Lecture 18: more Chapter 7, Section 3 Continuous Random Variables; Tails of the Normal Curve

- Preview Two Forms of Inference
- 68-95-99.7 Rule; Rule for Tails (90-95-98-99)
- Standard Normal Tail-Probability Problems
- Non-standard Tail-Probability Problems

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability
    - Finding Probabilities (discussed in Lectures 13-14)
    - Random Variables (introduced in Lecture 15)
      - Binomial (discussed in Lecture 16)
      - Normal
    - Sampling Distributions
  - Statistical Inference

Tails of Normal Curve in Inference

- **Goal:** Perform inference in 2 forms about unknown population proportion or mean:
  - Produce interval that has high probability (such as 90%, 95%, or 99%) of containing unknown population parameter
  - Test if proposed value of population proportion or mean is implausible (low probability---1% or 5%---of sample data)
- **Strategy:** Focus on tails of normal curve, in the vicinity of Z=+2 or Z=-2.

68-95-99.7 Rule for *Z (Review)*

For standard normal *Z*, the probability is

- 68% that *Z* takes a value in interval (-1, +1)
- 95% that *Z* takes a value in interval (-2, +2)
- 99.7% that *Z* takes a value in interval (-3, +3)

Need to fine-tune information for probability at or near 95%.

90-95-98-99 Rule for Standard Normal Z

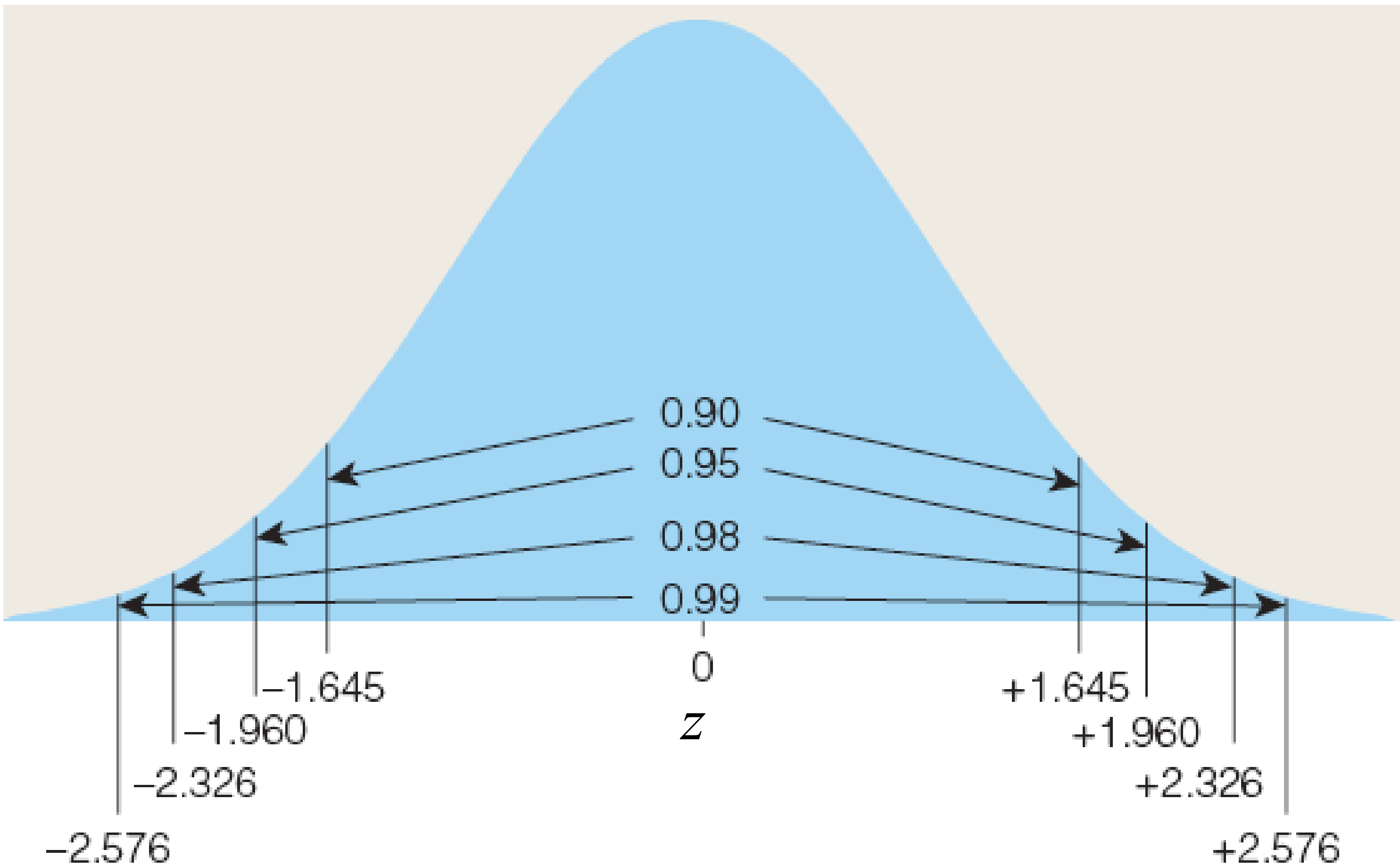
For standard normal Z, the probability is

- 0.90 that Z takes a value in interval (-1.645, +1.645)
- 0.95 that Z takes a value in interval (-1.960, +1.960)
- 0.98 that Z takes a value in interval (-2.326, +2.326)
- 0.99 that Z takes a value in interval (-2.576, +2.576)

Looking Back: *The 68-95-99.7 Rule rounded 0.9544 for 2 s.d.s to 0.95. For exactly 95%, need 1.96 s.d.s.*

90-95-98-99 Rule: "Inside" Probabilities

Looking Ahead: *This will be useful for "confidence intervals".*



90-95-98-99 Rule: "Outside" Probabilities

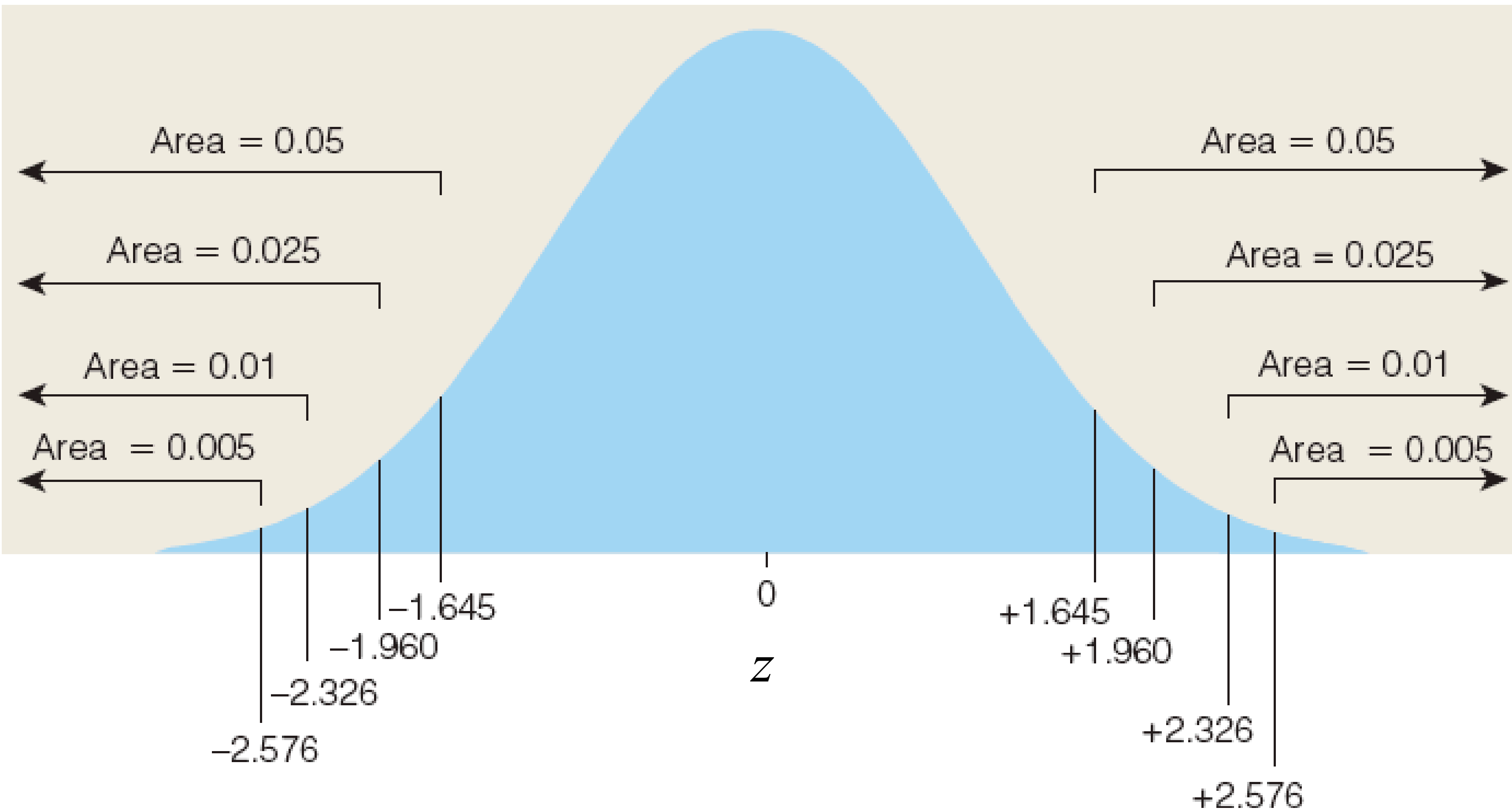
For standard normal Z, the probability is

