Independent of the Noise Voltage at Time t_2
- Gaussian Distribution: $P[v(t) < x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$
- Graphical Representation:



Martin B.H. Weiss

University of Pittsburgh

Model of Noise in the Communication Channel

- At Any Time t_i, the Noise Voltage is Drawn From a Random Variable
- Thus, We Need a Different Random Variable for Each Point in Time for Which $n(t)$ has a Value
  - Imagine $n(t)$ has a Value for m Points in Time
  - We Could Imagine m Identical Noise Generators Operating Simultaneously, Continuously, and Independently
  - Each is Sampled at the Appropriate Time
    - At Time t_1, Noise Generator 1 is Sampled
    - At Time t_i, Noise Generator i is Sampled
    - At Time t_m, Noise Generator m is Sampled
- Such a Collection of Random Variables is Called a *Random Process*

Ergodic Random Process

- The Mean Value of the Random Variables are Identical
- The Variance of the Random Variables are Identical
- The Average Across Random Variables in the Ensemble (the *Ensemble Average* of the Random Process) is Identical to the Time Average of Any Random Process in the Ensemble

Description of an Ergodic Random Process

- **Mean** $m = <n(t)> = E[n(t)]$
- **Variance** $\sigma^2 = <[n(t) - <n(t)>]^2> = <n^2(t)> - <n(t)>^2$
- **Covariance**
  - $m_{XY}(t,t+\tau) = <[n(t) - <n(t)>][n(t+\tau) - <n(t+\tau)>]>$
  - **or,** $m_{XY}(t,t+\tau) = <n(t)n(t+\tau)> - <n(t)><n(t+\tau)> = <n(t)n(t+\tau)>$
  - **Where**
    - X **Refers to the Value** $n(t)$ **may Take on at Time** t
    - Y **Refers to the Value** $n(t)$ **may Take on at Time** $t+\tau$

Description of an Ergodic Random Process

- $<n(t)>$ **is the DC Component**
- $<n(t)>^2$ **is the DC Power**
- σ^2 **is the AC Power**
- $<n^2(t)>$ **is the Total Power**
- $<n(t)n(t+\tau)>$ **is the *Autocorrelation Function*,** $R(\tau)$
  - **This is Only a Function of** τ
  - **For Ergodic Processes, this is independent of** t
  - **In General,** $R_n(\tau) = <n(t)n(t+\tau)> = \int_{-\infty}^{\infty} n(t)n(t+\tau)\,dt$
  - **Note That** $R(0) = <n^2(t)> =$ **Total Power**

Power Spectral Density of Noise

- The Power Spectral Density of Noise, $P_n(f) = F\{R(\tau)\}$ (Watts/Hz)
- If the Noise is White
  - It is Uniform Across all Frequencies
  - That is, $P(f) = \frac{N_0}{2}$
  - Thus, $R(\tau) = F^{-1}\{P_n(f)\}$
    - Or, $R(\tau) = \frac{N_0}{2}\delta(\tau)$
    - Where the Function $\delta(\tau) = \begin{cases} 1 & \tau = 0 \\ 0 & \tau \neq 0 \end{cases}$
  - White Noise is Completely Uncorrelated with Itself for Any Time Delay

Filtering Noise

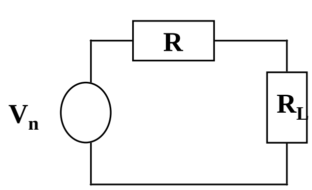
- Noise Behaves Like a Signal
- Therefore, $N_{out}(\omega)=|H(\omega)|N_{in}(\omega)$
- Assume a Low Pass Filter
  - $H(\omega) = \frac{1}{1 + j\omega RC}$
  - $N_{in}(\omega) = N/2$
  - $N_{out}(\omega) = \frac{N}{2}\frac{1}{1 + (\omega RC)^2}$
- In the Time Domain,
  - $R(\tau) = F^{-1}\{P_n(f)\}$
  - $R_{out}(\tau) = \frac{N}{4RC}e^{-\frac{|\tau|}{RC}}$
  - Recall That $R(0)$ = Total Power = $\frac{N}{4RC}$
  - Note that Output Noise is No Longer Uncorrelated

Characterization of Thermal Noise

- $P(f) = kT$
  - $P(f)$ = Power Spectral Density (Watts/Hz)
  - T = Temperature of the Conductor (in $^oK = 273 + {}^oC$)
  - k = Boltzman's constant = 1.38×10^{-23} joule/^oK
  - Example
    - Noise Power at Room Temperature (290^{o}K)
    - $P(f) = 1.38 \times 10^{-23} \times 290 = 4 \times 10^{-21}$ W/Hz
- In a Particular Bandwidth B
  - The Noise Power $N = P(f)\,B = kTB$
  - N is in Watts

Characterization of Thermal Noise

- The Existence of Power Presupposes
  - A Generator
  - A Circuit
  - A Load
- Circuit Diagram



