University of Pittsburgh | Mathematics Research Center
Dietrich School of Arts and Sciences

Workshop on PDEs in Fluid Dynamics

Department of Mathematics, University of Pittsburgh

April 12–13, 2025

Program

All talks are in *Thackerary Hall 704* in the Department of Mathematics.

Sponsors: Mathematics Research Center (MRC) of the University of Pittsburgh.

Organizers: Ming Chen and Dehua Wang.

Workshop on PDEs in Fluid Dynamics

University of Pittsburgh, April 12–13, 2025

All talks are in *Thackerary Hall 704* in the Department of Mathematics.

	Saturday, 04/12	**Sunday, 04/13**
❖ Morning Session		
09:00-09:45am	Alexis Vasseur	Jiahong Wu
09:45-10:30am	Cheng Yu	Gong Chen
10:30-11:00am	*Coffee break* ☕	
11:00-11:45pm	Mikhail Feldman	Toan Nguyen
12:00-2:00pm	*Lunch break* 🍴	
❖ Afternoon Session		
02:00-02:45pm	Samuel Walsh	Roman Shvydkoy
02:45-03:30pm	Miles Wheeler	Sameer Iyer
03:30-04:00pm	*Coffee break* ☕	
04:00-04:45pm	Ronghua Pan	Changhui Tan
04:45-05:30pm	Moon-jin Kang	Difan Yuan

Saturday, April 12

❖ Morning Session

- **09:00-09:45**: Alexis Vassuer, University of Texas at Austin
  Non-uniqueness for continuous solutions to 1D conservation laws

- **09:45-10:30**: Cheng Yu, University of Florida
  Non-uniqueness and incompressible limits in compressible Euler equations via convex integration

10:30-11:00: Coffee Break

- **11:15-11:45**: Mikhail Feldman, University of Wisconsin at Madison
  Self-similar solutions to two-dimensional Riemann problems with transonic shocks

12:00-02:00 pm: Lunch Break

❖ Afternoon Session

- **02:00-02:45**: Samuel Walsh, University of Missouri
  Finite-time self-similar implosion of hollow vortices

- **02:45-03:30**: Miles Wheeler, University of Bath
  Overhanging solitary water waves

03:30-04:00pm: Coffee Break

- **04:00-04:45**: Ronghua Pan, Georgia Institute of Technology
  Incompressible MHD without resistivity: structure and regularity

- **05:05-05:50**: Moon-Jin Kang, Korea Advanced Institute of Mathematical Sciences
  Long-time behavior for IBVP of barotropic Navier-Stokes system in 1D half space

Sunday, April 13

❖ Morning Session

- **09:00-09:45**: Jiahong Wu, University of Notre Dame
  A universal stabilizing phenomenon in fluid systems
- **09:45-10:30**: Gong Chen, Georgia Institute of Technology
  Asymptotic stability of the sine-Gordon kink outside symmetry

10:30-11:00: Coffee Break

- **11:15-11:45**: Toan Nguyen, Penn State University
  Landau damping below survival threshold

12:00-02:00 pm: Lunch Break

❖ Afternoon Session

- **02:00-02:45**: Roman Shvydkoy, University of Illinois at Chicago
  Existence and long time behavior of weak solutions to
  the Fokker-Planck-Alignment models
- **02:45-03:30**: Sameer Iyer, University of California at Davis
  Beyond the Goldstein singularity: reversal in the stationary Prandtl equations

03:30-04:00pm: Coffee Break

- **04:00-04:45**: Changhui Tan, University of South Carolina
  On the primitive equations with fractional horizontal dissipation
- **05:05-05:50**: Difan Yuan, Beijing Normal University
  Global solutions of the one-dimensional compressible Euler equations
  with nonloca interactions via the inviscid limit

THE END. ☺

Workshop on PDEs in Fluid Dynamics
University of Pittsburgh, April 12–13, 2025

Abstracts

Gong Chen, *Georgia Institute of Technology*

Title: Asymptotic stability of the sine-Gordon kink outside symmetry

Abstract: We consider scalar field theories on the line with Ginzburg-Landau (double-well) self-interaction potentials. Prime examples include the ϕ^4 model and the sine-Gordon model. These models feature simple examples of topological solitons called kinks. The study of their asymptotic stability leads to a rich class of problems owing to the combination of weak dispersion in one space dimension, low power nonlinearities, and intriguing spectral features of the linearized operators such as threshold resonances or internal modes. We present a perturbative proof of the full asymptotic stability of the sine-Gordon kink outside symmetry under small perturbations in weighted Sobolev norms. The strategy of our proof combines a space-time resonances approach based on the distorted Fourier transform to capture modified scattering effects with modulation techniques to take into account the invariance under Lorentz transformations and under spatial translations. A major difficulty is the slow local decay of the radiation term caused by the threshold resonances of the non-selfadjoint linearized matrix operator around the modulated kink. Our analysis hinges on two remarkable null structures that we uncover in the quadratic nonlinearities of the evolution equation for the radiation term as well as of the modulation equations. The entire framework of our proof, including the systematic development of the distorted Fourier theory, is general and not specific to the sine-Gordon model.

