Lecture 26: Chapter 10, Section 2 Inference for Quantitative Variable Confidence Interval with *t*

- *t* Confidence Interval for Population Mean
- Comparing *z* and *t* Confidence Intervals
- When neither *z* nor *t* Applies
- Other Levels of Confidence
- *t* Test vs. Confidence Interval

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability (discussed in Lectures 13-20)
  - Statistical Inference
    - 1 categorical (discussed in Lectures 21-23)
    - 1 quantitative: z CI, z test, t CI, t test
    - categorical and quantitative
    - 2 categorical
    - 2 quantitative

Behavior of Sample Mean *(Review)*

For random sample of size n from population with mean μ, standard deviation σ, sample mean $\bar{X}$ has

- mean μ
- standard deviation $\frac{\sigma}{\sqrt{n}}$
- shape approximately normal for large enough n

Sample Mean Standardizing to *z* *(Review)*

→If σ is known, standardized $\bar{X}$ follows *z* (standard normal) distribution:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

If σ is unknown but *n* is large enough (20 or 30), then $s \approx \sigma$ and

$$\frac{\bar{x} - \mu}{s / \sqrt{n}} \approx z$$

Sample mean standardizing to *t* *(Review)*

For σ unknown and *n* small, $\frac{\bar{x}-\mu}{s/\sqrt{n}} = t$

- *t* (like *z*) centered at 0 since $\bar{X}$ centered at μ
- *t* (like *z*) symmetric and bell-shaped if $\bar{X}$ normal
- *t* more spread than *z* (s.d.>1) [***s*** gives less info]

t has "*n*-1 degrees of freedom"(spread depends on *n*)

Inference by Hand or with Software: z or t?

	σ known	σ unknown
small sample ($n < 30$)	$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = z$	$\frac{\bar{x}-\mu}{s/\sqrt{n}} = t$
large sample ($n \geq 30$)	$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = z$	$\frac{\bar{x}-\mu}{s/\sqrt{n}} \approx z$

By Hand:

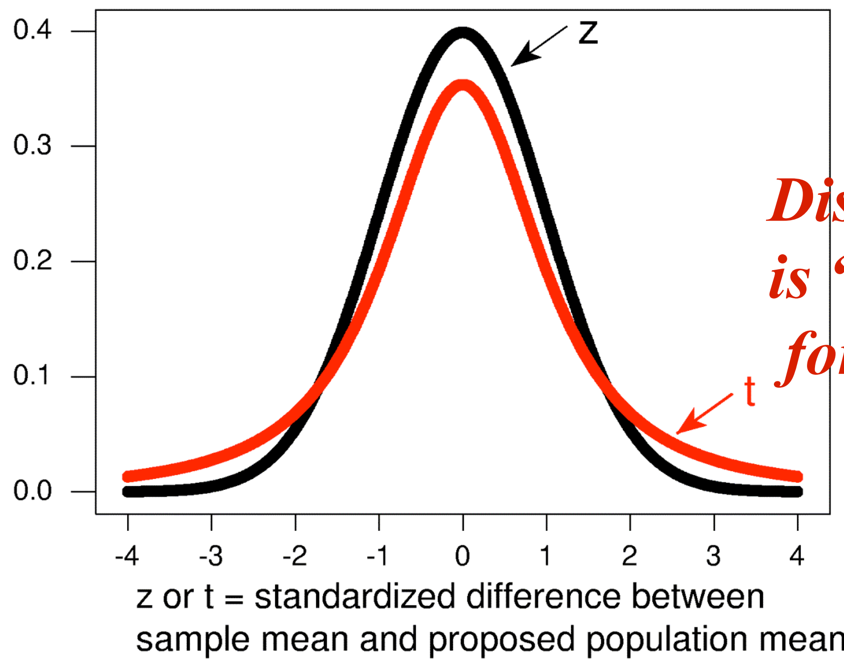
- z used if σ is known **or** n is large
- t used if σ is unknown **and** n is small

With Software:

- z used if σ is known
- t used if σ is unknown

Inference Based on z or t

	σ known	σ unknown
small sample $(n < 30)$	$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = z$	$\frac{\bar{x}-\mu}{s/\sqrt{n}} = t$
large sample $(n \geq 30)$	$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = z$	$\frac{\bar{x}-\mu}{s/\sqrt{n}} \approx z$



Distribution of t is "heavy tailed" for small n.