- 0.05 that $Z < -1.645$ and 0.05 that $Z > +1.645$
- 0.025 that $Z < -1.96$ and 0.025 that $Z > +1.96$
- 0.01 that $Z < -2.326$ and 0.01 that $Z > +2.326$
- 0.005 that $Z < -2.576$ and 0.005 that $Z > +2.576$

Looking Back: *These follow from the inside probabilities, using the fact that the normal curve is symmetric with total area 1.*

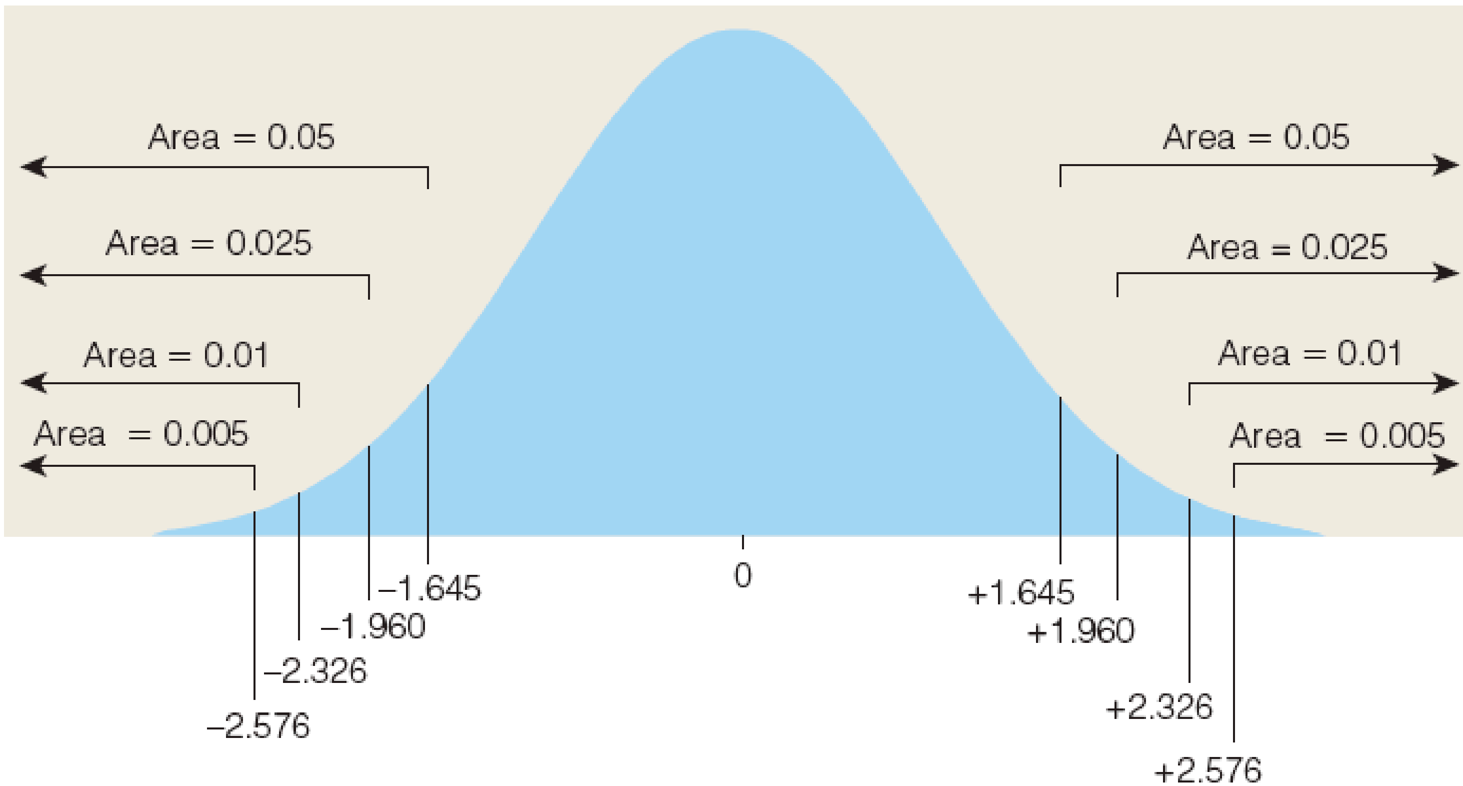
90-95-98-99 Rule: "Outside" Probabilities

Looking Ahead: *This will be useful for "hypothesis tests".*



Example: *Finding Tail Probabilities*

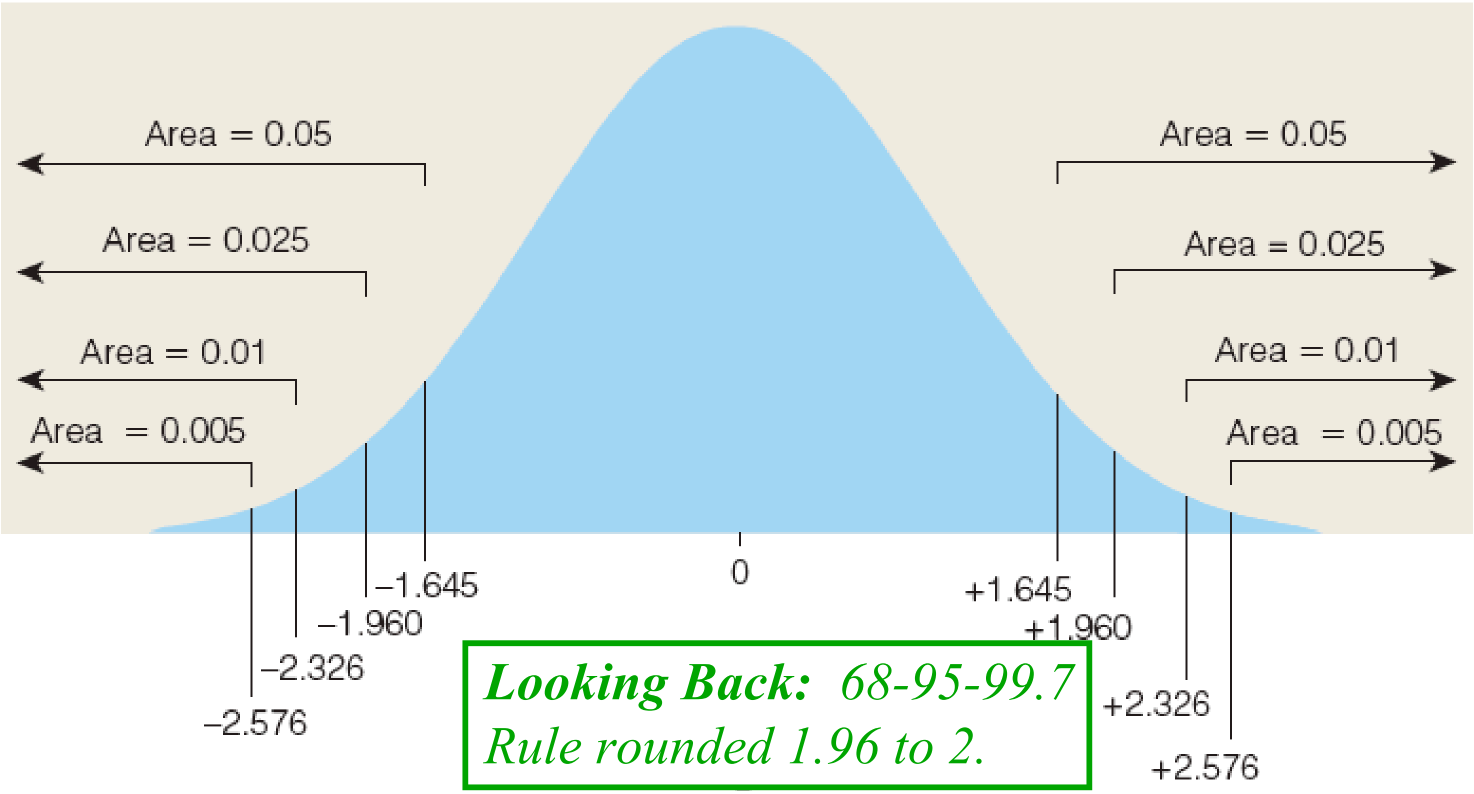
- **Background:** Refer to sketch.