This is a joint work with Jonas Lührmann (Texas A&M)

Mikhail Feldman, *University of Wisconsin at Madison*

Title: Self-similar solutions to two-dimensional Riemann problems with transonic shocks

Abstract: Multidimensional conservation laws is an active research area with open questions about existence, uniqueness, and stability of properly defined weak solutions, even for fundamental models such as the compressible Euler system. Understanding particular classes of weak solutions, such as Riemann problems, is crucial in this context. This talk focuses on self-similar solutions to two-dimensional Riemann problems involving transonic shocks for compressible Euler systems. Examples include regular shock reflection, Prandtl reflection, and four-shocks Riemann problem. We first review the results on existence, regularity, geometric properties and uniqueness of global self-similar solutions of regular reflection structure in the framework of potential flow equation. A significant open problem is to extend these results to compressible Euler system, i.e. to understand the effects of vorticity. We show that for the isentropic Euler system, solutions of regular reflection structure have low regularity. We further discuss existence, uniqueness and stability of renormalized solutions to the transport equation for vorticity in this low regularity setting.

Sameer Iyer, *University of California at Davis*

Title: Beyond the Goldstein singularity: reversal in the stationary Prandtl equations

Abstract: We will review some recent advances in the study of the Prandtl system and the inviscid limit. In particular, reversal occurs after the Goldstein singularity (the separation point), and is characterized by regions in which the velocity changes sign. The classical point of view of treating the stationary Prandtl system as an evolution equation in the tangential variable x completely breaks down. Instead, we view the problem as a quasilinear, mixed-type, free-boundary problem. Joint works with Nader Masmoudi.

Moon-Jin Kang, *Korea Advanced Institute of Mathematical Sciences*

Title: Long-time behavior for IBVP of barotropic Navier-Stokes system in 1D half space

Abstract: We will talk about the initial boundary value problem for barotropic Navier-Stokes system in 1D half space, when we impose one of the three boundary conditions : impermeable, inflow and outflow at the origin, together with a constant state at far-field. We are interested in the case where the two constant states at the origin and the far-field are connected by a Hugoniot curve possibly together with rarefaction curve and boundary layer curve. In this talk, we will present the long-time behavior towards the following cases: (i) Single shock for the Impermeable problem or Outflow problem; (ii) Superposition of the boundary layer solution, the 1-rarefaction wave, and the viscous 2-shock waves for the Inflow problem.

Toan Nguyen, *Penn State University*

Title: Landau damping below survival threshold

Abstract: In the collisionless Vlasov theory of excited electrons, plasmas oscillations arise due to their long-range meanfield interaction, and the classical Landau damping concerns decay of such an oscillation. While the damping mechanism is exponentially fast by phase mixing for short-wave perturbations, it's extremely slow or not available for long-wave perturbations and oscillations may survive as a Klein-Gordon's pure oscillatory wave (hence, a notion of survival threshold). Landau damping also plays a fundamental role in fluid dynamics, a.k.a. inviscid damping. This talk aims to provide an overview on Landau damping and present the recent nonlinear results, focusing on the newly discovered plasma oscillation regime.

Ronghua Pan, *Georgia Institute of Technology*

Title: Incompressible MHD without resistivity: structure and regularity

Abstract: We study the global existence of classical solutions to the incompressible viscous MHD system without magnetic diffusion in 2D and in 3D. The lack of resistivity (or magnetic diffusion) poses a major challenge to a global regularity theory even for small smooth initial data. However, the interesting nonlinear structure of the system not only leads to some significant challenges, but some useful stabilization properties, that shed the light on the possibility of a theory for global existence of classical solutions. This talk is based on joint works with Yi Zhou, Yi Zhu, Shijin Ding, and Xiaoying Zeng.

Roman Shvydkoy, *University of Illinois at Chicago*

Title: Existence and long time behavior of weak solutions to the Fokker-Planck-Alignment models

Abstract: In this talk we discuss global existence of weak solutions, their regularization, and global relaxation to Maxwellian for a broad class of Fokker-Planck-Alignment models which appear in collective dynamics. The main feature of these results, as opposed to previously known ones, is the lack of regularity or no-vacuum requirements on the initial data. With a particular application to the classical kinetic Cucker-Smale model, we demonstrate that any bounded data with finite energy, $(1+|v|^2)f_0 \in L^1$, $f_0 \in L^\infty$, and finite higher moment $|v|^q f \in L^2$, $q \gg 2$, gives rise to a global instantly smooth solution, satisfying entropy equality and relaxing exponentially fast.

The results are achieved through the use of a new thickness-based renormalization procedure, which circumvents the problem of degenerate diffusion in non-perturbative regime.