Characterization of Thermal Noise

- **If the "Source" Impedance is Matched to the "Load" Impedance for Maximum Power Transfer**
  - **In General, $P = V_s^2/4R_s$**
  - **Let $R_s = R$ (*i.e.*, the Resistor Under Investigation)**
  - **Let $P = kTB$, and $V_s^2 = V_n^2$**
    - $V_n^2/R=kTB$
    - **So $V_n^2 = 4RkTB$, by Substitution**
  - **Also, $V_n^2/B = 4kTR$ is the Voltage Spectral Density**
- **Generalization**
  - **This Applies to Imperfect Conductors as Well**
  - **Conductors Have Resistance**

Thermal Noise Example

- **Given**
  - $R = 10\text{K}\Omega$
  - $T = 290^{o}\text{K}$
  - $B = 1\text{MHz}$
- $V_n^2 = 4(4*10^{-21})(10^4)(10^6) = 16*10^{-11}$

Series Resistance

- Procedure
  - Sum the Resistances
  - Sum the Noise Powers
- Thus,
  - For Each Resistor, $V_n^2 = 4RkTB$
  - The Total Noise is $V_n^2 = V_{n1}^2 + V_{n2}^2 + \ldots$
  - Or, $V_n^2 = 4(R_1+R_2+ \ldots)\ kTB$
- A Similar Argument Holds for Parallel Circuits
  - Rely on Noise Current Instead of Noise Voltage
  - Use Conductances Instead of Resistances

Reactance

- Ideally, Reactance Does not Dissipate Heat
- Hence, No Noise Contribution
- Noise Bandwidth is Affected

Noise Bandwidth

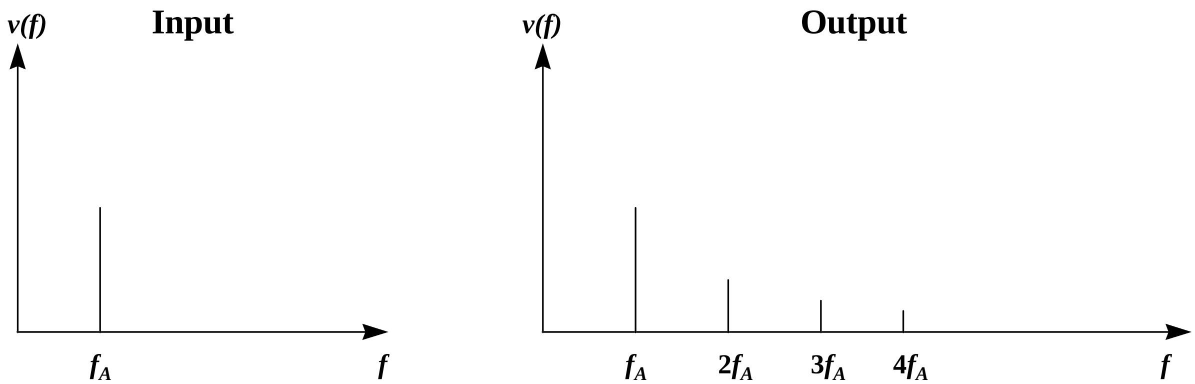
- **In General,**
  - $S_{n0} = |H(\omega)|^2 kT$
  - $B_{eff} = (\pi/2)(B_{3dB})$
- **This is Why Bandpass Filters are Often Included in the Front End of Communications Systems**

Correlated Noise

- **Result of System Non-Linearities**
- **Harmonic Distortion**
- **Intermodulation Distortion**

Harmonic Distortion

- **Result of Non-Linearities**
- **Occurs When the Result of a A Single Input Frequency Passed Through a System Contains the Fundamental and Its Harmonics**
- **%Total Harmonic Distorion = $(V_{higher})/(V_{fundamental})$ * 100**



Martin B.H. Weiss
University of Pittsburgh

Intermodulation Noise

- **Product of *Two or More* Signals are Passed Through a Non-Linear System**
- **Common Test is *Second Order IM Distortion***
  - **% IMD=(RMS of Second-Order Cross Products)/(Total RMS Amplitude of Input Frequencies) * 100**
- **Test**
  - **Use Four Frequecies**
    - **"A" Band: f_{a1} and f_{a2}**
    - **"B" Band: f_{b1} and f_{b1}**
  - **Second Order Cross Products (2A-B)**
    - $(2f_{a1} - f_{b1})$
    - $(2f_{a1} - f_{b2})$
    - $(2f_{a2} - f_{b1})$
    - $(2f_{a2} - f_{b2})$
    - $(f_{a1} + f_{a2}) - f_{b1}$
    - $(f_{a1} + f_{a2}) - f_{b2}$