- **Question:** What is P(Z >+2.326)?
- **Response:**

Example: *Finding Tail Probabilities*

- **Background:** Refer to sketch.

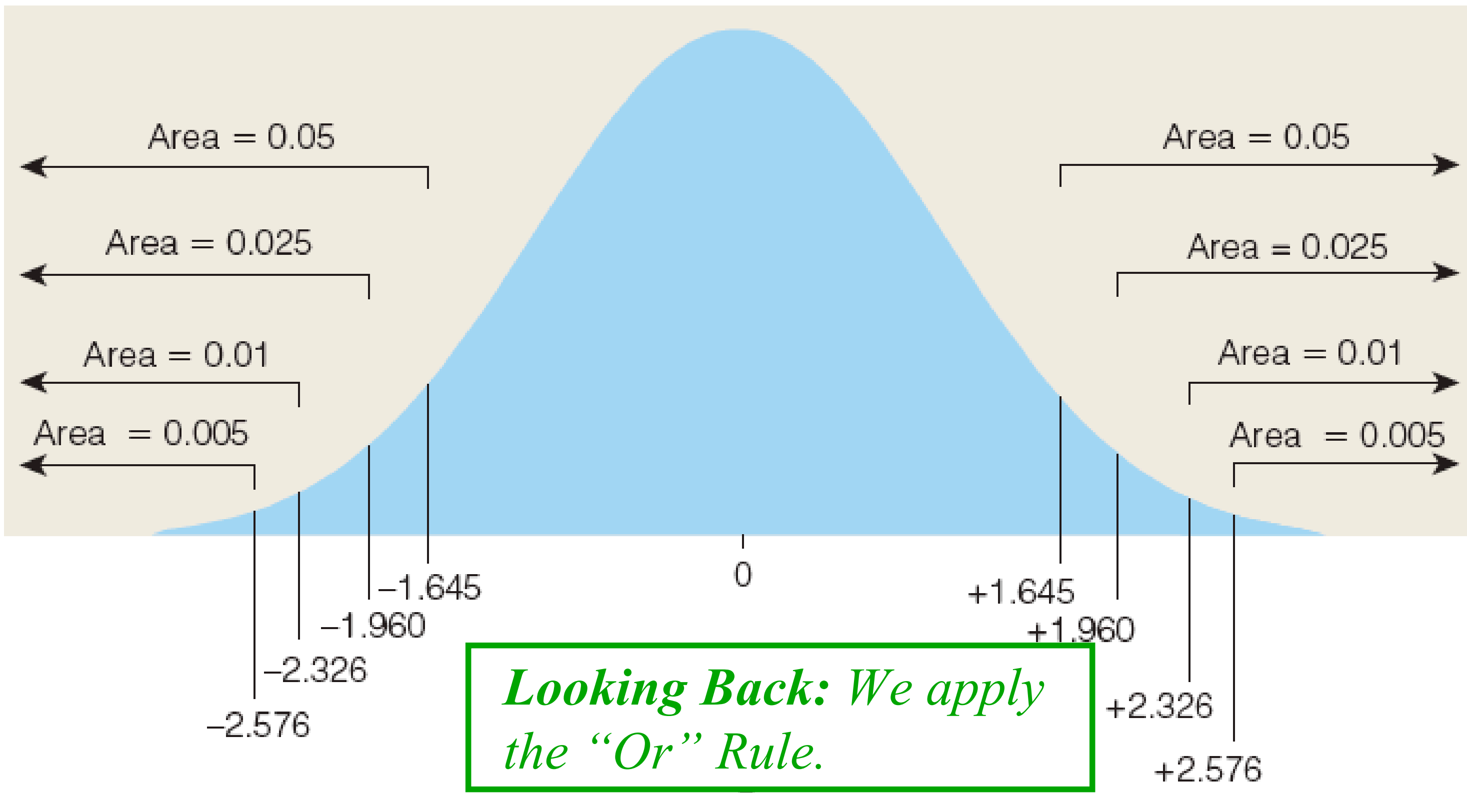


***Looking Back:* 68-95-99.7 Rule rounded 1.96 to 2.**

- **Question:** What is P($Z < -1.96$)?
- **Response:**

Example: *Finding Tail Probabilities*

- **Background:** Refer to sketch.