Confidence Interval for Mean *(Review)*

95% confidence interval for μ (σ known) is

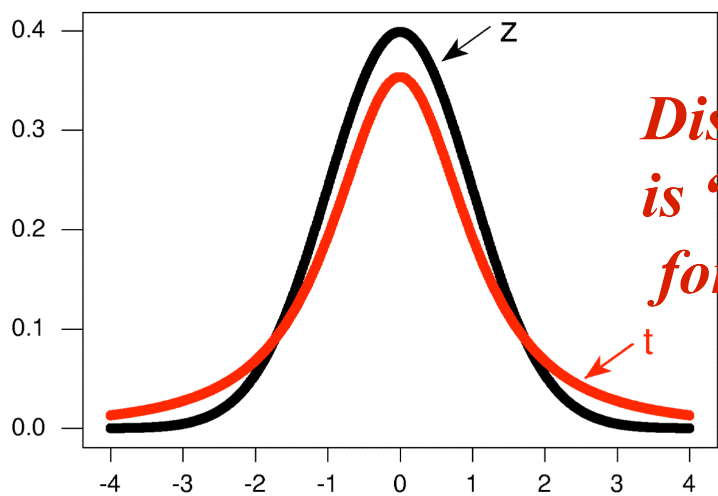
$$\bar{x} \pm 2\frac{\sigma}{\sqrt{n}}$$

- multiplier 2 is from *z* distribution

(95% of normal values within 2 s.d.s of mean)

For *n* small, σ unknown can't say 95% C.I. is

$$\bar{x} \pm 2\frac{s}{\sqrt{n}}$$



Distribution of t is "heavy tailed" for small n.

Confidence Interval for Mean: σ Unknown

95% confidence interval for μ is

$$\bar{x} \pm \text{multiplier} \left(\frac{s}{\sqrt{n}} \right)$$

- multiplier from *t* distribution with *n*-1 *degrees of freedom* (df)
- multiplier at least 2, closer to 3 for *very* small *n*

Degrees of Freedom

- **Mathematical** explanation of df: not needed for elementary statistics
- **Practical** explanation of df: several useful distributions like *t*, *F*, chi-square are *families* of similar curves; df tells us which one applies (depends on sample size *n*).

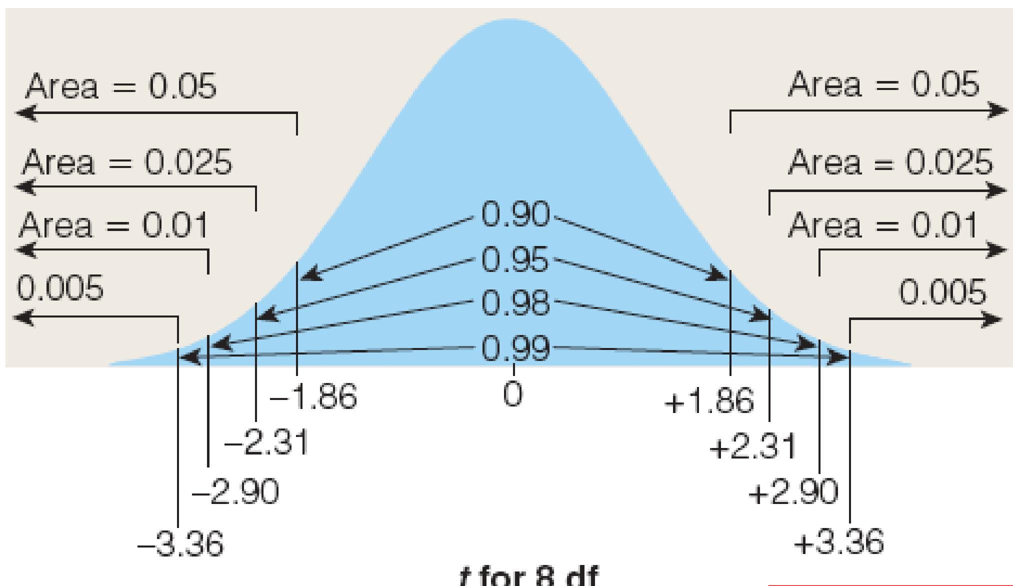
z or t: Which to Concentrate On?