Martin B.H. Weiss
University of Pittsburgh

Noise and Amplifiers

- **General**
  - ❍ **Amplifiers Amplify Noise as well as Signal**
  - ❍ **Amplifiers Add Noise to the Signal**
- **Signal to Noise Ratio**
  - ❍ **Definitions**
    - ❑ **Voltage Ratio: $S/N = V_S^2/V_N^2$**
    - ❑ **Power Ratio: $S/N = P_S/P_N$**
  - ❍ **Common Form: SNR = 10 log (S/N)**
- **Common Mode Rejection**
  - ❍ **Applies to Differential Amplifiers**
  - ❍ **Differential Amplifiers Amplify the *Difference* Between the Inputs**
  - ❍ **The Elements of the Signal that the Inputs Have in Common Should Not Be Amplified**

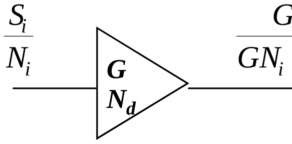
Noise Figure

- **Qualitative**
  - ❍ **Measure of the Amount of Noise Added by a System**
  - ❍ **May be Frequency Dependent**
- **Analytical Definition**
  - ❍ **Ratio of S/N at Input to S/N at Output**
  - ❍ **Noise Factor:** $\mathrm{F} = \dfrac{\left(S/N\right)_i}{\left(S/N\right)_o}$
  - ❍ **Noise Figure (NF)**
    - ❑ **Noise Ratio with S/N in Decibels ($SNR_{dB} = 10 \log_{10} (S/N)$)**
    - ❑ **Thus, $SNR_{dB}(i) = SNR_{dB}(o) - NF$**

Noise Figure

- NF ≥ 0
  - **Perfect Systems Add No Noise**
  - **Imperfect Systems Add Noise (i.e., the SNR Decreases)**
  - **Analog Systems Cannot Remove Noise Due to its Random Nature**
- **More Specifically,**
  - **Gain, $G = S_o/S_i$**
  - **NR = $F = (N_o/N_i)(S_i/S_o) = N_o/GN_i$**
  - **Or $N_o = FGN_i$**
  - **But $N_i = kTB$, so**
  - **$N_o = FGkTB$**

$\frac{S_i}{N_i}$



$$\frac{GS_i}{GN_i + N_d} = \frac{S_i}{N_i + N_d/G}$$

Martin B.H. Weiss

University of Pittsburgh

Amplifiers in Cascade

- **Given a Series of Amplifiers, a_1, a_2, a_3, . . .**
- **With Gains G_1, G_2, G_3, . . .**
- **And With Noise Ratios F_1, F_2, F_3, . . .**
- **Then the Noise Ratio for the Series is Given by Friiss's Formula**

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

Martin B.H. Weiss

University of Pittsburgh

Noise Temperature

- General
  - Another Way of Measuring Noise
  - Since Noise is Associated with Temperature,
    - Its Level Can be Represented as an Equivalent Temperature
    - *i.e.*, the System Seems to be Operating Hotter Than It Is
  - Used Frequently in Microwave Systems with Receiver Input Noise
- Analytical Representation
  - Let $kT_eB = (F - 1)kT_0B$
  - Or, $T_e = (F - 1)T_0$

Implications for Communications Systems

- In Analog Systems,
  - The Repeaters Have a Noise Ratio > 1
  - A Repeatered Network Consists of Amplifiers in Cascade
    - This Implies that Noise Multiplies
    - This Limits the Length of Transmission Systems
  - Thus, in Addition to Amplifying the Signal and the Noise, they add Noise
- In Digital Systems, the Regenerative Repeaters Do Not Add Noise
  - That is, $F = 1$
  - Thus, Noise Does not Accumulate
  - In Addition, The Repeaters Can Repeat Only Signal

Sample Noise Calculation

- **Problem**
  - **Three Telephone Circuits in Series Have SNR = 44dB**
  - **A Fourth Circuit is Added with SNR = 34dB**
  - **What is the Overall SNR?**
- **Solution**
  - **Recall that $SNR_{dB} = 10 \log_{10}[P_s/P_n]$**
  - **The Signal Remains Constant Across All Circuits**
  - **Thus,** $SNR_{dB} = 10\log_{10}\left(\frac{P_s}{P_{n_1} + P_{n_2} + P_{n_3} + P_{n_4}}\right)$
  - **We Can Compute the Noise Power for Each Circuit:**
    - $\frac{P_s}{P_n} = 10^{\frac{SNR_{dB}}{10}}$
    - **Thus, For the First Three Circuits,** $\frac{P_s}{P_n} = 10^{\frac{44}{10}} \cong 25 \times 10^3$
    - **For the Last Circuit,** $\frac{P_s}{P_n} = 10^{\frac{34}{10}} \cong 2.5 \times 10^3$
  - **Combining,**

$$SNR_{dB} = 10\log_{10}\left(\frac{P_s}{3P_{n_a} + P_{n_b}}\right) = 10\log_{10}\left(\frac{P_s}{P_s\left[\frac{3}{25\times10^3} + \frac{1}{2.5\times10^3}\right]}\right) = 33dB$$