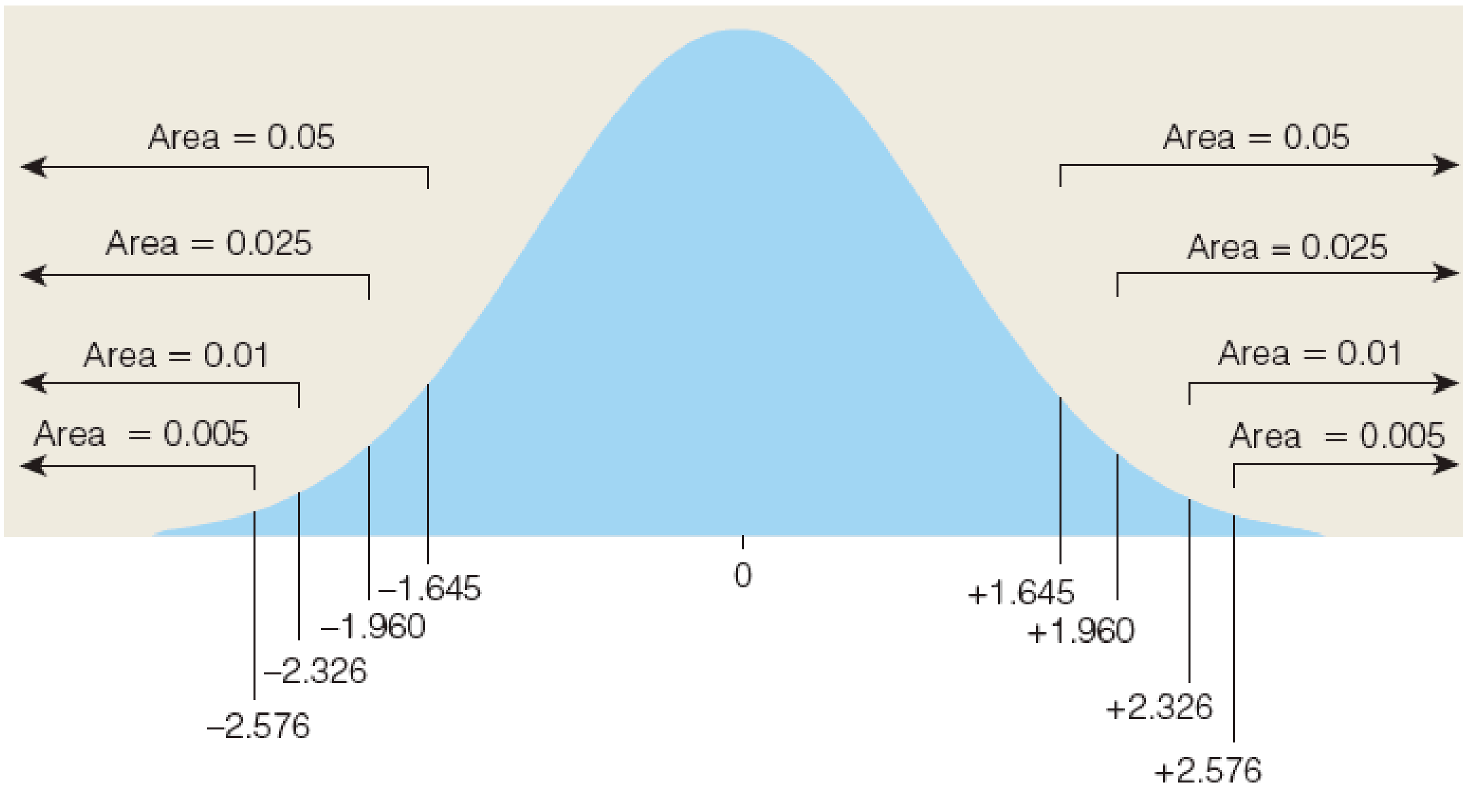


Looking Back: *We apply the "Or" Rule.*

- **Question:** What is $P(|Z| > 1.96)$?
- **Response:**

Example: *Given Probability, Find z*

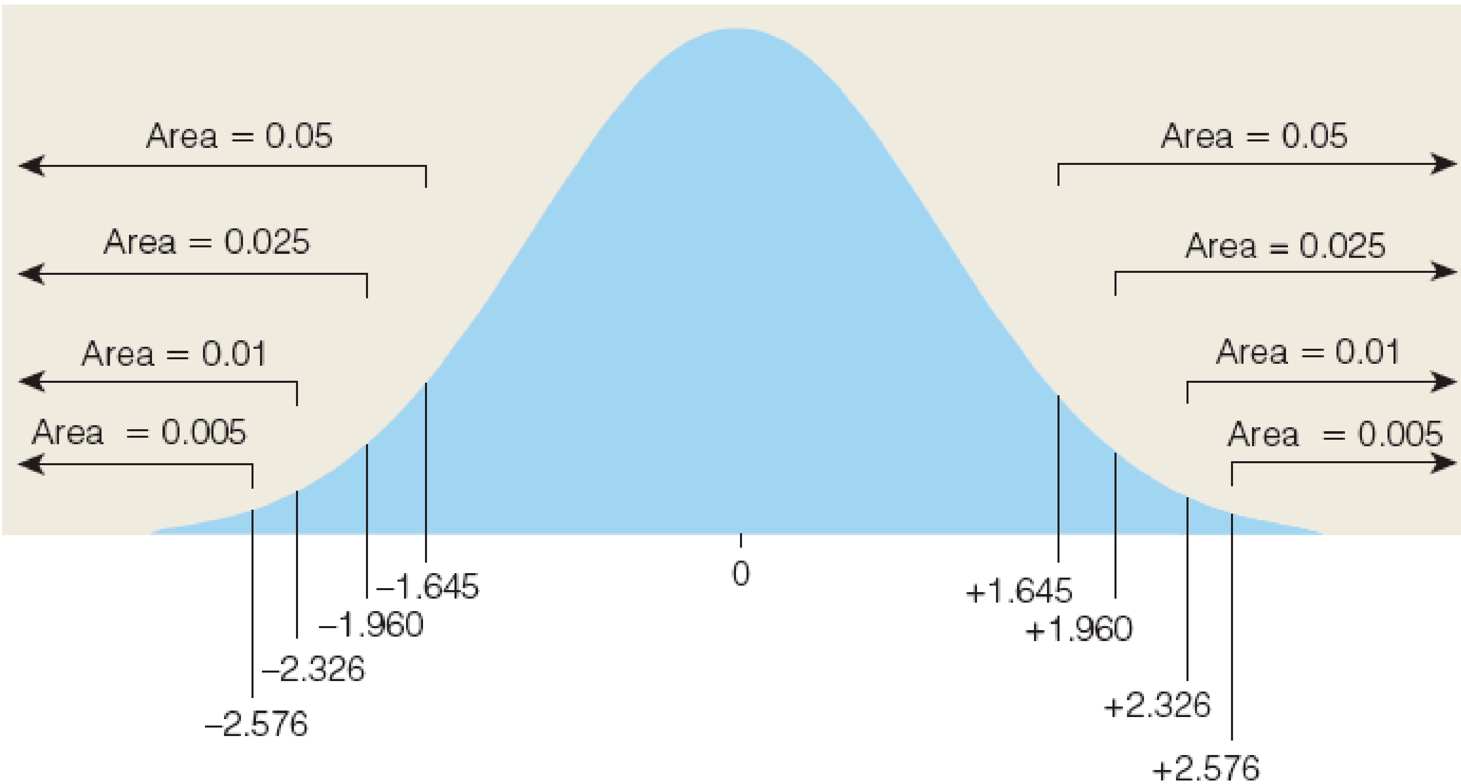
- **Background**: Refer to sketch.



- **Question**: 0.05=P(Z < ?)
- **Response**:

Example: *Given Probability, Find z*

- **Background:** Refer to sketch.



- **Question:** $0.005 = P(Z > ?\ )$
- **Response:**

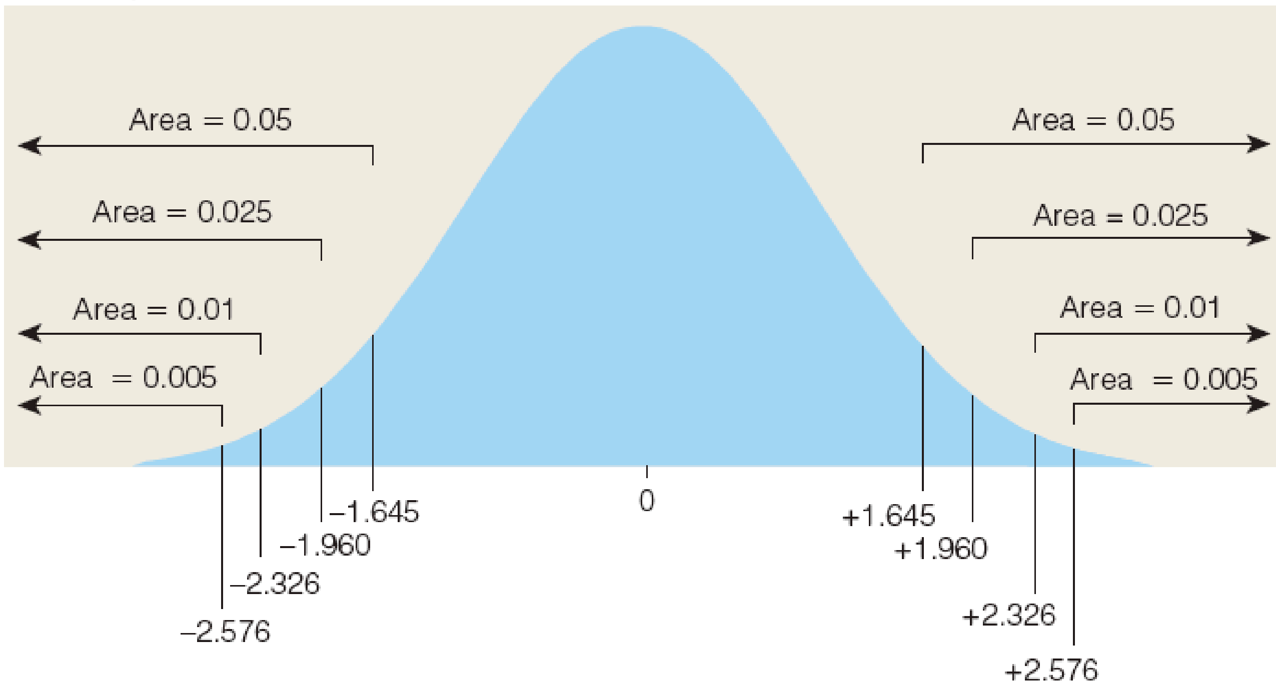
Non-Standard Normal Problems *(Review)*

To find probability, given non-standard normal x, first standardize: $z = \frac{x-\mu}{\sigma}$
then find probability (area under z curve).

To find non-standard x, given probability, find z then unstandardize: $x = \mu + z\sigma$

Example: *Given x, Find Probability*

- **Background**: Women's waist circumference X (in.) normal; $\mu = 32,\ \sigma = 5$.