- For purpose of **learning**, start with z (know what to expect from 68-95-99.7 Rule, etc.) (only one z distribution)
- For **practical** purposes, t more realistic (usually don't know population s.d. σ)

Software automatically uses appropriate t distribution with n-1 df: just enter data.

Example: *Confidence Interval with t Curve*

- **Background**: Random sample of shoe sizes for 9 college males: 11.5, 12.0, 11.0, 15.0, 11.5, 10.0, 9.0, 10.0, 11.0
- **Question:** What is 95% C.I. for population mean?
- **Response:** Mean 11.222, s= 1.698, n=9, multiplier 2.31:



Example: *t Confidence Interval* ***with Software***

- **Background**: Random sample of shoe sizes for 9 college males: 11.5, 12.0, 11.0, 15.0, 11.5, 10.0, 9.0, 10.0, 11.0
- **Question:** How do we find a 95% C.I. for the population mean, using software?
- **Response:**

```
One-Sample T: Shoe
Variable          N      Mean     StDev   SE Mean          95.0% CI
Shoe              9    11.222     1.698     0.566   (   9.917,  12.527)
```

Example: *Compare t and z Confidence Intervals*

- **Background**: Random sample of shoe sizes for 9 college males: 11.5, 12.0, 11.0, 15.0, 11.5, 10.0, 9.0, 10.0, 11.0
  We produced 95% *t* confidence interval:

$$11.222 \pm 2.31\left(\frac{1.698}{\sqrt{9}}\right) = 11.222 \pm 1.307 = (9.92, 12.53)$$

  If 1.698 had been population s.d., would get *z* C.I.:

$$11.222 \pm 1.96\left(\frac{1.698}{\sqrt{9}}\right) = 11.222 \pm 1.109 = (10.11, 12.33)$$

- **Question:** How do the *t* and *z* intervals differ?
- **Response:** *t* multiplier is 2.31, *z* multiplier is 1.96:

t interval width about

z interval width about

σ known→ info→ interval

Practice: 10.16b p.489

Example: *t vs. z Confidence Intervals, Large n*

- **Background**: Earnings for sample of 446 students at a university averaged \$3,776, with s.d. \$6,500. The *t* multiplier for 95% confidence and 445 df is 1.9653.
- **Question:** How different are the *t* and *z* intervals?
- **Response:** The intervals will be , whether we use
  - *t* multiplier 1.9653
  - precise *z* multiplier 1.96
  - approximate *z* multiplier 2

Interval approximately

Behavior of Sample Mean *(Review)*

For random sample of size n from population with mean μ, standard deviation σ, sample mean $\bar{X}$ has

- mean μ
- standard deviation $\frac{\sigma}{\sqrt{n}}$
- shape approx. normal for large enough n

