Lecture 18: Chapter 10, Sections 2-3 Inference for Quantitative Variable Finish CI; Hypothesis Test with *t*

- *t*: Other Levels of Confidence; Test vs. CI
- Compare *z* and *t*; *t* Test with Software
- How Large is "Large" *t*?
- *t* Test with Small *n*
- What Leads to Rejecting Ho; Errors, Multiple Tests
- Relating Confidence Interval and Test Results, Review

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-3)
  - Displaying and Summarizing (Lectures 3-8)
  - Probability (discussed in Lectures 9-14)
  - Statistical Inference
    - 1 categorical (discussed in Lectures 14-16)
    - 1 quantitative (began L16) : z CI, z test, t CI, t test
    - categorical and quantitative
    - 2 categorical
    - 2 quantitative

t Intervals at Other Levels of Confidence

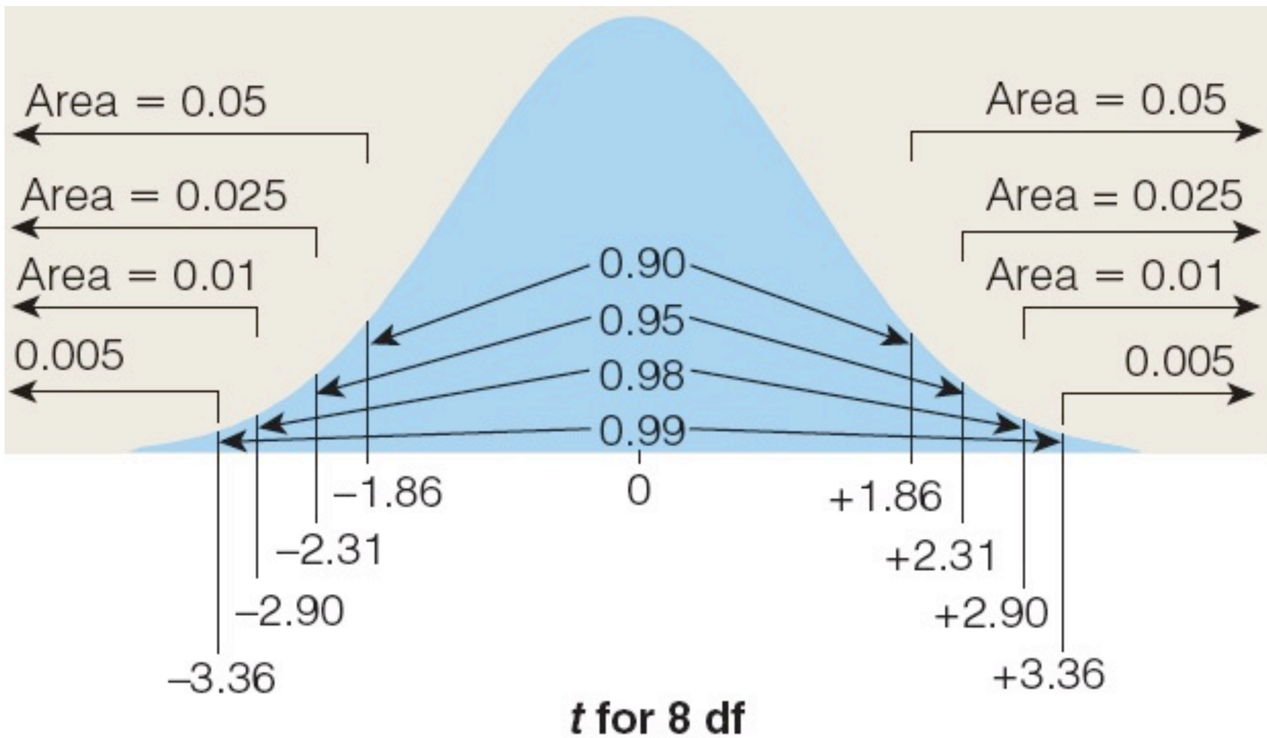
Confidence Level

	90%	95%	98%	99%
***z* (infinite *n*)**	1.645	1.960 or 2	2.326	2.576
***t*: *df* = 19 (*n* = 20)**	1.73	2.09	2.54	2.86
***t*: *df* = 11 (*n* = 12)**	1.80	2.20	2.72	3.11
***t*: *df* = 3 (*n* = 4)**	2.35	3.18	4.54	5.84

- Lower confidence→smaller *t* multiplier
- Higher confidence→larger *t* multiplier
- Table excerpt→at any given level, $t > z$ mult→ using s not σ gives wider interval (less info)
- *t* multipliers decrease as df (and *n*) increase

Example: *Intervals at Other Confidence Levels*

- **Background**: Random sample of shoe sizes for 9 college males: 11.5, 12.0, 11.0, 15.0, 11.5, 10.0, 9.0, 10.0, 11.0



- **Question:** What is *t* multiplier for 99% confidence?
- **Response:**

Example: *Intervals at Other Confidence Levels*

- **Background**: Random sample of shoe sizes for 9 college males: 11.5, 12.0, 11.0, 15.0, 11.5, 10.0, 9.0, 10.0, 11.0 We can produce 95% confidence interval:

$$11.2 \pm 2.31 \frac{1.7}{\sqrt{9}} = (9.9, 12.5)$$

- **Question:** What would 99% confidence interval be, and how does it compare to 95% interval? (Use the fact that *t* multiplier for 8 df, 99% confidence is 3.36.)
- **Response:** 99% interval interval is

  - Width ______________ for 95%
  - Width ______________ for 99%

Practice: 10.60b p.512

Summary of *t* Confidence Intervals

Confidence interval for μ is $\bar{x} \pm \text{multiplier}\left(\frac{s}{\sqrt{n}}\right)$

where multiplier depends on

- df: smaller for larger *n*, larger for smaller *n*
- level: smaller for lower level, larger for higher

Note: margin of error is larger for larger *s*.