Changhui Tan, *University of South Carolina*

Title: On the primitive equations with fractional horizontal dissipation

Abstract: In this talk, I will introduce the primitive equations, also known as the hydrostatic Navier-Stokes equations, which model large-scale atmospheric and oceanic flows. The inviscid dynamics are notoriously ill-posed (worse than the Euler equations), while the introduction of horizontal viscosity yields global well-posedness, exhibiting better behavior than the Navier-Stokes equations. We investigate the two-dimensional primitive equations with fractional horizontal dissipation, examining ill-posedness, local well-posedness, and global well-posedness under varying strengths of dissipation. This is joint work with Elie Abdo and Quyuan Lin.

Alexis Vasseur, *Univeristy of Texas at Austin*

Title: Non-uniqueness for continuous solutions to 1D conservation laws

Abstract: In this talk, we show that a geometrical condition on 2×2 systems of conservation laws leads to non-uniqueness in the class of 1D continuous functions. This demonstrates that the Liu Entropy Condition alone is insufficient to guarantee uniqueness, even within the mono-dimensional setting. We provide examples of systems where this pathology holds, even if they verify stability and uniqueness for small BV solutions. Our proof is based on the convex integration process. Notably, this result represents the first application of convex integration to construct non-unique continuous solutions in one dimension. This is a joint work with Robin Ming Chen, and Cheng Yu.

Samuel Walsh, *University of Missouri*

Title: Finite-time self-similar implosion of hollow vortices

Abstract: In this talk, we will present some recent results on the finite-time blowup of hollow vortices. These are solutions of the two-dimensional Euler equations with the fluid domain being the complement of finitely many Jordan curves $\Gamma_1, \ldots, \Gamma_M$. The flow is irrotational and incompressible, but with a nonzero circulation around each boundary component. The "vortex core" bounded by each Γ_k is modeled as a bubble of ideal gas: the pressure is constant in space and inversely proportional to the area of the vortex. This can be thought of as the isobaric approximation.

The existence of collapsing configurations of point vortices is classical. We prove that generically, these can be desingularized to yield a families of hollow vortex configurations that exhibit self-similar finite-time implosion. Specific examples of an imploding trio and quartet of hollow vortices are given.

This is joint work with Ming Chen (University of Pittsburgh) and Miles Wheeler (University of Bath)

Miles Wheeler, *University of Bath*

Title: Overhanging solitary water waves

Abstract: We construct gravity water waves with constant vorticity having the approximate shape of a disk joined to a strip by a thin neck. This is the first rigorous existence result for such waves, which have been seen in numerics since the 80s and 90s. Our method is related to the construction of constant mean curvature surfaces through gluing, and involves combining three explicit solutions to related problems: a disk of fluid in rigid rotation, a linear shear flow in a strip, and a rescaled version of an "exceptional domain" discovered by Hauswirth, Hélein, and Pacard.

Jiahong Wu, *University of Notre Dame*

Title: A universal stabilizing phenomenon in fluid systems

Abstract: The magnetic field stabilizes and damps electrically conducting fluids, a phenomenon observed in many physical experiments and numerical simulations. The temperature tames and stabilizes buoyancy driven fluids and helps the formation of stable structures in turbulent Rayleigh-Benard convection. These are just two examples of a universal stabilizing phenomenon in fluid dynamics. There is a rigorous mathematical mechanism behind this stabilizing phenomenon. In magnetohydrodynamics (MHD), the interaction between fluid velocity and the magnetic field near a background state gives rise to a damped wave equation, which reveals the smoothing and stabilizing effects. In the Boussinesq system modeling buoyancy-driven fluids, the interaction between velocity and temperature near hydrostatic balance also leads to a wave equation. Such wave structures appear in many fluid stability problems. This talk presents several stability results that derive and make use of the underlying wave structures.

Cheng Yu, *University of Florida*

Title: Non-uniqueness and incompressible limits in compressible Euler equations via convex integration

Abstract: In this talk, I will discuss the non-uniqueness of global weak solutions to the compressible Euler equations using convex integration. Additionally, I will explore how any L^2-bounded weak solution of the incompressible Euler equations can be obtained as a limit of solutions to the compressible Euler equations in the vanishing Mach number, also within the convex integration framework.

Difan Yuan, *Beijing Normal University*

Title: Global solutions of the one-dimensional compressible Euler equations with nonlocal interactions via the inviscid limit

Abstract: In this talk, I will present the global existence of finite-energy entropy solutions of the one-dimensional compressible Euler equations with (possibly) damping, alignment forces, and nonlocal interactions: Newtonian repulsion and quadratic confinement. Both

the polytropic gas law and the general gas law are analyzed. This is achieved by constructing a sequence of solutions of the one-dimensional compressible Navier-Stokes-type equations with density-dependent viscosity under the stress-free boundary condition and then taking the vanishing viscosity limit. The main difficulties in this paper arise from the appearance of the nonlocal terms. In particular, some uniform higher moment estimates for the compressible Navier-Stokes equations on expanding intervals with stress-free boundary conditions are obtained by careful design of the approximate initial data. This is a joint work with J. A. Carrillo, G-Qiang Chen, E. Zatorska.