→If σ is unknown and n small,

$$\frac{\bar{x}-\mu}{s/\sqrt{n}} = t$$

Guidelines for $\bar{X}$ Approx. Normal *(Review)*

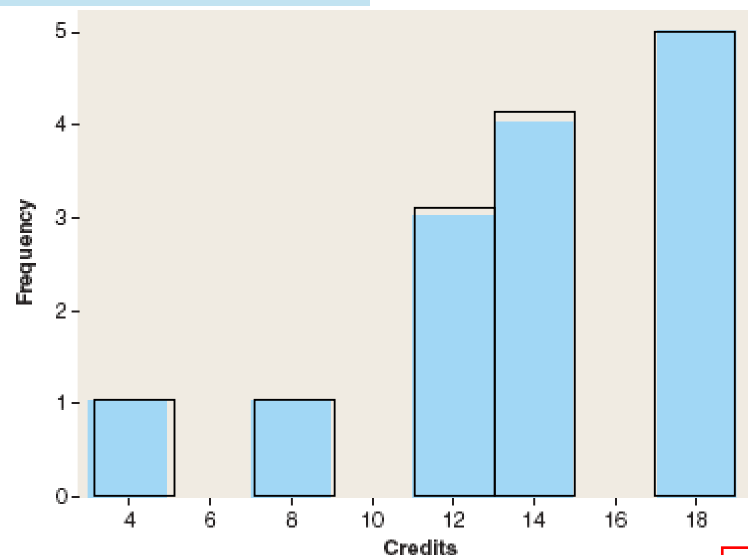
Can assume shape of $\bar{X}$ for random samples of size n is approximately normal if

- Graph of sample data appears normal; or
- Sample data fairly symmetric, n at least 15; or
- Sample data moderately skewed, n at least 30; or
- Sample data very skewed, n much larger than 30

If $\bar{X}$ is not normal, $\frac{\bar{x}-\mu}{s/\sqrt{n}}$ is not *t*.

Example: *Small, Skewed Data Set*

- **Background**: Credits taken by 14 non-traditional students: 4, 7, 11, 11, 12, 13, 13, 14, 14, 17, 17, 17, 17, 18
- **Question:** What is a 95% confidence interval for population mean?
- **Response:** *n* small, shape of credits left-skewed →



Looking Ahead:
Non-parametric methods can be used for small n, skewed data.

