Lecture 12: more Chapter 7, Secs. 2-3 Continuous Random Variables; Tails of the Normal Curve

- Standard/Nonstandard Normal RV Probabilities
- Preview Two Forms of Inference
- 68-95-99.7 Rule; Rule for Tails (90-95-98-99)
- Standard Normal Tail-Probability Problems
- Non-standard Tail-Probability Problems

1

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-3)
  - Displaying and Summarizing (Lectures 3-8)
  - Probability
    - Finding Probabilities (discussed in Lectures 9-10)
    - Random Variables (introduced in Lecture 10)
      - Binomial (discussed in Lecture 11)
      - Normal
    - Sampling Distributions
  - Statistical Inference

2

Definition *(Review)*

- ***z*-score**, or **standardized value**, tells how many standard deviations below or above the mean the original value is:

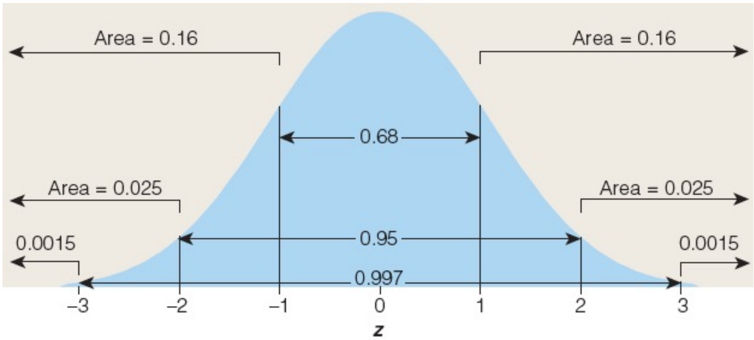
$$z = \frac{\text{value}-\text{mean}}{\text{standard deviation}}$$

- Notation for Population: $z = \frac{x-\mu}{\sigma}$
  - $z>0$ for x above mean
  - $z<0$ for x below mean
- Unstandardize: $x = \mu + z\sigma$

3

Standardizing Values of Normal R.V.s

Standardizing to z lets us avoid sketching a different curve for every normal problem: we can always refer to same standard normal (z) curve:



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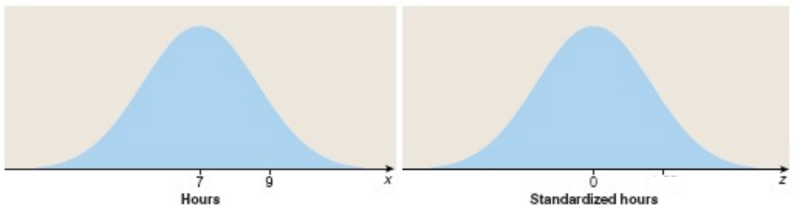
Example: *Standardized Value of Normal R.V.*

- **Background**: Typical nightly hours slept by college students normal; $\mu = 7, \sigma = 1.5$
- **Question:** How many standard deviations below or above mean is 9 hours?
- **Response:** Standardize to $z =$ ________________
(9 is ______ standard deviations above mean)



Example: *Standardizing/Unstandardizing Normal R.V.*

- **Background**: Typical nightly hours slept by college students normal; $\mu = 7, \sigma = 1.5$
- **Questions:**
  - What is standardized value for sleep time 4.5 hours?
  - If standardized sleep time is +2.5, how many hours is it?
- **Responses:**
  - $z =$ ______________________________
  - ______________________________

Interpreting *z*-scores *(Review)*

This table classifies ranges of *z*-scores informally, in terms of being unusual or not.

Size of z	Unusual?
$\|z\|$ greater than 3	extremely unusual
$\|z\|$ between 2 and 3	very unusual
$\|z\|$ between 1.75 and 2	unusual
$\|z\|$ between 1.5 and 1.75	maybe unusual (depends on circumstances)
$\|z\|$ between 1 and 1.5	somewhat low/high, but not unusual
$\|z\|$ less than 1	quite common

Looking Ahead: *Inference conclusions will hinge on whether or not a standardized score can be considered "unusual".*

Example: *Characterizing Normal Values Based on z-Scores*

- **Background**: Typical nightly hours slept by college students normal; $\mu = 7, \sigma = 1.5$.
- **Questions:** How unusual is a sleep time of 4.5 hours ($z = -1.67$)? 10.75 hours ($z = +2.5$)?
- **Responses:**
  - Sleep time of 4.5 hours ($z = -1.67$):
  - Sleep time of 10.75 hours ($z = +2.5$):