→ interval **narrower** for

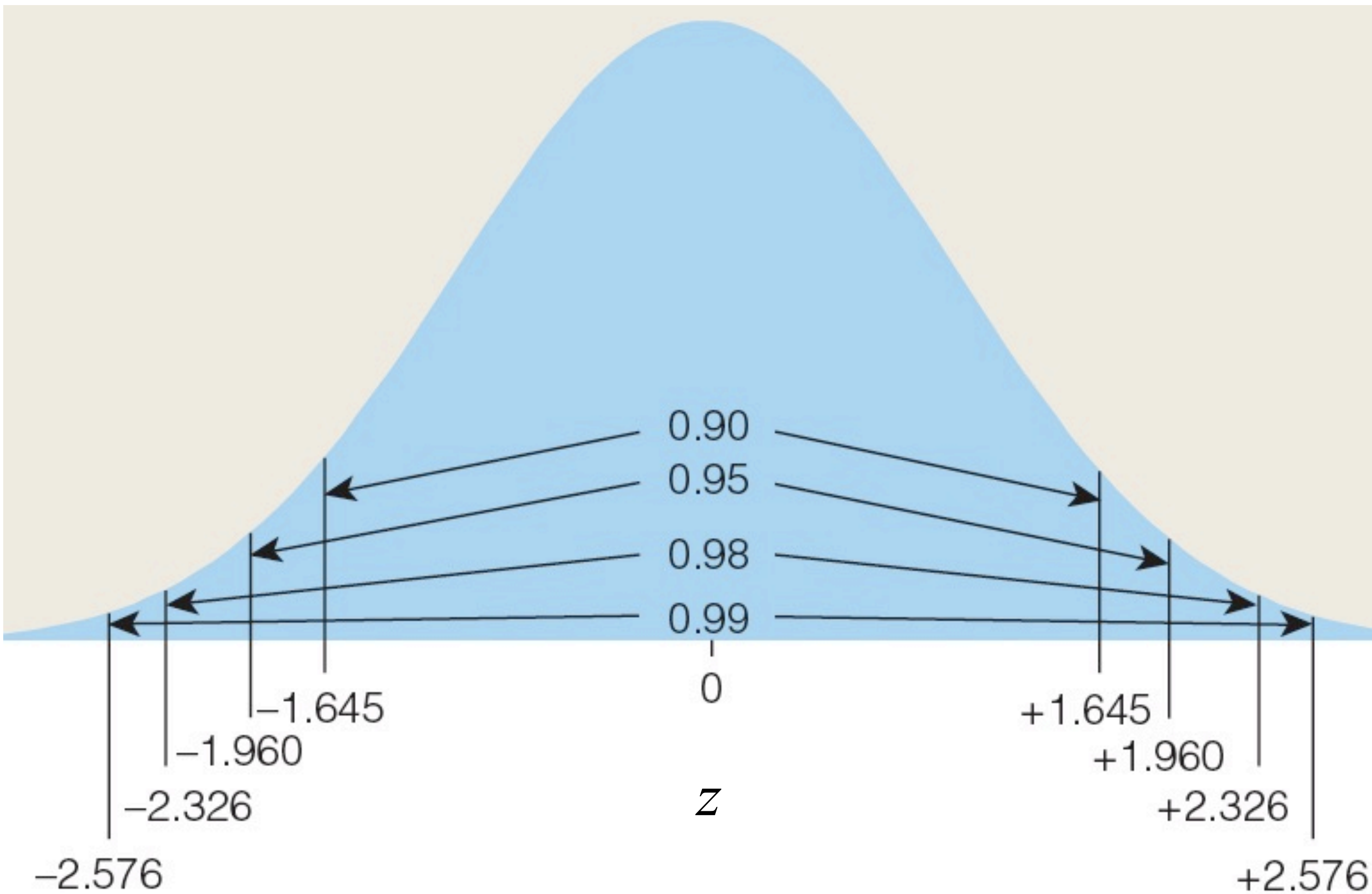
- larger *n* (via df *and* $\sqrt{n}$ in denominator)
- lower level of confidence
- smaller s.d. (distribution with less spread)

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (already discussed)
  - Displaying and Summarizing (already discussed)
  - Probability
  - Statistical Inference
    - 1 categorical
    - 1 quantitative: z CI, z test, t CI, t test
    - categorical and quantitative
    - 2 categorical
    - 2 quantitative