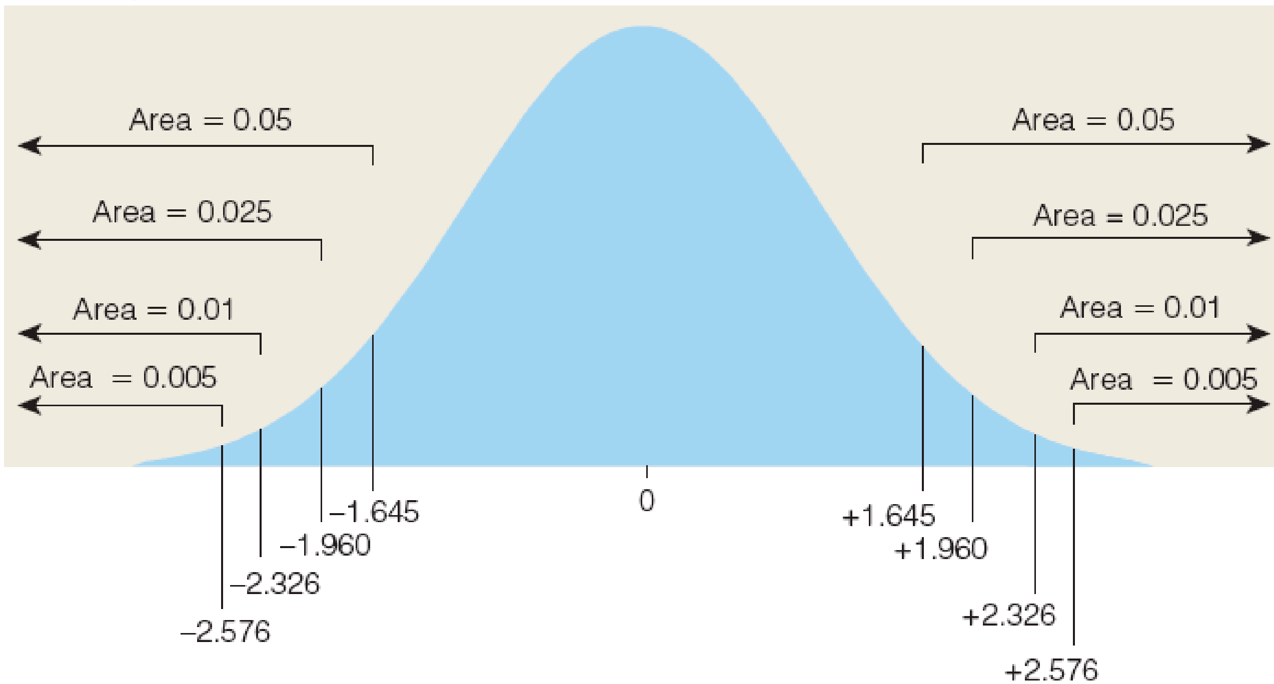


- **Question:** What is P(X > 43)?
- **Response:** $z =$ between and
  so P(X > 43) is between and .

Practice: 7.63 p.333

Example: *Given x, Find Probability*

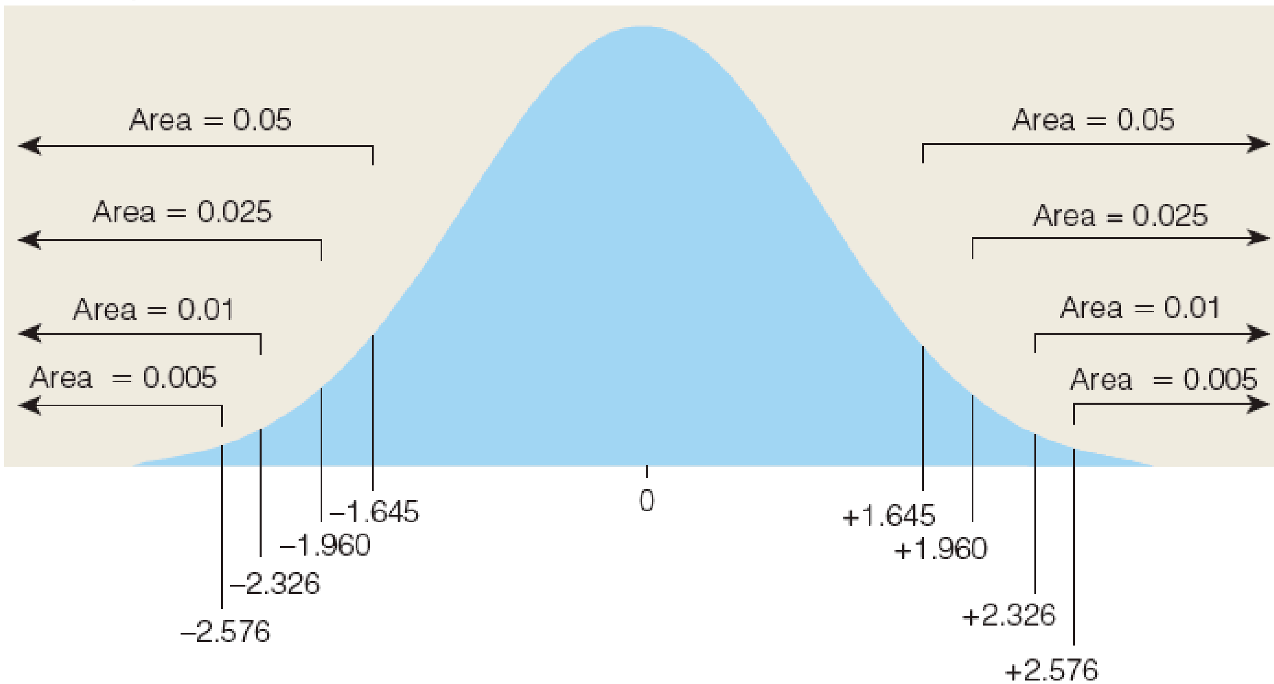
- **Background**: Women's waist circumference X (in.) normal; $\mu = 32,\ \sigma = 5$.



- **Question:** What is P(X < 23)?
- **Response:** $z =$ ______ between ______ and ______ so P(X < 23) is between ______ and ______

Example: *Given x, Find Probability*

- **Background**: Women's waist circumference X (in.) normal; $\mu = 32,\ \sigma = 5$.

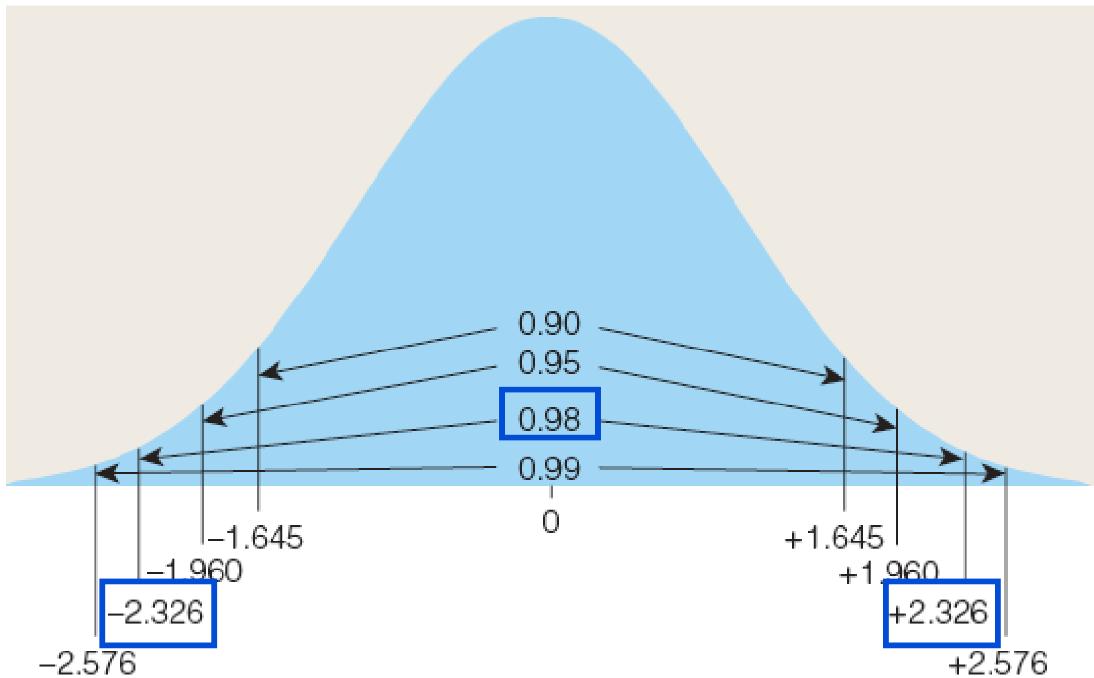


- **Question:** What is P(X > 39)?
- **Response:** $z =$

  so P(X > *39*) is

Example: *Given Probability, Find x*

- **Background**: Math SAT score X for population of college students normal; $\mu = 610$, $\sigma = 72$.