t Intervals at Other Levels of Confidence

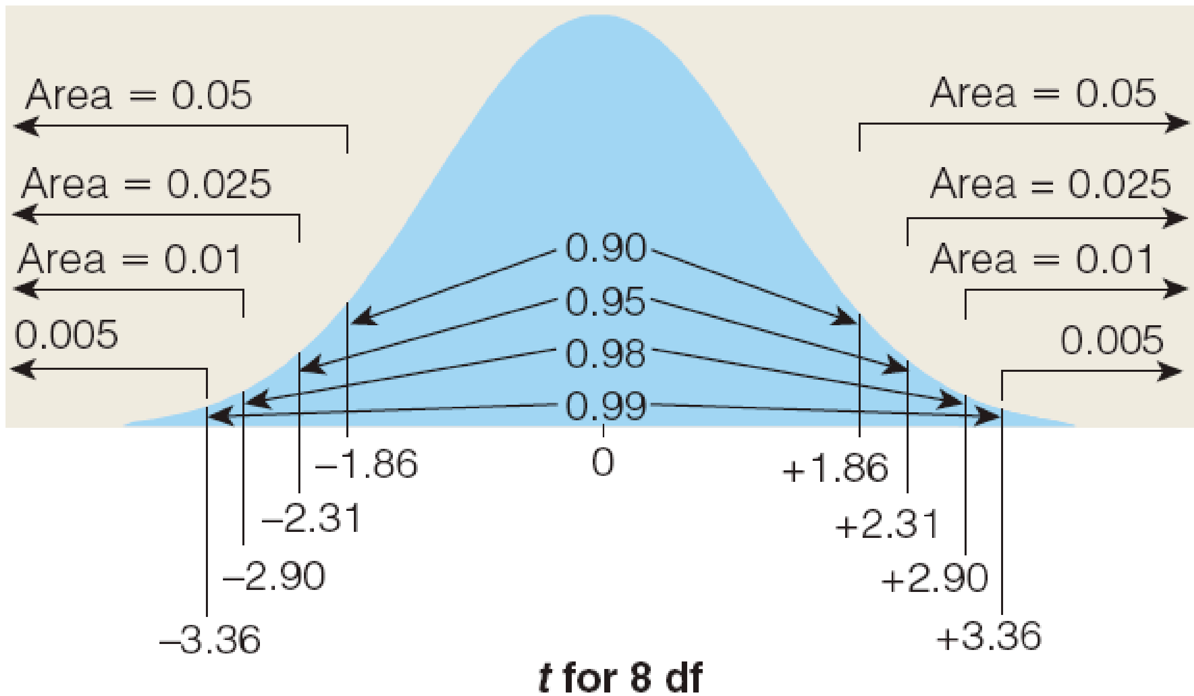
Confidence Level

	90%	95%	98%	99%
z (infinite n)	1.645	1.960 or 2	2.326	2.576
t: df = 19 (n = 20)	1.73	2.09	2.54	2.86
t: df = 11 (n = 12)	1.80	2.20	2.72	3.11
t: df = 3 (n = 4)	2.35	3.18	4.54	5.84

- Lower confidence→smaller t multiplier
- Higher confidence→larger t multiplier
- Table excerpt→at any given level, $t > z$ mult→ using s not σ gives wider interval (less info)
- t multipliers decrease as df (and n) increase

Example: *Intervals at Other Confidence Levels*

- **Background**: Random sample of shoe sizes for 9 college males: 11.5, 12.0, 11.0, 15.0, 11.5, 10.0, 9.0, 10.0, 11.0



- **Question:** What is *t* multiplier for 99% confidence?
- **Response:**

Example: *Intervals at Other Confidence Levels*

- **Background**: Random sample of shoe sizes for 9 college males: 11.5, 12.0, 11.0, 15.0, 11.5, 10.0, 9.0, 10.0, 11.0 We can produce 95% confidence interval:

$$11.2 \pm 2.31 \frac{1.7}{\sqrt{9}} = (9.9, 12.5)$$

- **Question:** What would 99% confidence interval be, and how does it compare to 95% interval? (Use the fact that *t* multiplier for 8 df, 99% confidence is 3.36.)
- **Response:** 99% interval interval is

  - Width for 95%
  - Width for 99%

Practice: 10.60b p.512

Summary of *t* Confidence Intervals

Confidence interval for μ is $\bar{x} \pm \text{multiplier}\left(\frac{s}{\sqrt{n}}\right)$

where multiplier depends on

- df: smaller for larger *n*, larger for smaller *n*
- level: smaller for lower level, larger for higher