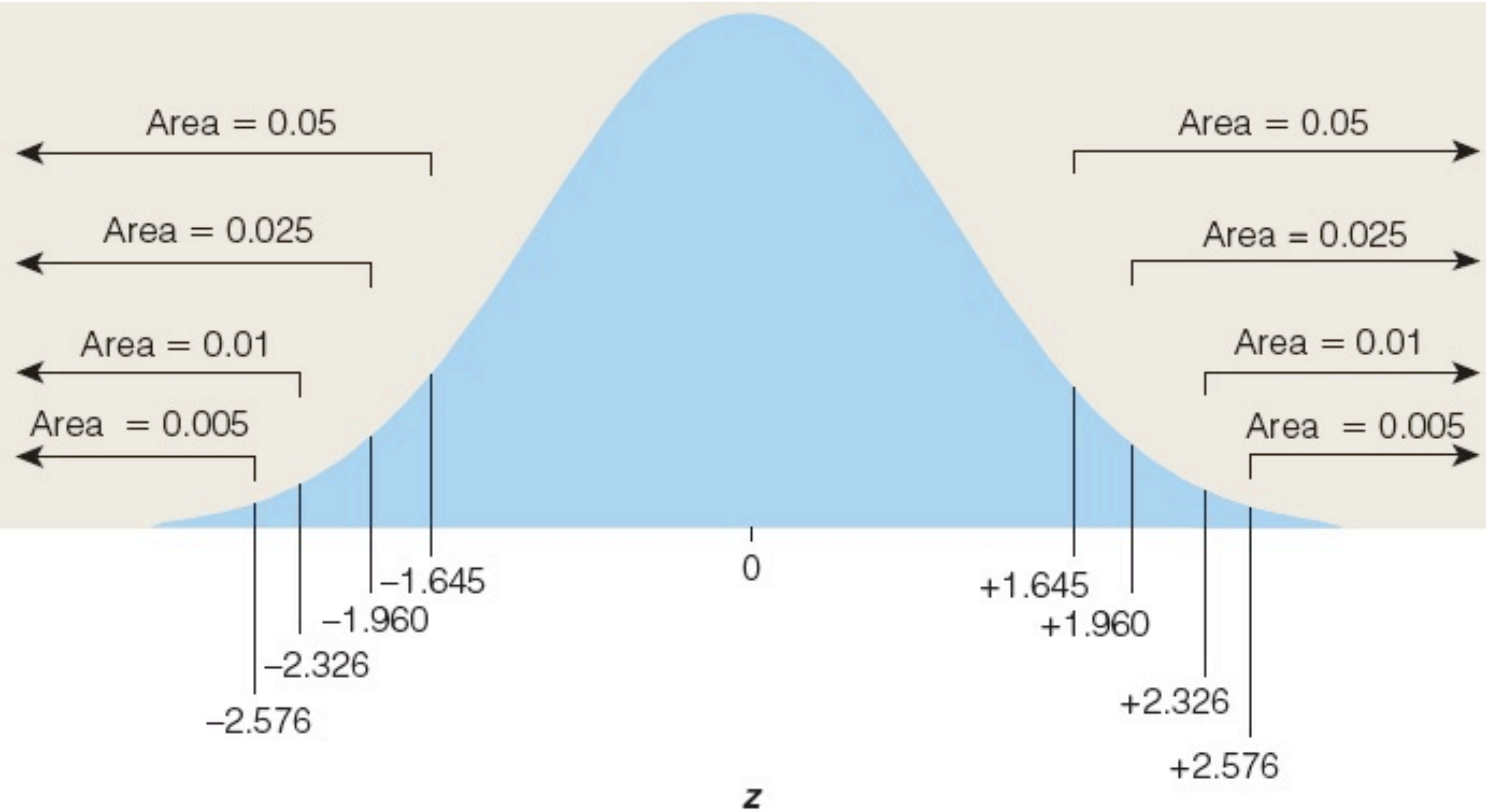
From z Confidence Intervals to Tests *(Review)*

For confidence intervals, used "inside" probabilities.



From z Confidence Intervals to Tests *(Review)*

For hypothesis tests, used "outside" probabilities.