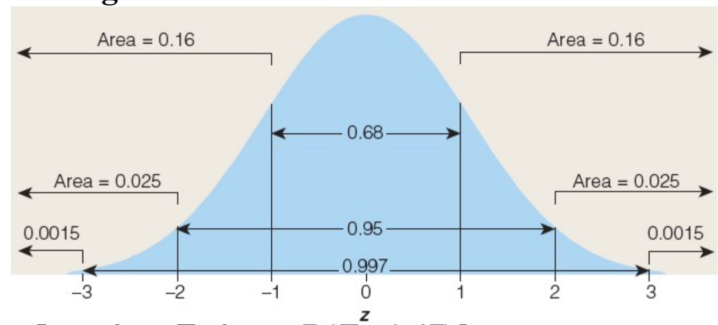
Size of z	Unusual?
$\|z\|$ greater than 3	extremely unusual
$\|z\|$ between 2 and 3	very unusual
$\|z\|$ between 1.75 and 2	unusual
$\|z\|$ between 1.5 and 1.75	maybe unusual (depends on circumstances)
$\|z\|$ between 1 and 1.5	somewhat low/high, but not unusual
$\|z\|$ less than 1	quite common

Normal Probability Problems

- Estimate probability given *z*
  - Probability close to 0 or 1 for extreme *z*
- Estimate *z* given probability
- Estimate probability given non-standard *x*
- Estimate non-standard *x* given probability

Example: *Estimating Probability Given z*

- **Background**: Sketch of 68-95-99.7 Rule for *Z*

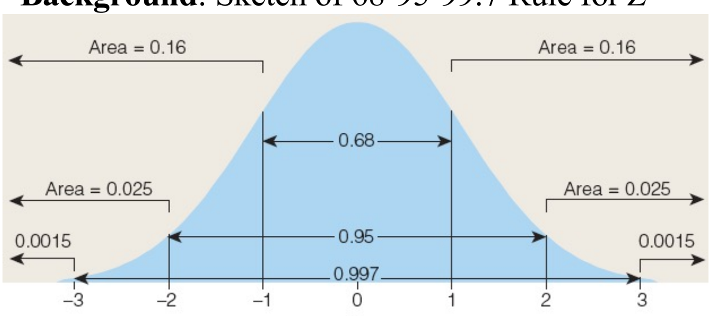


- **Question:** Estimate **P(*Z*<-1.47)**?
- **Response:**

Practice: 7.53 p.332

Example: *Estimating Probability Given z*

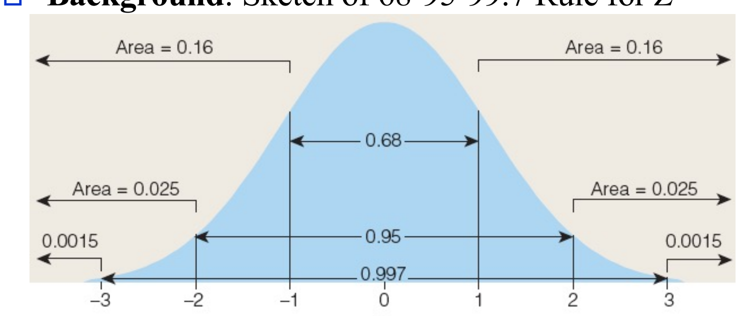
- **Background**: Sketch of 68-95-99.7 Rule for *Z*



- **Question:** Estimate **P(*Z*>+0.75)**?
- **Response:**

Example: *Estimating Probability Given z*

- **Background**: Sketch of 68-95-99.7 Rule for *Z*



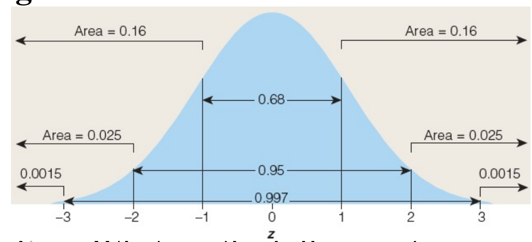
- **Question:** Estimate **P(*Z*<+2.8)**?
- **Response:**

Normal Probability Problems

- Estimate probability given *z*
  - Probability close to 0 or 1 for extreme *z*
- Estimate *z* given probability
- Estimate probability given non-standard *x*
- Estimate non-standard *x* given probability

Example: *Probabilities for Extreme z*

- **Background**: Sketch of 68-95-99.7 Rule for *Z*



- **Question:** What are the following (approximately)?

a. P(Z<-14.5) b. P(Z<+13) c. P(Z>+23.5) d. P(Z>-12.1)

- **Response:**

a. _________ b. ________ c. _________ d. _________

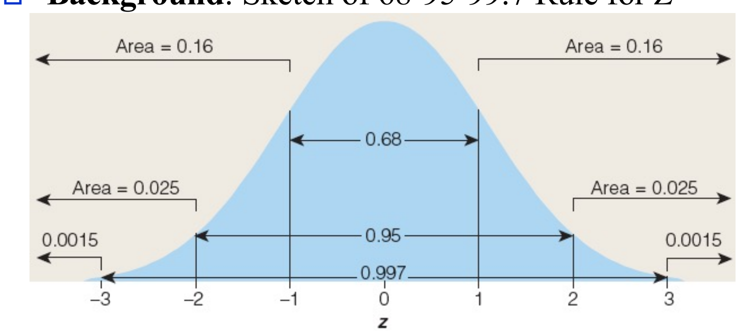
 Practice: 7.54 p.332

Normal Probability Problems

- Estimate probability given *z*
  - Probability close to 0 or 1 for extreme *z*
- Estimate *z* given probability
- Estimate probability given non-standard *x*
- Estimate non-standard *x* given probability