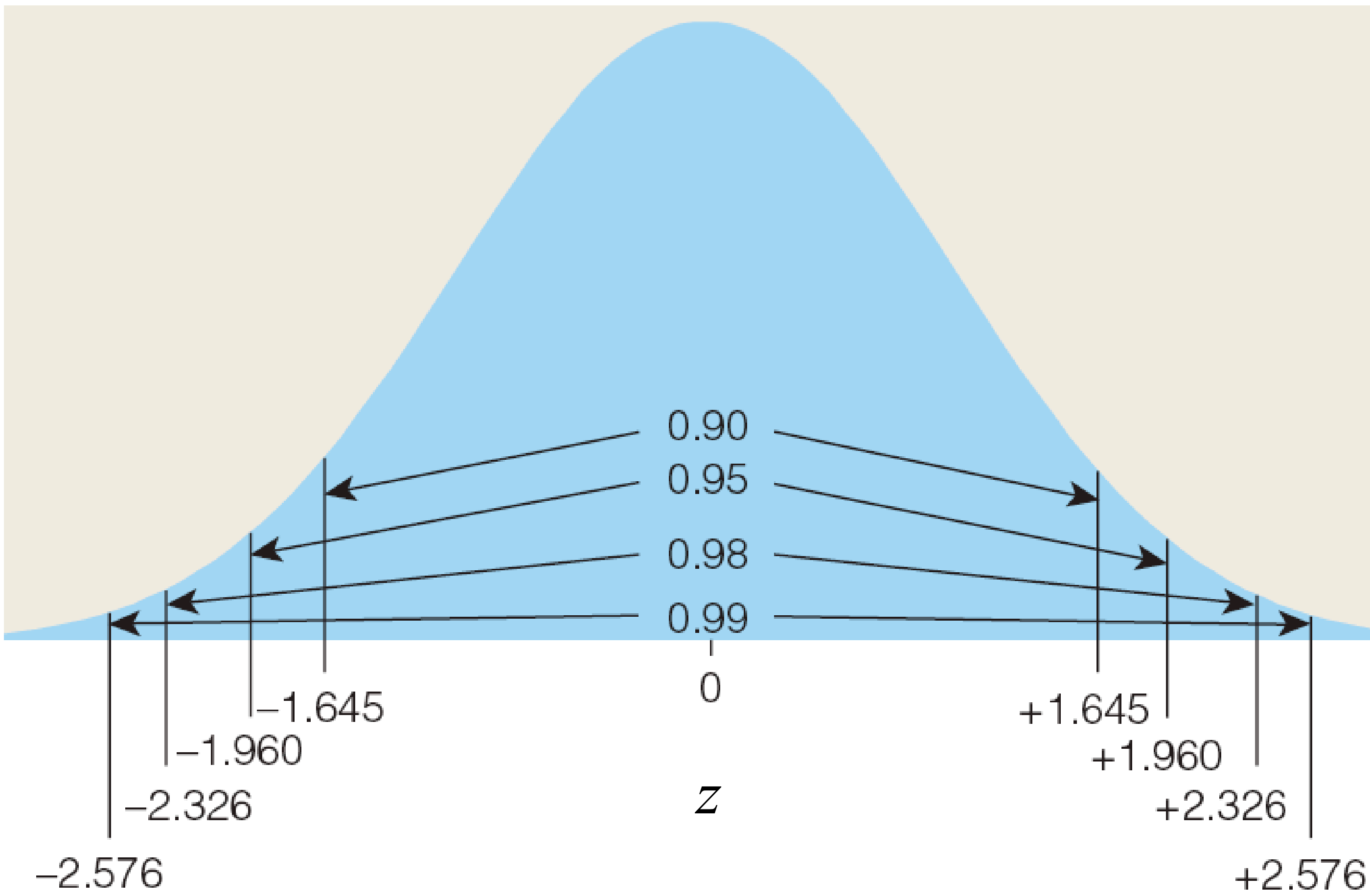
Note: margin of error is larger for larger *s*.

→ interval **narrower** for

- larger *n* (via df *and* $\sqrt{n}$ in denominator)
- lower level of confidence
- smaller s.d. (distribution with less spread)