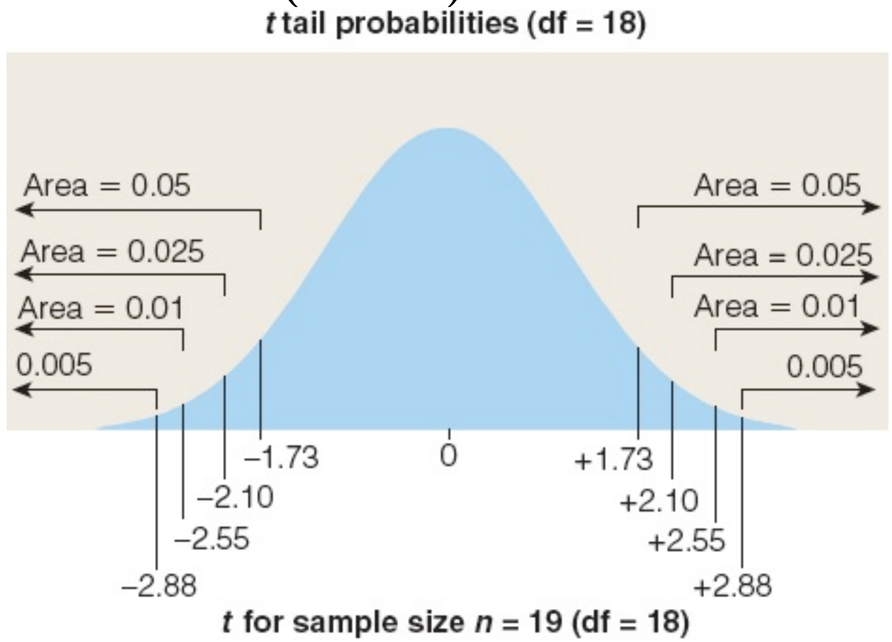
From *t* Confidence Intervals to Tests

Confidence interval: use multiplier for t dist, n-1 df

Hypothesis test: P-value based on tail of t dist, n-1 df

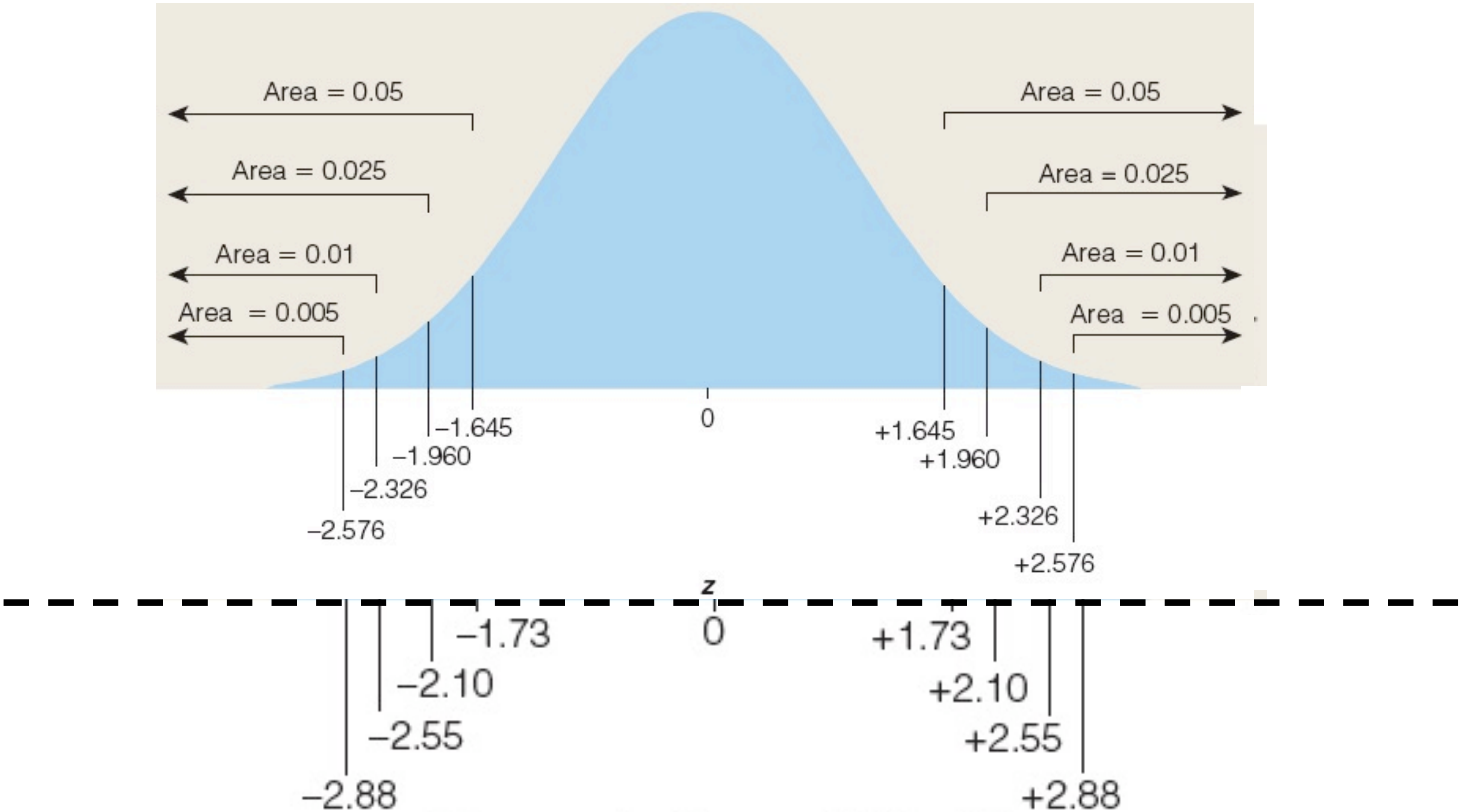
Example: *Hypothesis Test: t vs. z*

- **Background**: Suppose one test with very large n has z =2; another test with n=19 (18 df) has t =2.



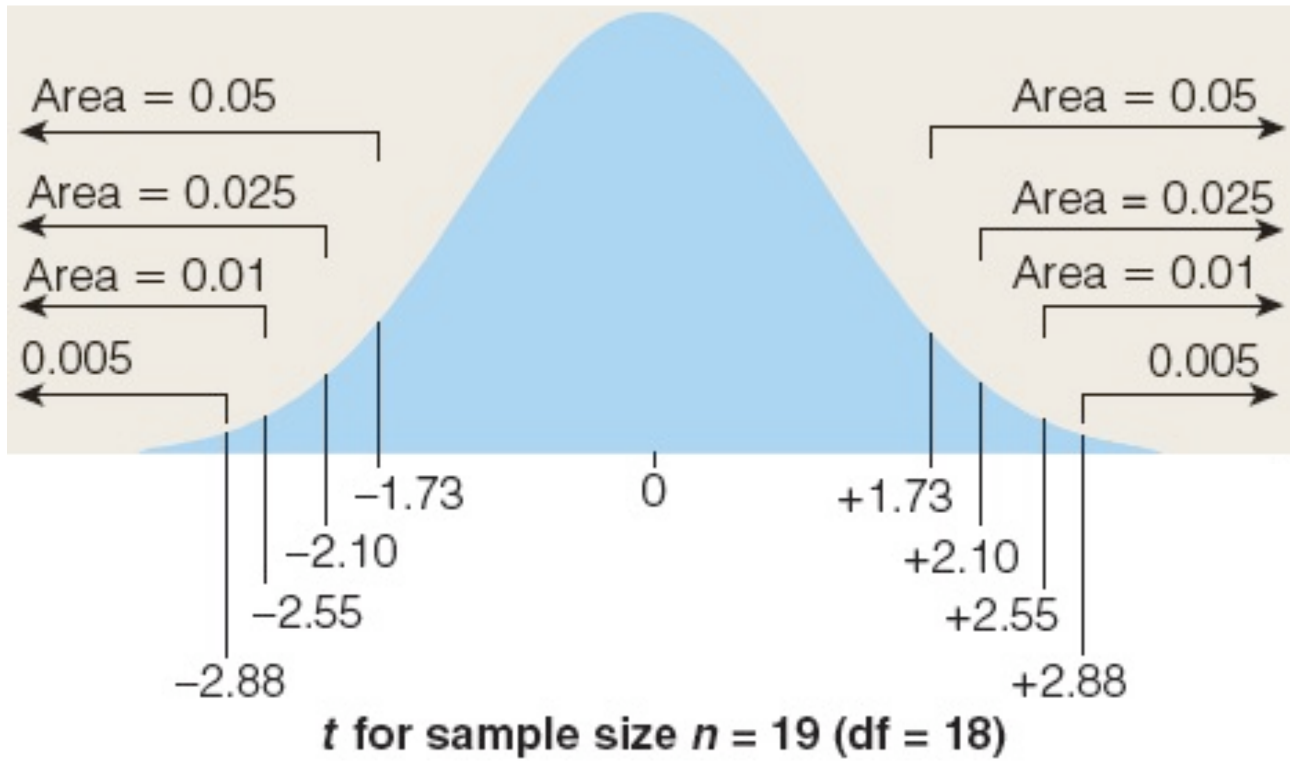
- **Question:** How do P-values compare for z and t? (Assume alternative is "greater than".)
- **Response:** 90-95-98-99 Rule→z P-value ________________.
  t curve for 18 df→t P-value ________________

Comparing Critical Values, *z* with ***t* for 18 df**



Example: *Hypothesis Test: t vs. z*

- **Background**: Consider t curve for 18 df.