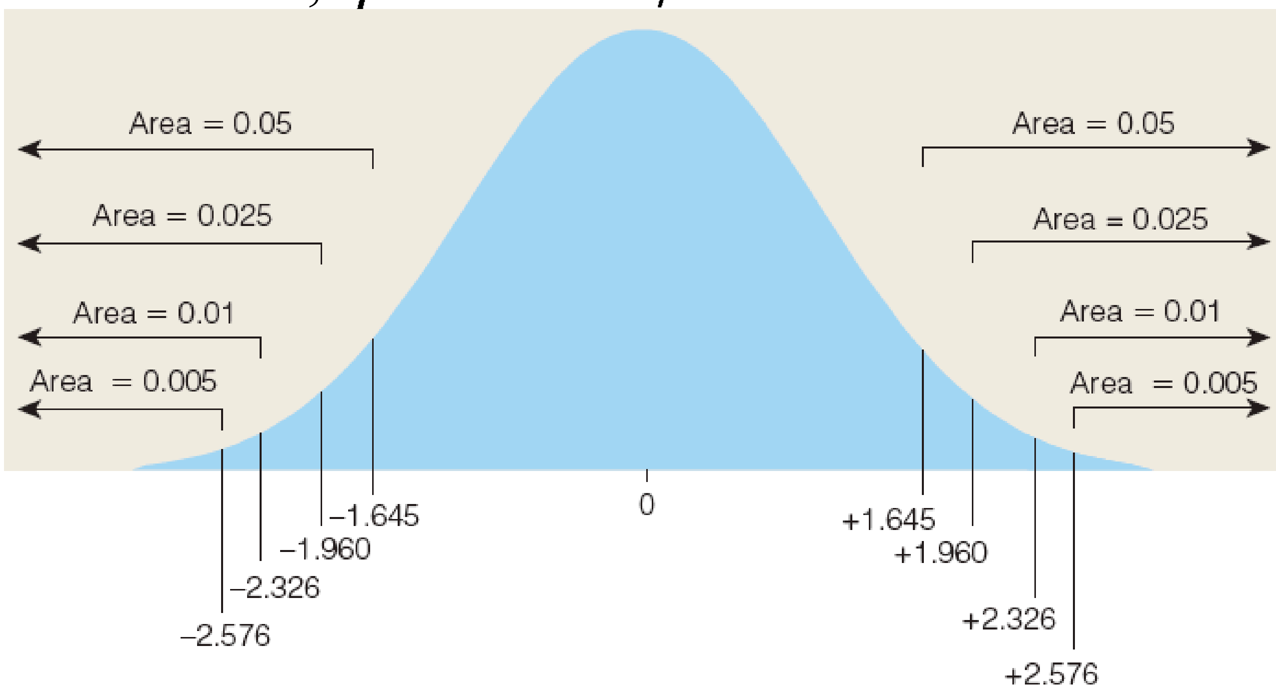


- **Question:** 0.98 is probability of X in what interval?
- **Response:** Prob. 0.98 has z from _ to _ so x is from _ to _

Practice: 7.69 p.334

Example: *Given Probability, Find x*

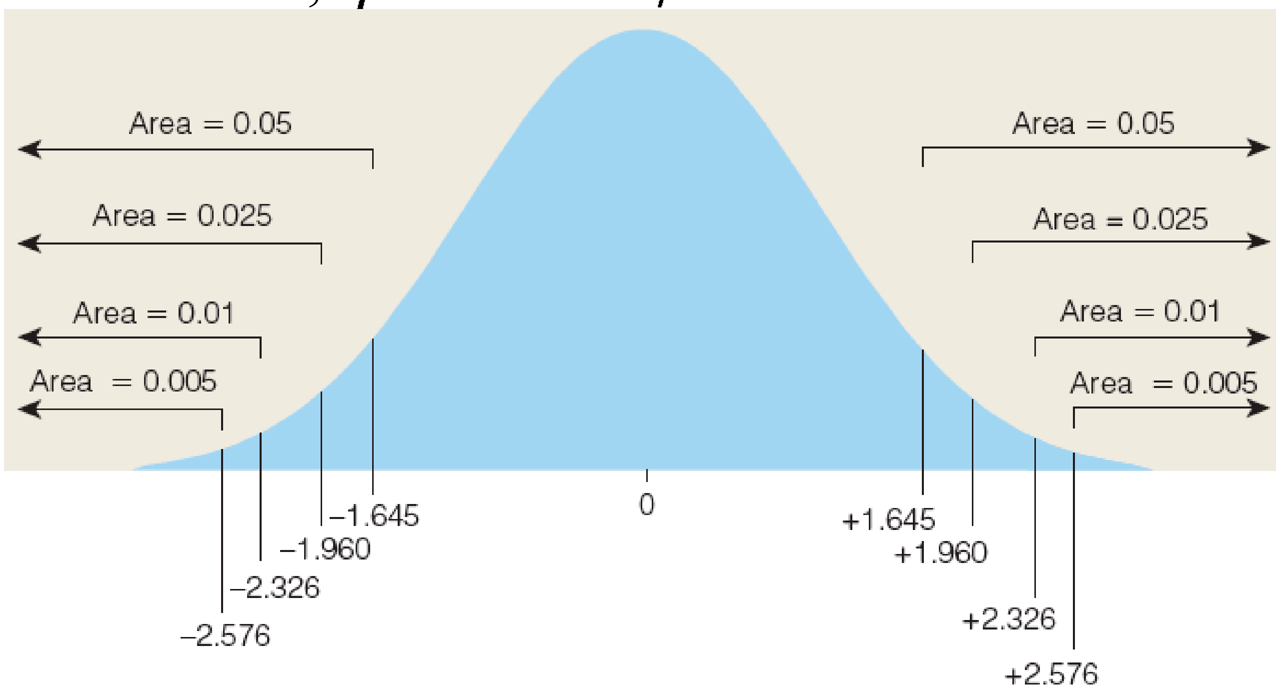
- **Background**: Math SAT score X for population of college students normal; $\mu = 610$, $\sigma = 72$.



- **Question:** Bottom 5% are below what score?
- **Response:** Bottom 0.05 has $z =$
  so $x =$

Example: *Given Probability, Find x*

- **Background**: Math SAT score *X* for population of college students normal; $\mu = 610$, $\sigma = 72$.

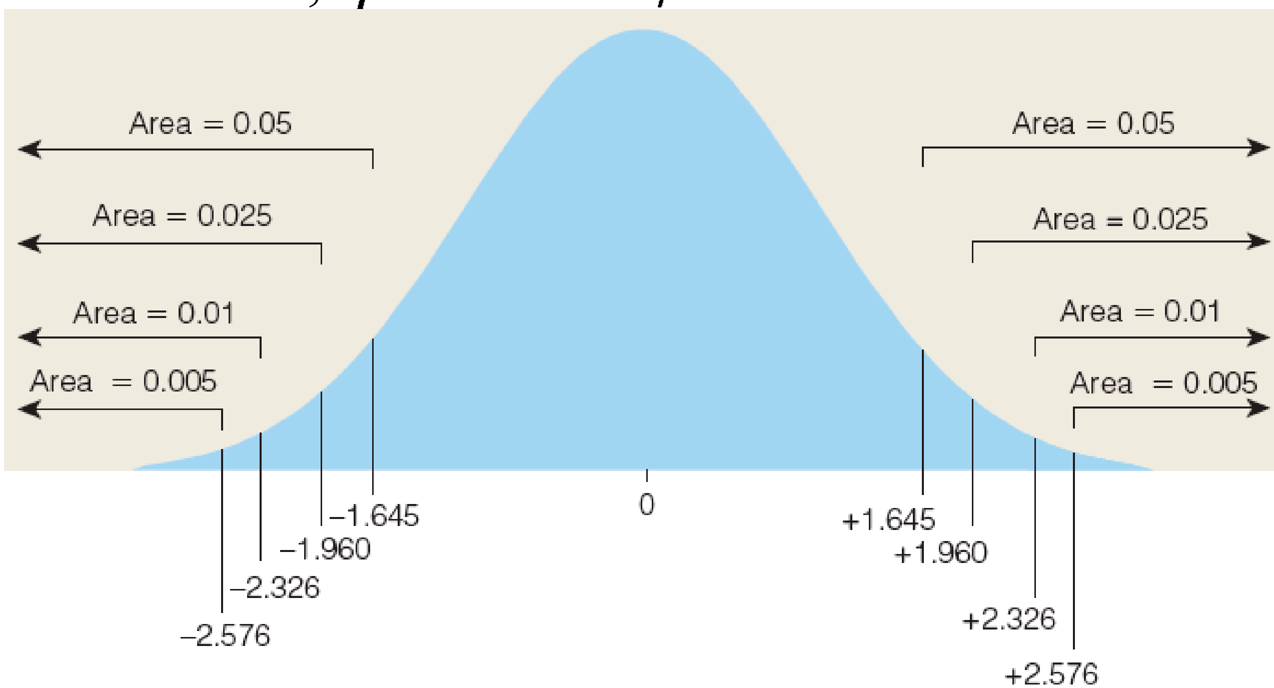


- **Question:** Top half a percent were above what score?
- **Response:** Top 0.005 has $z =$

  so $x =$

Example: *Comparing to a Given Probability*

- **Background**: Math SAT score *X* for population of college students normal; $\mu = 610, \sigma = 72$.