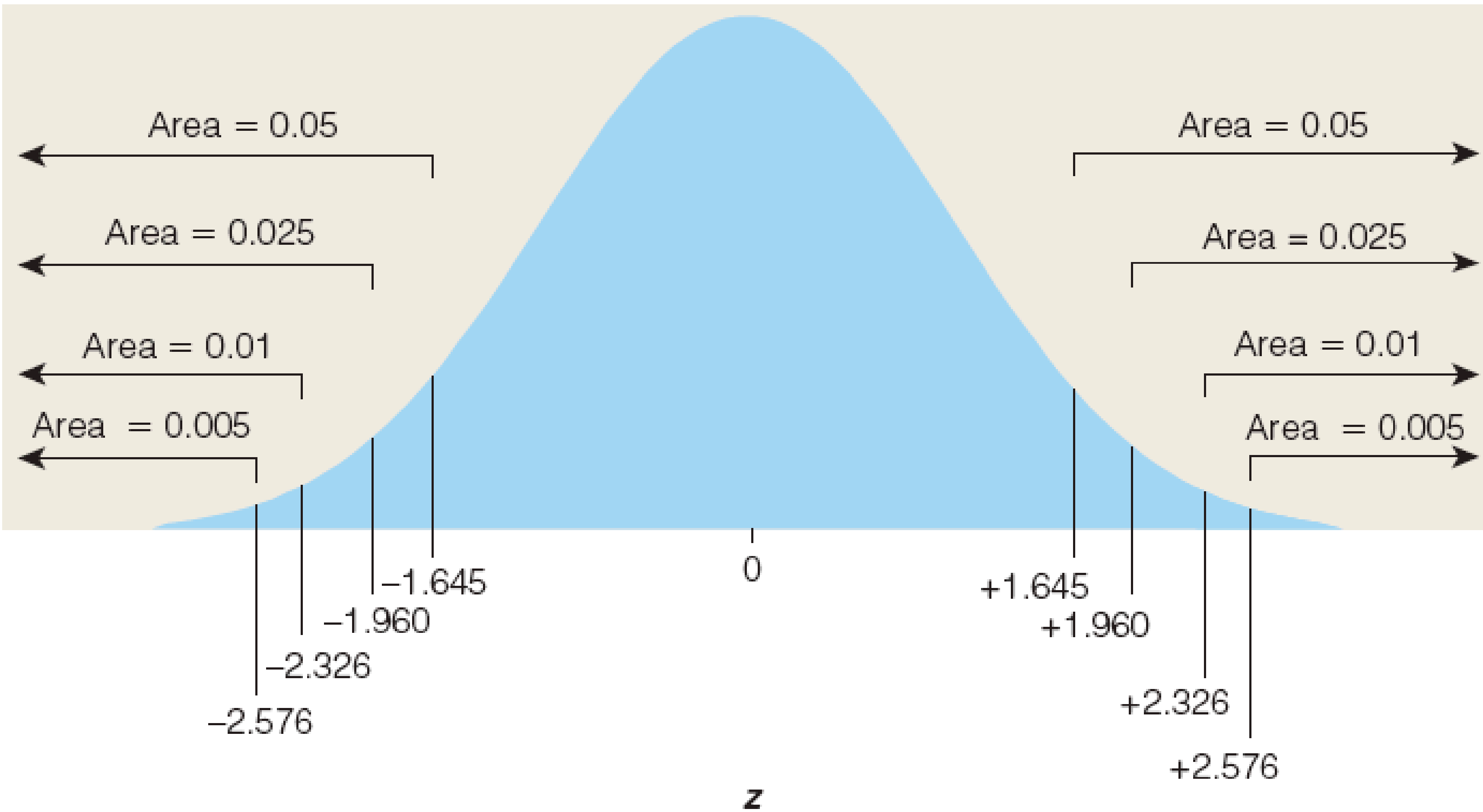
From *z* Confidence Intervals to Tests *(Review)*

For confidence intervals, used “inside” probabilities.



From z Confidence Intervals to Tests *(Review)*

For hypothesis tests, used "outside" probabilities.



From t Confidence Intervals to Tests

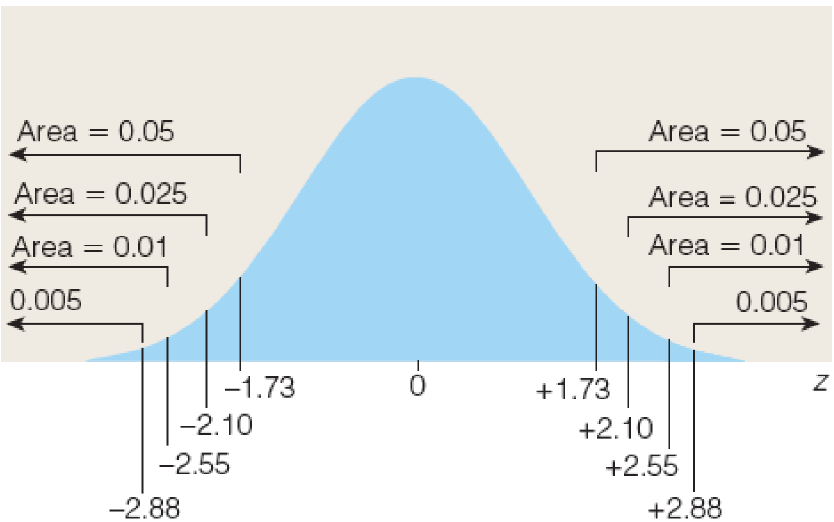
Confidence interval: use multiplier for t dist, n-1 df

Hypothesis test: P-value based on tail of t dist, n-1 df

Example: *Hypothesis Test: t vs. z*

- **Background**: Suppose one test with very large n has z =2; another test with n=19 (18 df) has t =2.