- **Question:** Would a value of $t = 3$ be considered extreme?
- **Response:** ____; $|t|$ for 18 df almost never exceeds _____.

Example: *t Test (by Hand)*

- **Background**: Wts. of 19 female college students:

110 110 112 120 120 120 125 125 130 130 132 133 134 135 135 135 145 148 159

- **Question:** Is pop. mean 141.7 reported by NCHS plausible, or is there evidence that we've sampled from pop. with lower mean (or that there is bias due to under-reporting)?
- **Response:**

1. Pop.≥10(19); shape of weights close to normal→n=19 OK
2. $\bar{x} = 129.36, s = 12.82, t =$
3. P-value =__________ small because $|t|$ more extreme than 3 can be considered unusual for most n; in particular, for 18 df, P(t<-2.88) is less than 0.005.
4. Reject H_0? ____ Conclude?

 Practice: 10.68 p.590

Example: *t Test (with Software)*

- **Background**: Random sample of shoe sizes for 9 college males: 11.5, 12.0, 11.0, 15.0, 11.5, 10.0, 9.0, 10.0, 11.0
- **Question:** Can we believe mean shoe size of all college males is 11?
- **Response:** Use software: enter values, specify proposed mean 11 and "not-equal" alternative.

```
One-Sample T: Shoe
Test of mu = 11 vs mu not = 11
Variable             N       Mean      StDev   SE Mean
Shoe                 9     11.222      1.698     0.566
Variable               95.0% CI              T       P
Shoe             (   9.917,  12.527)      0.39   0.705
```

Note: small sample is OK because shoe sizes are normal.

Is *t* large? ____ Is *P*-value small? ____

Believe population mean=11? ____

Practice: 10.22c-e p.498

How Large is "Large" for *z* Statistic *(Review)*

68-95-99.7 Rule→guidelines for "unusual" *z*:

	common	not unusual	borderline	quite unusual	extremely improbable
$\|z\|$	less than 1	closer to 1 than to 2	around 2	considerably greater than 2	greater than 3

Values near **2** may be considered borderline.

How Large is “Large” for *t* Statistic

Excerpts from *t* table→

- May call values near **2** borderline for df>10
- May call values near **3** borderline for df< 5

Confidence Level

	90%	95%	98%	99%
z **(infinite** ***n*****)**	1.645	1.960 or 2	2.326	2.576
t**:** ***df*** **= 19 (*****n*** **= 20)**	1.73	2.09	2.54	2.86
t**:** ***df*** **= 11 (*****n*** **= 12)**	1.80	2.20	2.72	3.11
t**:** ***df*** **= 3 (*****n*** **= 4)**	2.35	3.18	4.54	5.84

Use of *t* with Very Small Samples

Can assume shape of $\bar{X}$ for random samples of **any** size *n* is approximately normal if graph of sample data appears normal.

Normal population → $\dfrac{\bar{x}-\mu}{s/\sqrt{n}}$ is exactly *t*

Example: *t Test with Small n*

- **Background**: Random sample of 4 Math SATs (570, 580, 640, 760) have mean 637.5, s.d. 87.3.
- **Question:** Do they represent population with mean greater than 500? (Use cut-off alpha=0.05.)
- **Response:** *n* is small but *t* procedure is OK because SATs are normal:

```
One-Sample T: MathSAT
Test of mu = 500 vs mu > 500
Variable          N      Mean     StDev   SE Mean
MathSAT           4     637.5      87.3      43.7
Variable      95.0% Lower Bound       T         P
MathSAT                 534.7      3.15     0.026
```

P-value = ______________________

Using cutoff 0.05, small enough to reject H_0? _____

Conclude population mean > 500? _____

Practice: 10.20 p.490

Example: *t Test with Small n, 2-Sided Alternative*

- **Background**: Random sample of 4 Math SATs (570, 580, 640, 760) have mean 637.5, s.d. 87.3.
- **Question:** Do they represent population with mean different from 500? (Use cut-off alpha=0.05.)
- **Response:** Now use ≠ alternative:

```
One-Sample T: MathSAT
Test of mu = 500 vs mu not = 500
Variable          N      Mean      StDev   SE Mean
MathSAT           4     637.5       87.3      43.7
Variable            95.0% CI            T        P
MathSAT        (   498.6,    776.4)     3.15   0.051
```

P-value = ____________________

Using cutoff 0.05, small enough to reject H_0? ______

Conclude population mean ≠ 500? ____________________

A Closer Look: *t near 3 can be considered borderline for very small n.*

One-sided vs. Two-sided Results

- Tested $H_o : \mu = 500$ vs. $H_a : \mu > 500$

  P-value=0.026→rejected H_0
- Tested $H_o : \mu = 500$ vs. $H_a : \mu \neq 500$

  P-value=0.051→ did not reject H_0

Suspecting mean > 500 got us significance

Example: *Concerns about 2-Sided Test*

- **Background**: Random sample of 4 Math SATs (570, 580, 640, 760) have mean 637.5, s.d. 87.3. The *t* test failed to reject H_0: μ =500 vs. 2-sided H_a because *P*-value=0.051.
- **Question:** Should we believe 500 is a plausible value for the population mean?
- **Response:** Several concerns:
  - If these were students admitted to university, should have used ">" alternative.
  - n=4 very small→vulnerable to Type____ Error
  - MUST we stick to 0.05 as cut-off for small *P*-value?____
  - Maybe could have found out σ and done ___test instead.
  - Does μ=500 seem plausible when smallest value is 570? ___

Factors That Lead to Rejecting H_0

Statistically significant data produce *P*-value small enough to reject H_0. *t* plays a role:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{(\bar{x} - \mu_0)\sqrt{n}}{s}$$