Example: *Estimating z Given Probability*

- **Background**: Sketch of 68-95-99.7 Rule for *Z*

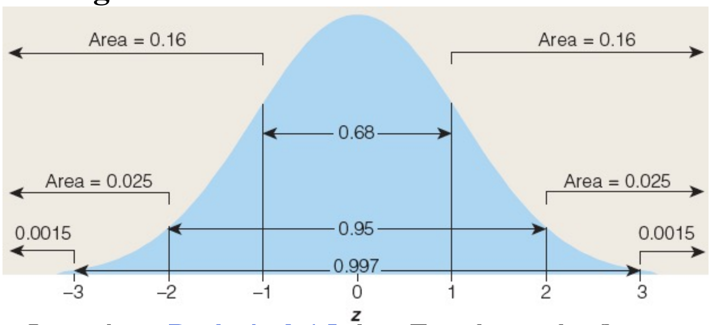


- **Question:** Prob. is 0.01 that *Z* < what value?
- **Response:**

 Practice: 7.57 p.332

Example: *Estimating z Given Probability*

- **Background**: Sketch of 68-95-99.7 Rule for *Z*



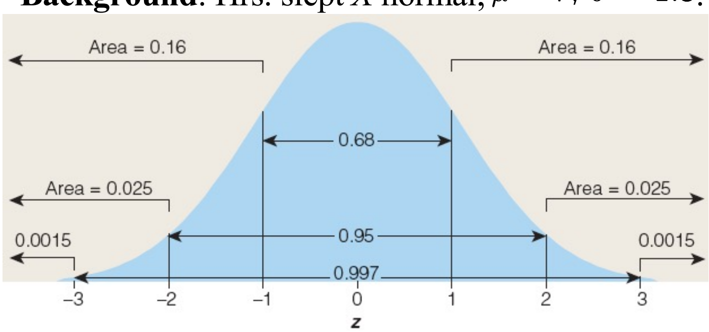
- **Question:** Prob. is 0.15 that *Z* >what value?
- **Response:**

Normal Probability Problems

- Estimate probability given *z*
  - Probability close to 0 or 1 for extreme *z*
- Estimate *z* given probability
- Estimate probability given non-standard *x*
- Estimate non-standard *x* given probability

Example: *Estimating Probability Given x*

- **Background**: Hrs. slept *X* normal; $\mu = 7$, $\sigma = 1.5$.

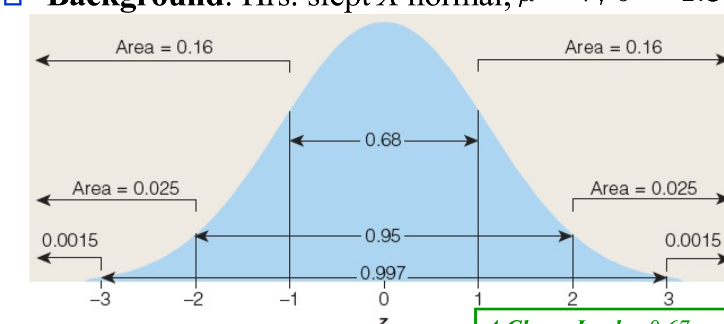


- **Question:** Estimate **P(*X*>9)**?
- **Response:**

 Practice: 7.59 p.333

Example: *Estimating Probability Given x*

- **Background**: Hrs. slept *X* normal; $\mu = 7$, $\sigma = 1.5$.



- **Question:** Estimate **P(6<*X*<8)**?

*A Closer Look: -0.67 and +0.67 are the **quartiles** of the z curve.*

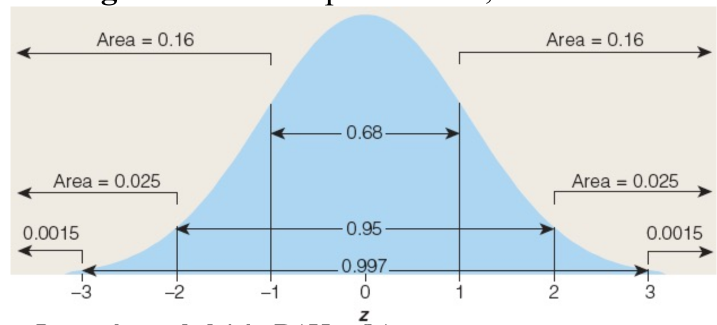
- **Response:**

Normal Probability Problems

- Estimate probability given z
  - Probability close to 0 or 1 for extreme z
- Estimate z given probability
- Estimate probability given non-standard x
- Estimate non-standard x given probability

Example: *Estimating x Given Probability*

- **Background**: Hrs. slept X normal; $\mu = 7$, $\sigma = 1.5$.