- **Question:** Is P(X<480) more or less than 0.01?
- **Response:** 480 has $z =$
  Since -1.81 is prob. is

Example: *More Comparisons to Given Probability*

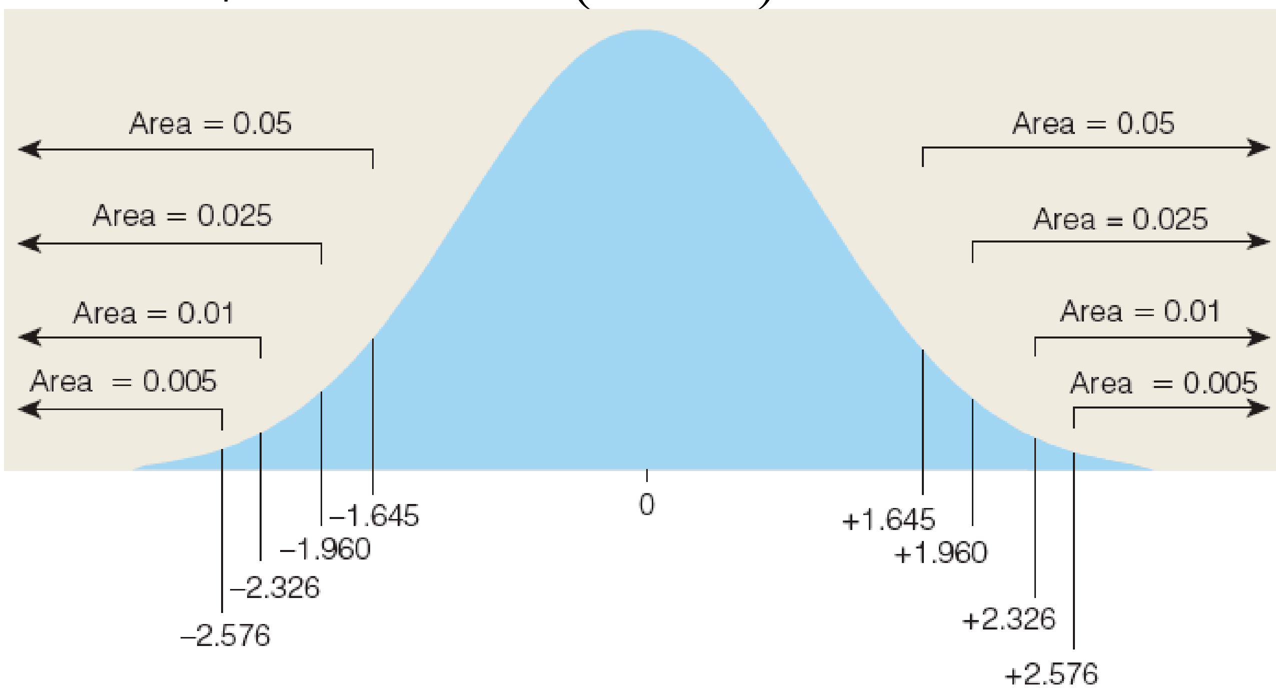
- **Background**: 0.01= P(*Z*<-2.326) = P(*Z*>+2.326)
- **Question:** Are the following >0.01 or <0.01?
  - P(*Z*>+2.4); P(*Z*>+1.9); P(*Z*<-3.7); P(*Z*<-0.4)
- **Response:**
  - P(*Z*>+2.4) ____ 0.01, since +2.4 is ____ extreme than +2.326
  - P(*Z*>+1.9) ____ 0.01, since +1.9 is ____ extreme than +2.326
  - P(*Z*<-3.7) ____ 0.01, since -3.7 is ____ extreme than -2.326
  - P(*Z*<-0.4) ____ 0.01, since -0.4 is ____ extreme than -2.326

***A Closer Look:** As z gets **more** extreme, the tail probability gets ____.*

***Looking Ahead:** When we perform inference in Part 4, some key decisions will be based on how a normal probability compares to a set value like 0.01 or 0.05.*

Example: *Practice with 90-95-98-99 Rule*

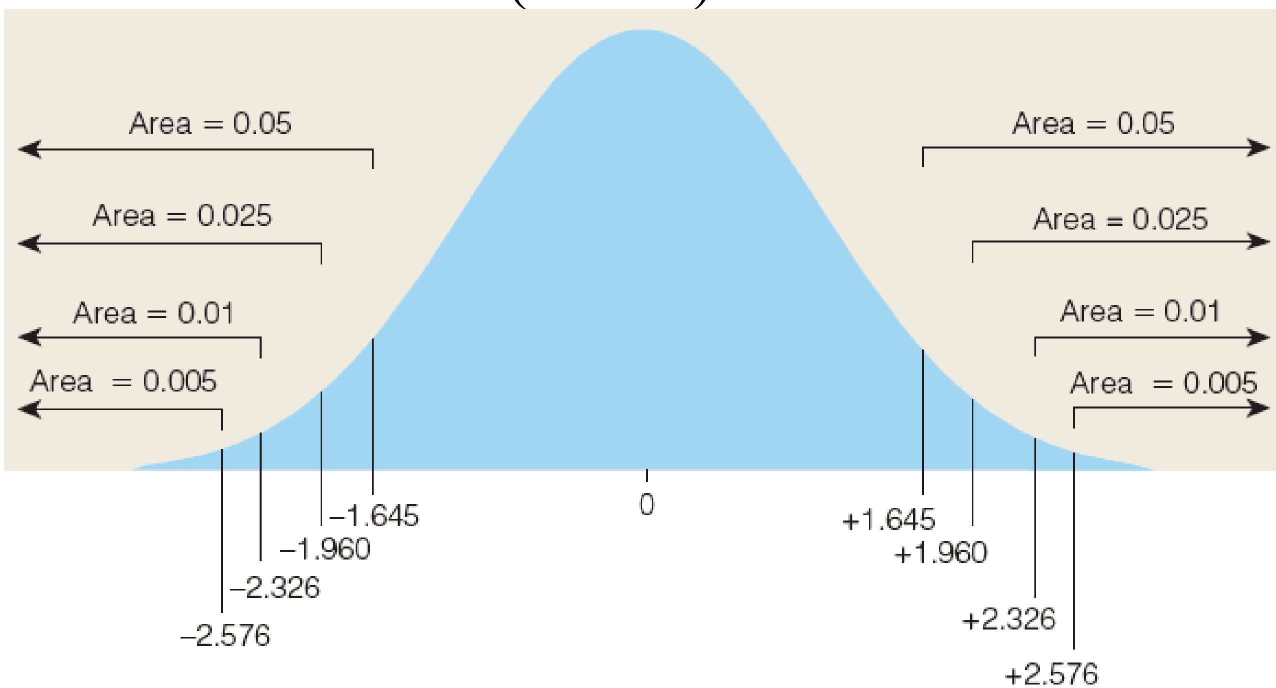
- **Background**: Male chest sizes X normal; $\mu = 37.35,\ \sigma = 2.64$ (inches).



- **Question:** P(X > 45) is in what range?
- **Response:** 45 has z =
  so P(X > 45) is between _____ and _____.

Example: *More Practice with 90-95-98-99 Rule*

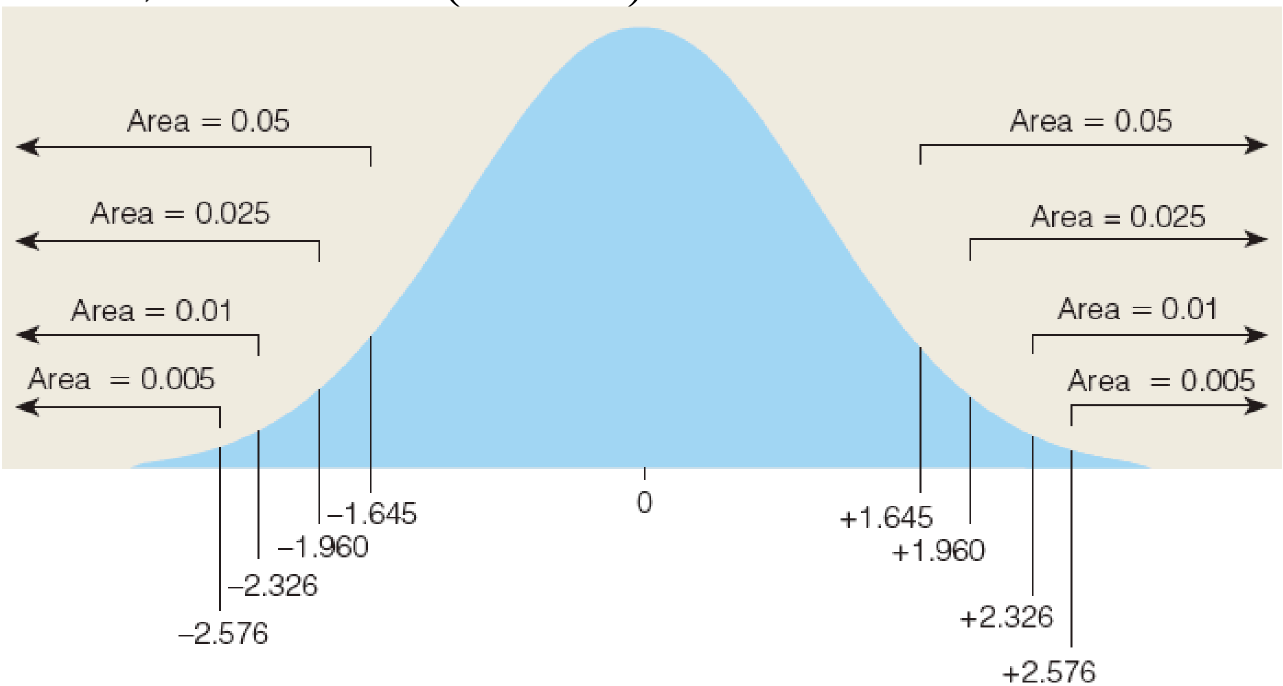
- **Background**: Female chest sizes X normal; $\mu = 35.15,\ \sigma = 2.64$ (inches).