Reject H_0 if *P*-value small; if $|t|$ large; if…

- Sample mean far from μ_0
- Sample size n large
- Standard deviation s small

Factors That Lead to *Not* Rejecting H_0

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{(\bar{x} - \mu_0)\sqrt{n}}{s}$$

Can't reject H_0 if P-value not small; if $|t|$ not large; if…

- Sample mean close to μ_0
- Sample size n small
- Standard deviation s large

Types I and II Error

- Small *n* can lead to Type II Error (Fail to reject false H_0) *(Sampled only 4 SATs.)*
- Multiple tests can lead to Type I Error (Reject true H_0)…

Example: *Multiple Tests*

- **Background**: Suppose all Verbal SATs have mean 500. Sample n=20 scores each in 100 schools, each time test $H_o : \mu = 500$ vs. $H_a : \mu < 500$.
- **Question:** If we reject H_0 in 4 of those schools, can we conclude that mean Verbal SAT in those 4 schools is significantly lower than 500?
- **Response:** If we set 0.05 as cut-off for small *P*-value then long-run probability of committing Type I Error (rejecting true H_0) is _____.

  Even if all 100 schools actually have mean 500, by chance alone some samples will produce a sample mean low enough to reject H_0 ____% of the time.

Practice: 10.26 p.500

Confidence Interval and Hypothesis Test Results

- Confidence Interval: range of plausible values
- Hypothesis Test: decides if a value is plausible

Informally,

- If μ_o is in confidence interval, don't reject $H_o : \mu = \mu_o$
- If μ_o is outside confidence interval, reject $H_o : \mu = \mu_o$

Example: *Relating Confidence Interval to Test*

- **Background**: Consider these confidence intervals:
  - 95% CI for pop mean earnings (3171, 4381)
  - 95% CI for pop mean shoe size (9.9, 12.5)
  - 95% CI for pop mean Math SAT (498.6, 776.4)
- **Question:** What to conclude about hypotheses...?
  - $H_o: \mu = 5000$ vs. $H_a: \mu < 5000$
  - $H_o: \mu = 11$ vs. $H_a: \mu \neq 11$
  - $H_o: \mu = 500$ vs. $H_a: \mu \neq 500$
- **Response:** Check if proposed mean is in interval:
  - Reject H_0? ______________________________
  - Reject H_0? ______________________________
  - Reject H_0? ______________________________

Practice: 10.27c,e p.500

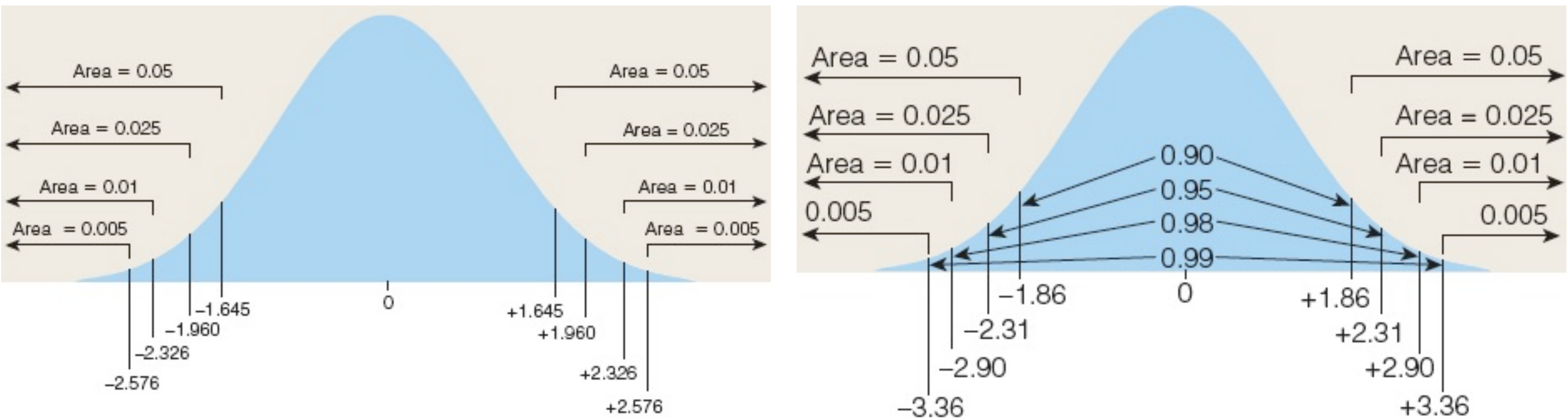
Examples: *Reviewing z and t Tests (#1-#4)*

- **Background**: Sample mean and standard deviation of amount students spent on textbooks in a semester is being used to test if the mean for all students exceeds \$500. The null hypothesis will be rejected if the *P*-value is less than 0.01. We want to draw conclusions about mean amount spent by all students at a particular college.

Looking Back: *If the sample is biased, or n is too small to guarantee $\bar{X}$ to be approximately normal, neither z nor t is appropriate. Otherwise, use **z** if population standard deviation is known or n is large. Use **t** if population standard deviation is unknown and n is small.*