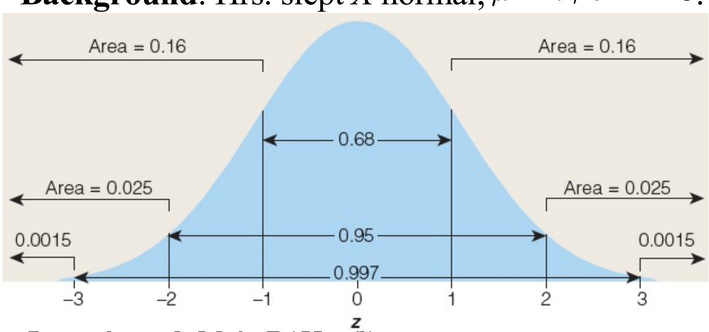


- **Question:** 0.04 is P($X <$?)
- **Response:**

 Practice: 7.61 p.333

Example: *Estimating x Given Probability*

- **Background**: Hrs. slept X normal; $\mu = 7$, $\sigma = 1.5$.



- **Question:** 0.20 is P($X >$?)
- **Response:**

Strategies for Normal Probability Problems

- Estimate probability given non-standard x
  - Standardize to z
  - Estimate probability using Rule
- Estimate non-standard x given probability
  - Estimate z
  - Unstandardize to x

Tails of Normal Curve in Inference

- **Goal:** Perform inference in 2 forms about unknown population proportion or mean:
  - Produce interval that has high probability (such as 90%, 95%, or 99%) of containing unknown population parameter
  - Test if proposed value of population proportion or mean is implausible (low probability---1% or 5%---of sample data)
- **Strategy:** Focus on tails of normal curve, in the vicinity of *Z*=+2 or *Z*=-2.

68-95-99.7 Rule for *Z (Review)*

For standard normal *Z*, the probability is

- 68% that *Z* takes a value in interval (-1, +1)
- 95% that *Z* takes a value in interval (-2, +2)
- 99.7% that *Z* takes a value in interval (-3, +3)

Need to fine-tune information for probability at or near 95%.

90-95-98-99 Rule for Standard Normal Z

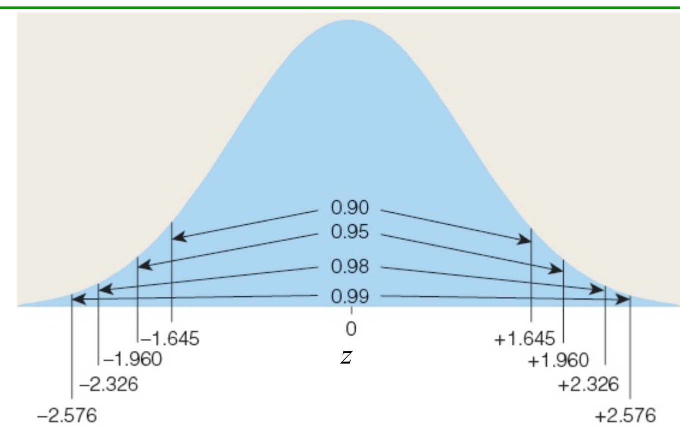
For standard normal Z, the probability is

- 0.90 that Z takes a value in interval (-1.645, +1.645)
- 0.95 that Z takes a value in interval (-1.960, +1.960)
- 0.98 that Z takes a value in interval (-2.326, +2.326)
- 0.99 that Z takes a value in interval (-2.576, +2.576)

Looking Back: *The 68-95-99.7 Rule rounded 0.9544 for 2 s.d.s to 0.95. For exactly 95%, need 1.96 s.d.s.*

90-95-98-99 Rule: "Inside" Probabilities

Looking Ahead: *This will be useful for "confidence intervals".*



90-95-98-99 Rule: “Outside” Probabilities

For standard normal Z, the probability is

- 0.05 that $Z < -1.645$ and 0.05 that $Z > +1.645$
- 0.025 that $Z < -1.96$ and 0.025 that $Z > +1.96$
- 0.01 that $Z < -2.326$ and 0.01 that $Z > +2.326$
- 0.005 that $Z < -2.576$ and 0.005 that $Z > +2.576$

Looking Back: *These follow from the inside probabilities, using the fact that the normal curve is symmetric with total area 1.*