- **Question:** P(X < 28.8) is in what range?
- **Response:** 28.8 has $z =$ ______
so P(X < 28.8) is between _____ and _____.

Example: *90-95-98-99 Rule, Given Probability*

- **Background**: Male ear lengths X normal; $\mu = 2.45, \sigma = 0.17$ (inches).

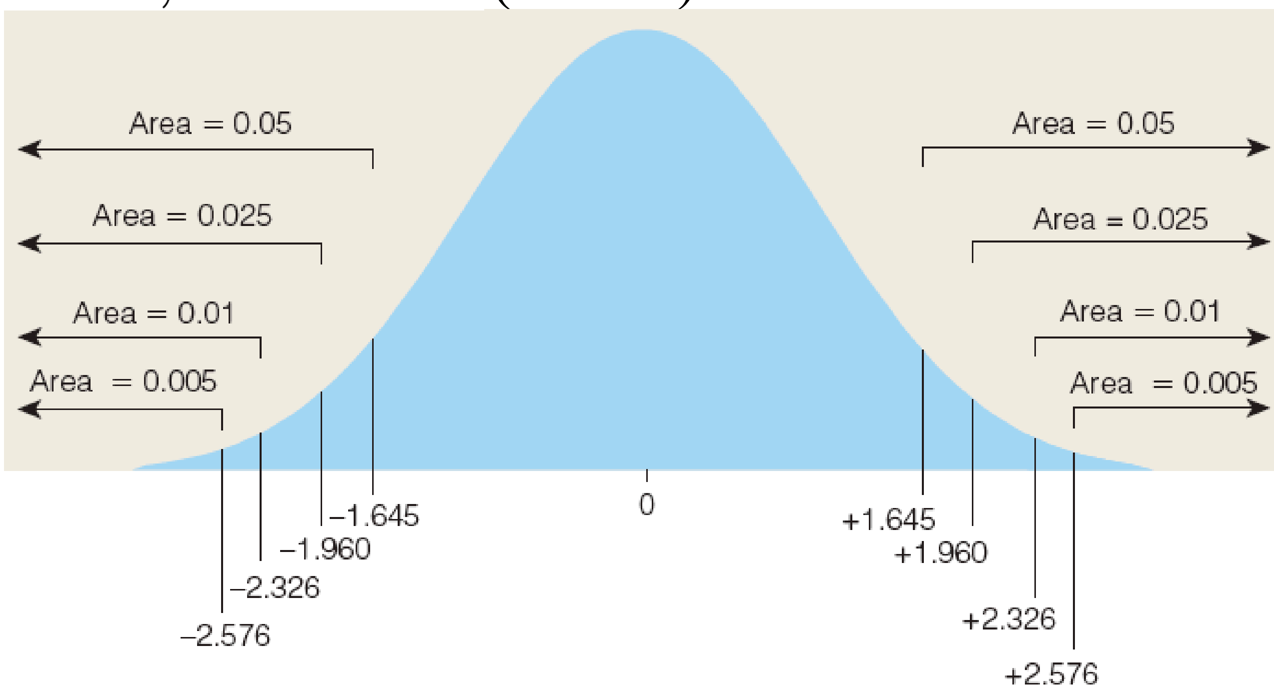


- **Question:** Top 5% are greater than what value?
- **Response:** Top 5% are above $z =$

  so $x =$

Example: *More Use of Rule, Given Probability*

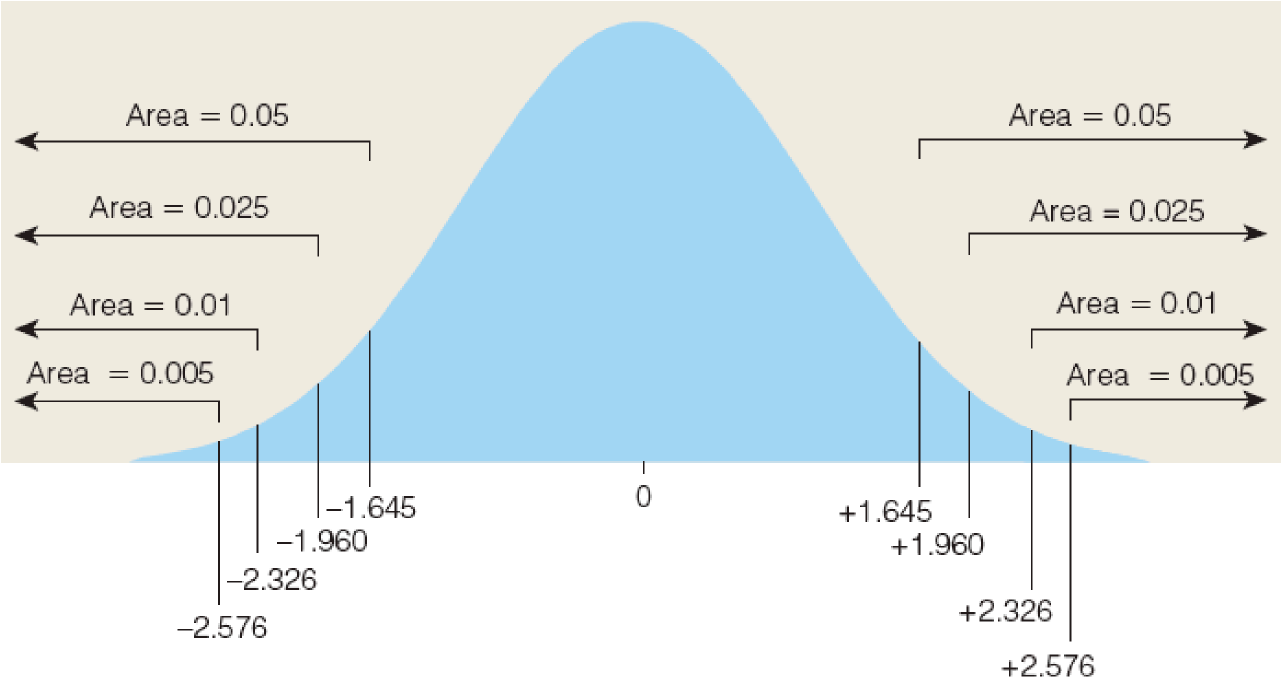
- **Background**: Female ear lengths X normal; $\mu = 2.06, \sigma = 0.17$ (inches).



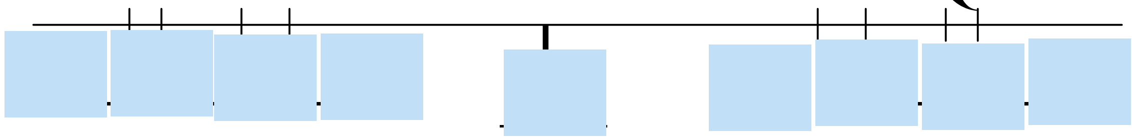
- **Question:** Bottom 2.5% are less than what value?
- **Response:** Bottom 2.5% are below $z =$
  so $x =$

Example: *Sketching Curve with 90-95-98-99 Rule*

- **Background**: IQs X are normal; $\mu = 100,\ \sigma = 15$.



- **Question:** What does the Rule tell us about the IQ curve?
- **Response:**



Lecture Summary
(Tails of Normal Curve)

- Two forms of inference
  - Interval estimate
  - Test if value is plausible
- 68-95-99.7 Rule and Rule for tails of normal curve
- Reviewing normal probability problems
  - Given x, find probability
  - Given probability, find x
- Focusing on tails of normal curve
  - Standard normal problems
  - Non-standard normal problems