Example: *Reviewing z and t Tests (#1)*

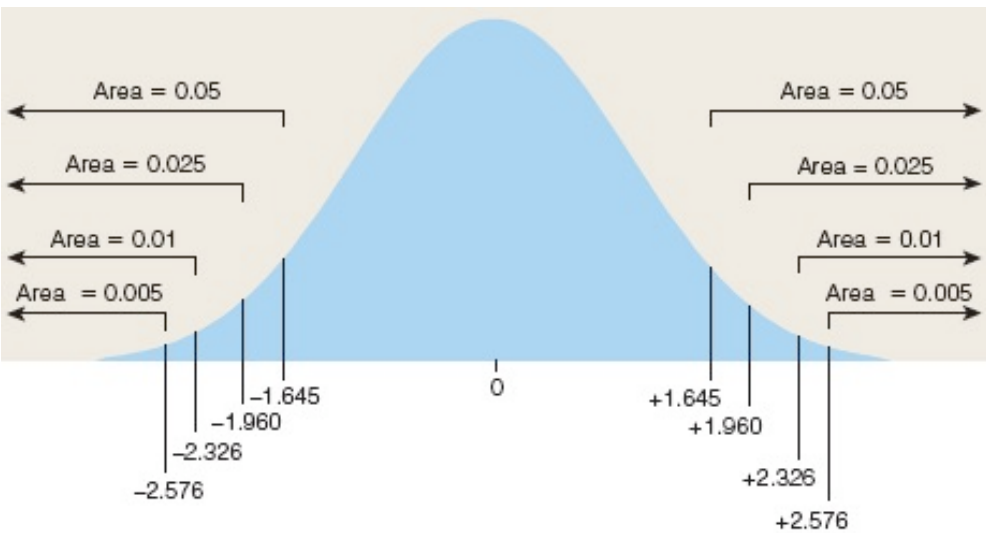
- **Background**: Sample mean and s.d. of textbook costs are used to test if $\mu > 500$ (reject H_0 if P-value < 0.01). Refer to *z* (on left) or *t* for 8 df (on right) or neither.

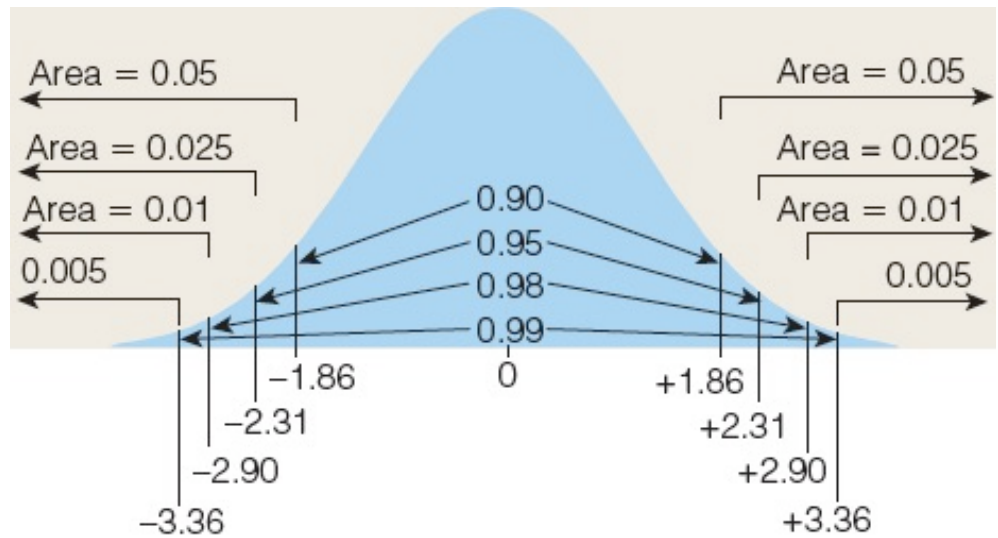


- **Question:** What do we conclude if a **representative** sample of **9** students have t=+2.5? There is an **outlier** in the data set.
- **Response:** ____________________

Example: *Reviewing z and t Tests (#2)*

- **Background**: Sample mean and s.d. of textbook costs are used to test if $\mu > 500$ (reject H_0 if P-value < 0.01). Refer to z (on left) or t for 8 df (on right) or neither.