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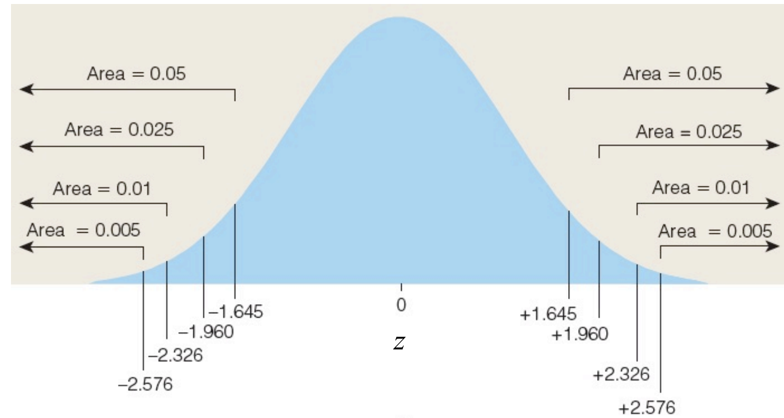
90-95-98-99 Rule: “Outside” Probabilities

Looking Ahead: *This will be useful for “hypothesis tests”.*



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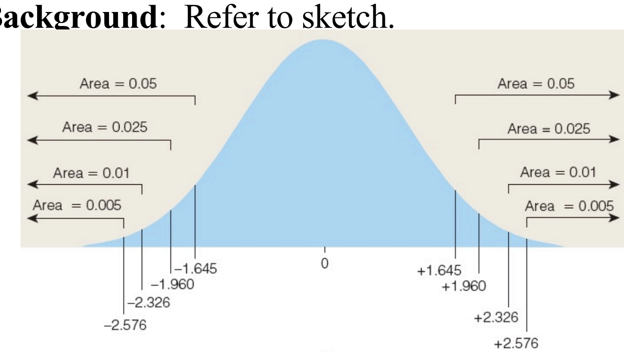
Example: *Finding Tail Probabilities*

- **Background**: Refer to sketch.



Question: What is $P(Z > +2.326)$?

- **Response:**

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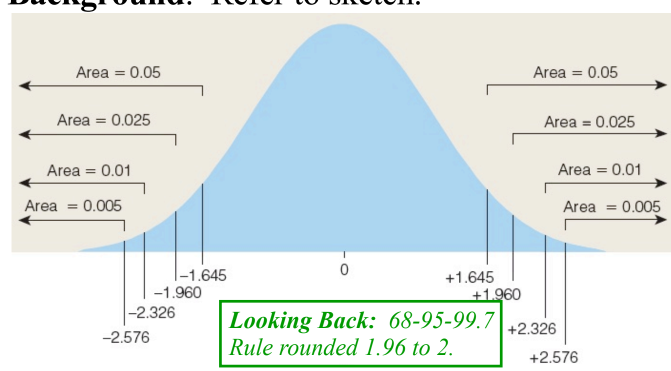
Example: *Finding Tail Probabilities*

- **Background**: Refer to sketch.



Looking Back: *68-95-99.7 Rule rounded 1.96 to 2.*

- **Question:** What is $P(Z < -1.96)$?
- **Response:**

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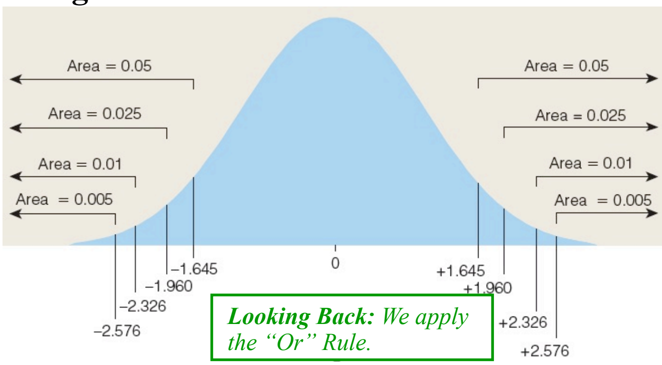
Example: *Finding Tail Probabilities*

- **Background**: Refer to sketch.