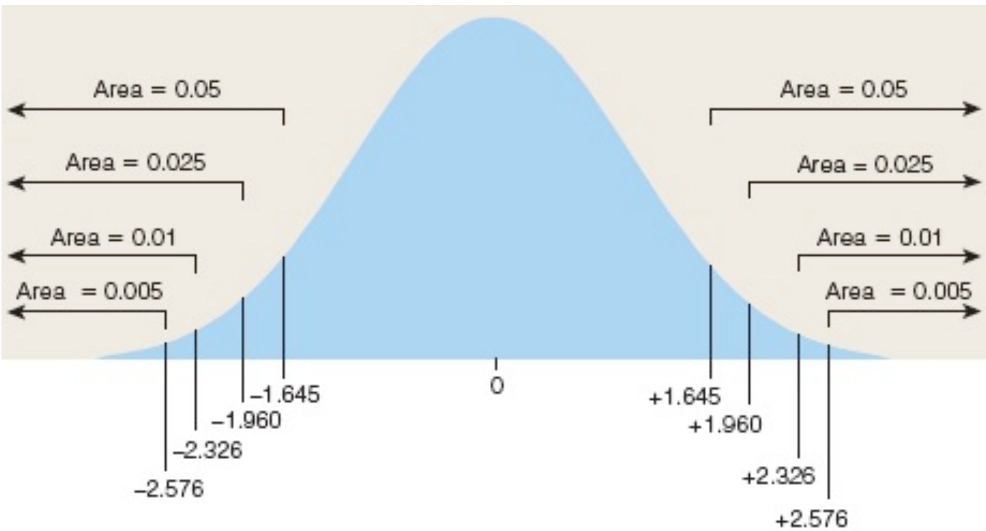


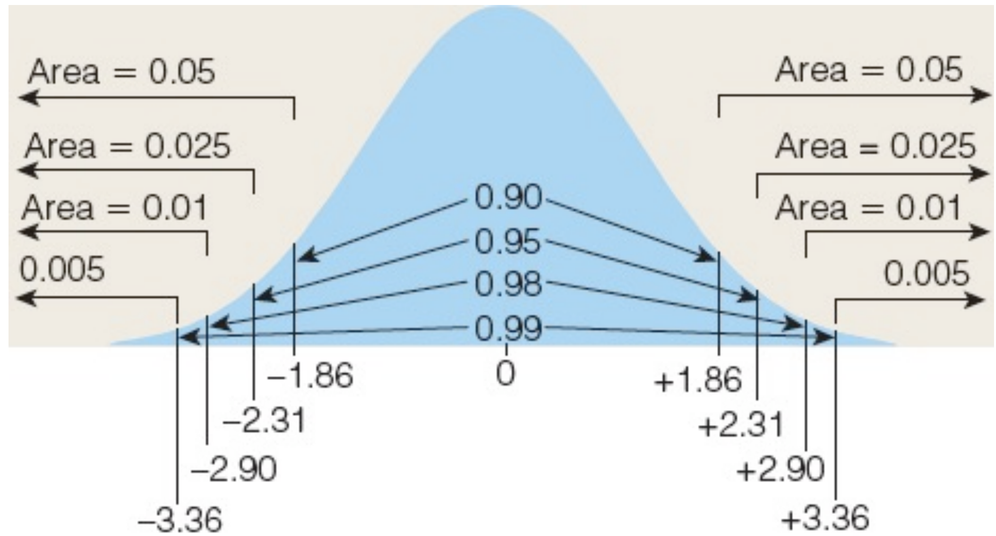


- **Question:** What do we conclude if a **representative** sample of **9** students have t=+2.5? The data set appears **normal**.
- **Response:** ____ H_0__

Example: *Reviewing z and t Tests (#3)*

- **Background**: Sample mean and s.d. of textbook costs are used to test if $\mu > 500$ (reject H_0 if P-value < 0.01). Refer to z (on left) or t for 8 df (on right) or neither.

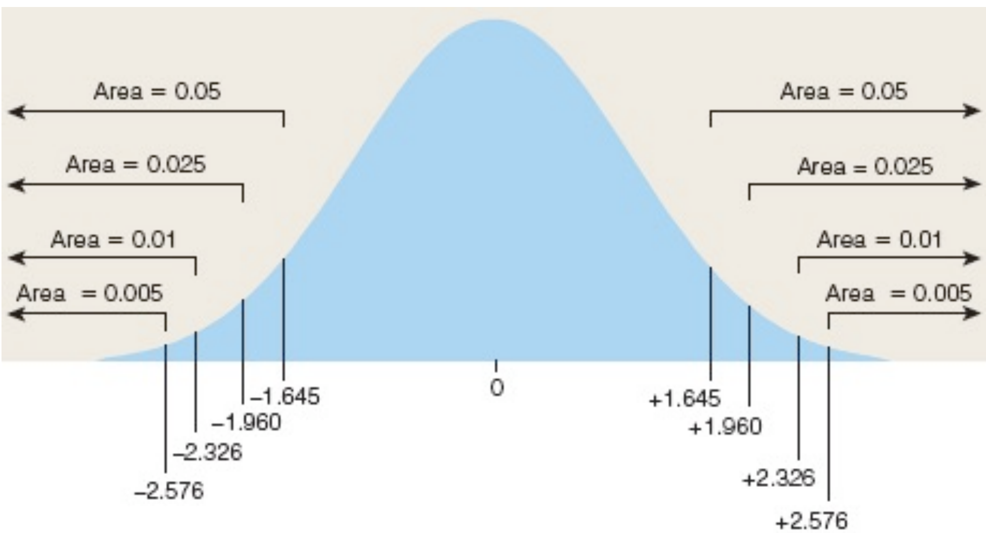


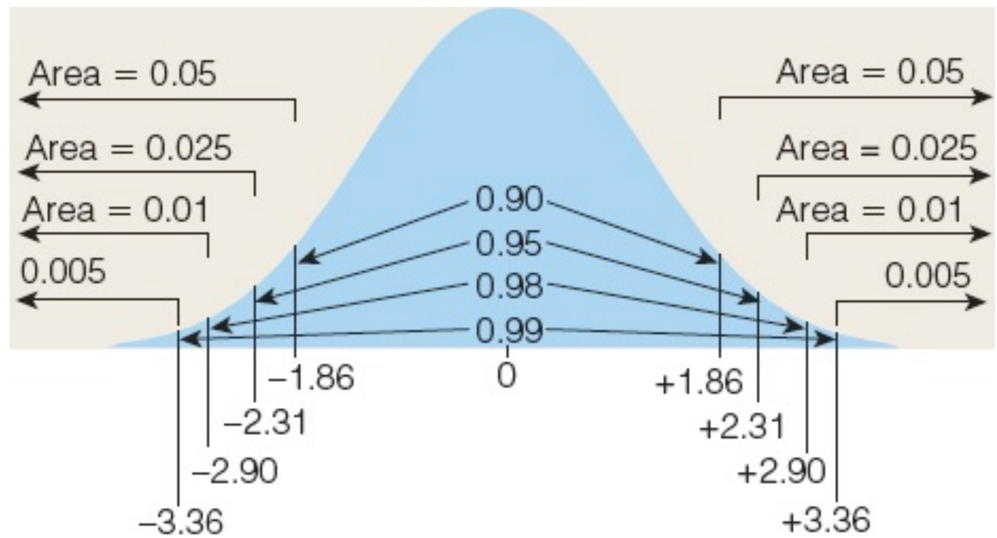


- **Question:** What do we conclude if a **representative** sample of **90** students have t=+2.5? There is an outlier in the data set.
- **Response:** ____________________

Example: *Reviewing z and t Tests (#4)*

- **Background**: Sample mean and s.d. of textbook costs are used to test if $\mu > 500$ (reject H_0 if P-value < 0.01). Refer to z (on left) or t for 8 df (on right) or neither.





- **Question:** What do we conclude if a sample of **90 biology majors** have t=+2.5? The data set appears **normal**.
- **Response:** ______________________

Lecture Summary

(Inference for Means: t Confidence Intervals)

- Other levels of confidence
- from confidence interval to hypothesis test
- *t* test by hand

Lecture Summary
(Inference for Means: t Hypothesis Test)

- Comparing *z* and *t* distributions
- *t* test with software
- How large is "large" *t*?
- *t* test with small *n* (one-sided or two-sided alternative)
- Factors that lead to rejecting null hypothesis
- Type I or II Error; multiple tests
- Relating confidence interval and test results
- Examples for review