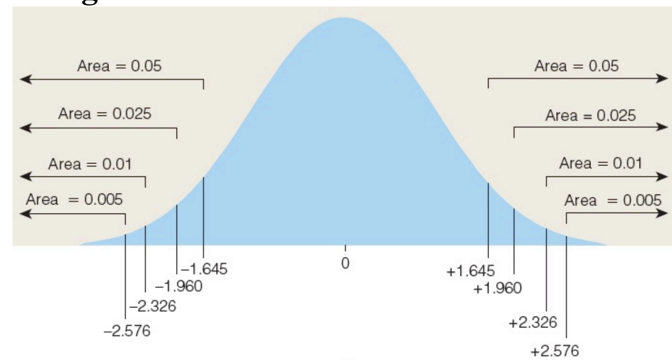


Looking Back: *We apply the "Or" Rule.*

- **Question:** What is $P(|Z| > 1.96)$?
- **Response:**

Example: *Given Probability, Find z*

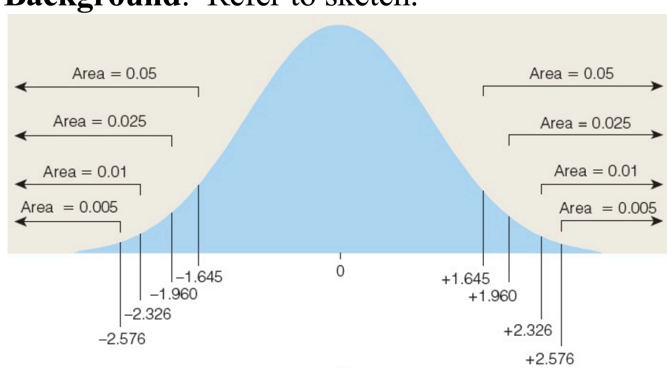
- **Background**: Refer to sketch.



- **Question:** $0.05 = P(Z <$ **?** $)$
- **Response:**

Example: *Given Probability, Find z*

- **Background**: Refer to sketch.



- **Question:** $0.005 = P(Z >$ **?** $)$
- **Response:**

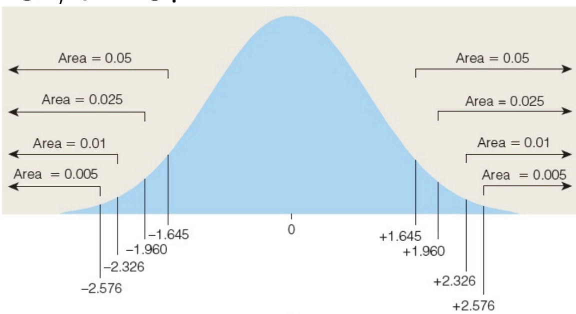
Non-Standard Normal Problems *(Review)*

To find probability, given non-standard normal x, first standardize: $z = \frac{x-\mu}{\sigma}$ then find probability (area under z curve).

To find non-standard x, given probability, find z then unstandardize: $x = \mu + z\sigma$

Example: *Given x, Find Probability*

- **Background**: Women's waist circumference X (in.) normal; $\mu = 32,\ \sigma = 5$.

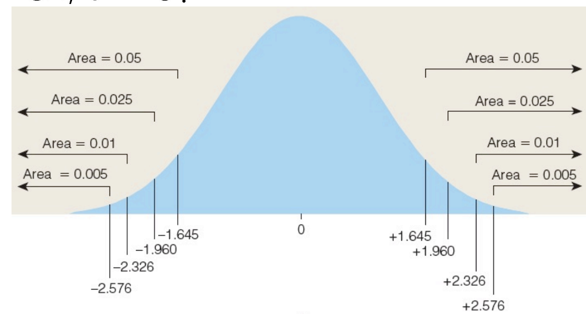


- **Question:** What is P(X > 43)?
- **Response:** z = ______________, between ______ and ______
  so P(X > 43) is between ______ and ______.

Practice: 7.63 p.333

Example: *Given x, Find Probability*

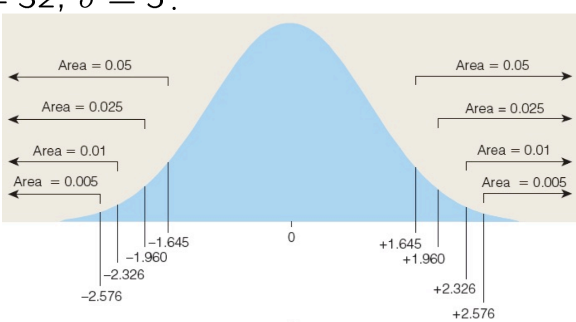
- **Background**: Women's waist circumference X (in.) normal; $\mu = 32,\ \sigma = 5$.



- **Question:** What is P(X < 23)?
- **Response:** z = ______________, between ______ and ______
  so P(X < 23) is between ______ and ______.

Example: *Given x, Find Probability*

- **Background**: Women's waist circumference X (in.) normal; $\mu = 32,\ \sigma = 5$.



- **Question:** What is P(X > 39)?
- **Response:** z = ____________________________________
  so P(X > *39*) is ______________________

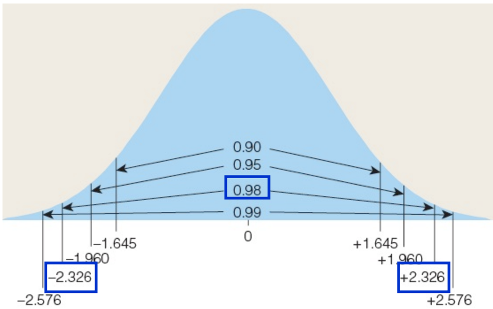
Non-Standard Normal Problems *(Review)*

To find probability, given non-standard normal x, first standardize: $z = \frac{x-\mu}{\sigma}$
then find probability (area under z curve).

To find non-standard x, given probability, find z then unstandardize: $x = \mu + z\sigma$

Example: *Given Probability, Find x*

- **Background**: Math SAT score *X* for population of college students normal; $\mu = 610,\ \sigma = 72$.

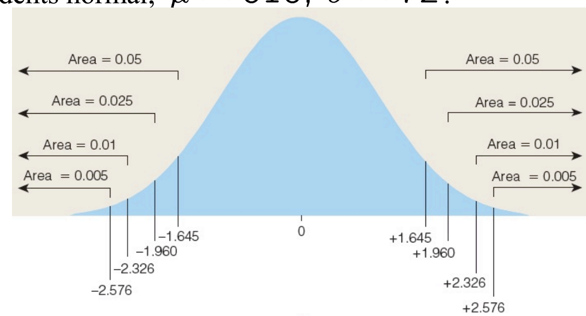


- **Question:** 0.98 is probability of *X* in what interval?
- **Response:** Prob. 0.98 has *z* from ________ to _________ so *x* is from ____________________ to _____________________

Practice: 7.69 p.334

Example: *Given Probability, Find x*

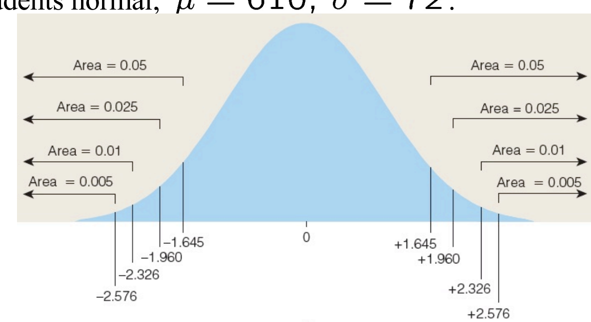
- **Background**: Math SAT score *X* for population of college students normal; $\mu = 610,\ \sigma = 72$.



- **Question:** Bottom 5% are below what score?
- **Response:** Bottom 0.05 has $z =$ _________ so $x =$ __________________

Example: *Given Probability, Find x*

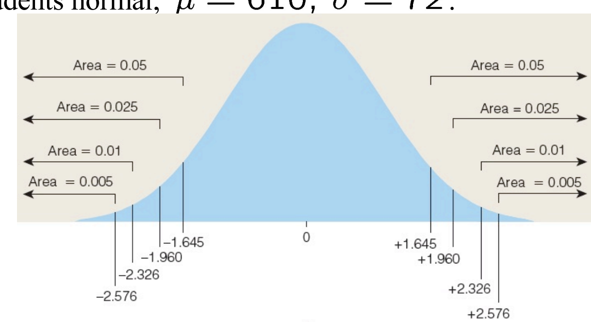
- **Background**: Math SAT score *X* for population of college students normal; $\mu = 610,\ \sigma = 72$.



- **Question:** Top half a percent were above what score?
- **Response:** Top 0.005 has $z =$ __________ so $x =$ ________________________________

Example: *Comparing to a Given Probability*

- **Background**: Math SAT score *X* for population of college students normal; $\mu = 610,\ \sigma = 72$.



- **Question:** Is P(X<480) more or less than 0.01?
- **Response:** 480 has $z =$ ____________________ Since -1.81 is ______________________, prob. is ______________

Example: *More Comparisons to Given Probability*

- **Background**: 0.01= P(*Z*<-2.326) = P(*Z*>+2.326)
- **Question:** Are the following >0.01 or <0.01?
  - P(*Z*>+2.4); P(*Z*>+1.9); P(*Z*<-3.7); P(*Z*<-0.4)
- **Response:**
  - P(*Z*>+2.4) ___0.01, since +2.4 is _____extreme than +2.326
  - P(*Z*>+1.9) __0.01, since +1.9 is _____ extreme than +2.326
  - P(*Z*<-3.7) __0.01, since -3.7 is ______ extreme than -2.326
  - P(*Z*<-0.4) __0.01, since -0.4 is ______ extreme than -2.326

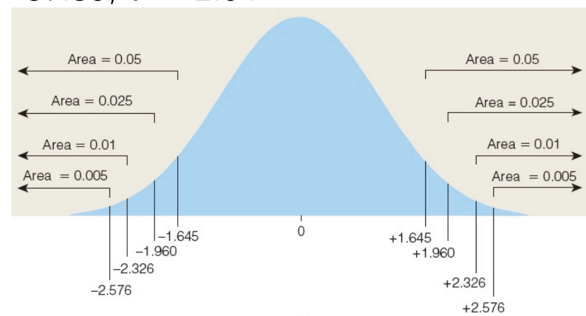
***A Closer Look:** As z gets **more** extreme, the tail probability gets __________.*

***Looking Ahead:** When we perform inference in Part 4, some key decisions will be based on how a normal probability compares to a set value like 0.01 or 0.05.*

Practice: 7.67 p.334

Example: *Practice with 90-95-98-99 Rule*

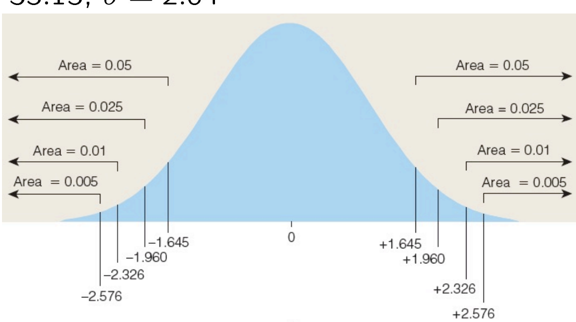
- **Background**: Male chest sizes *X* (in inches) normal; $\mu = 37.35,\ \sigma = 2.64$



- **Question:** P(*X* > 45) is in what range?
- **Response:** 45 has *z* = ____________
  so P(*X* > 45) is between ______ and _______.

Example: *More Practice with 90-95-98-99 Rule*

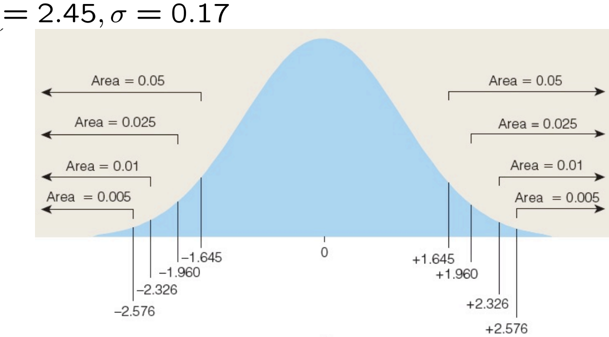
- **Background**: Female chest sizes *X* (in inches) normal; $\mu = 35.15,\ \sigma = 2.64$



- **Question:** P(*X* < 28.8) is in what range?
- **Response:** 28.8 has *z* = ______________
  so P(*X* < 28.8) is between _______ and _______.

Example: *90-95-98-99 Rule, Given Probability*

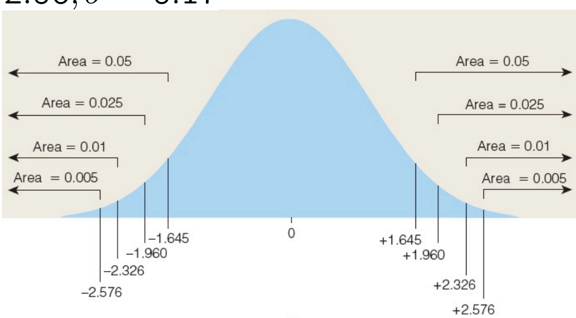
- **Background**: Male ear lengths *X* (in inches) normal; $\mu = 2.45, \sigma = 0.17$



- **Question:** Top 5% are greater than what value?
- **Response:** Top 5% are above *z* = _______
  so *x* = ______________________________

Example: *More Use of Rule, Given Probability*

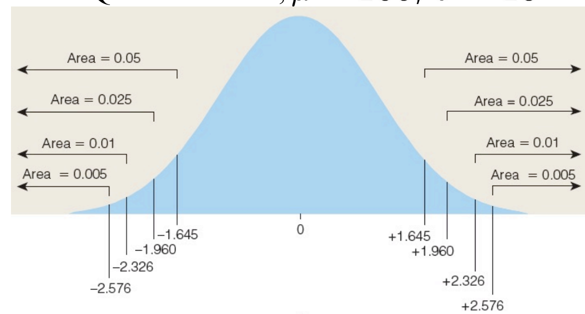
- **Background**: Female ear lengths X (in inches) normal; $\mu = 2.06, \sigma = 0.17$



- **Question:** Bottom 2.5% are less than what value?
- **Response:** Bottom 2.5% are below $z =$ ________
  so $x =$ ______________________________

Example: *Sketching Curve with 90-95-98-99 Rule*

- **Background**: IQs X are normal; $\mu = 100, \ \sigma = 15$.



- **Question:** What does the Rule tell us about the IQ curve?
- **Response:** ___ ___ ___ ___ ___ ___ ___ ___ ___

Lecture Summary
(Normal Random Variables)

- Standardizing/unstandardizing
- Probability problems
  - Find probability given z
  - Find z given probability
  - Find probability given x
  - Find x given probability

Lecture Summary
(Tails of Normal Curve)

- Two forms of inference
  - Interval estimate
  - Test if value is plausible
- 68-95-99.7 Rule and Rule for tails of normal curve
- Reviewing normal probability problems
  - Given x, find probability
  - Given probability, find x
- Focusing on tails of normal curve
  - Standard normal problems
  - Non-standard normal problems