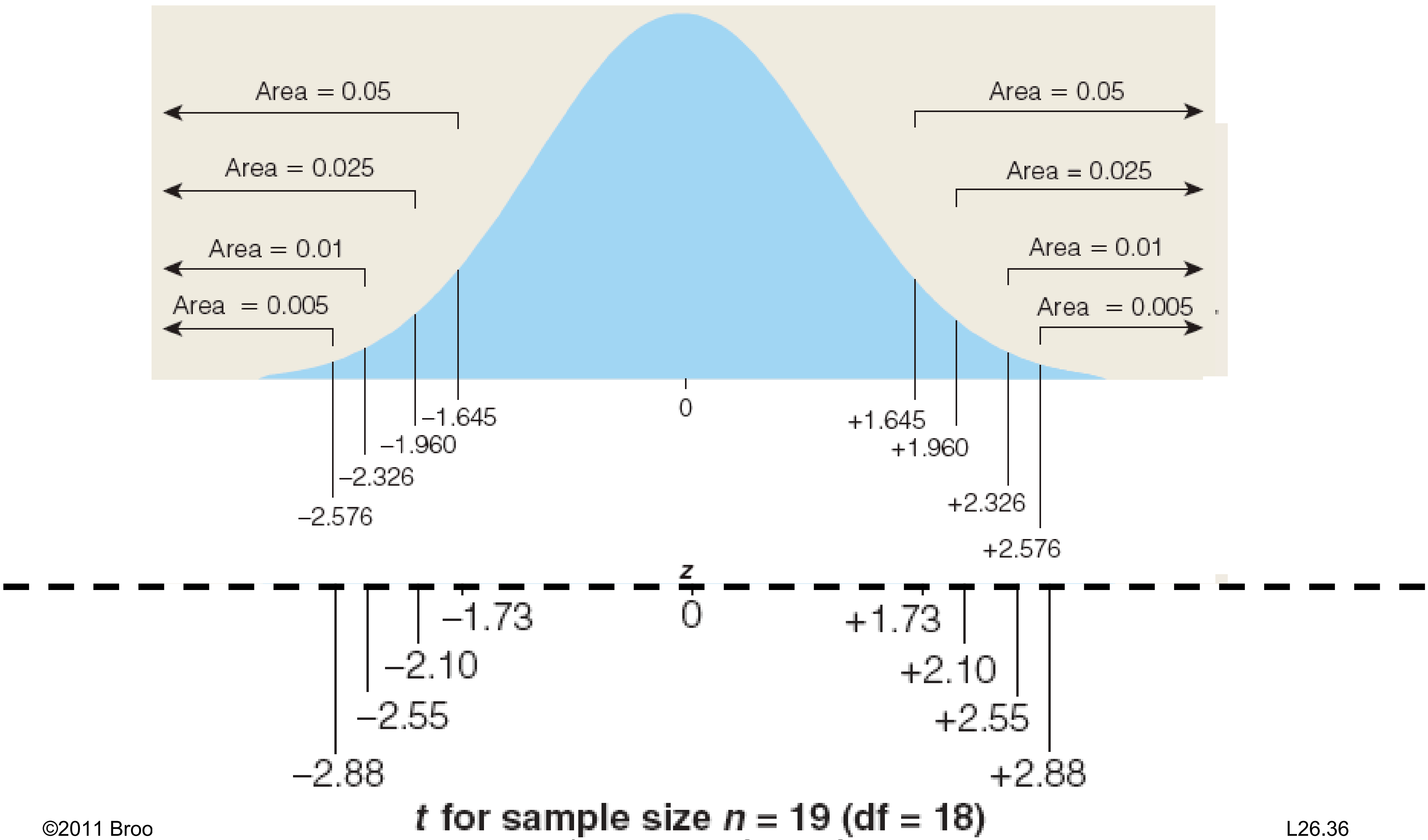
t tail probabilities (df = 18)



t for sample size *n* = 19 (df = 18)

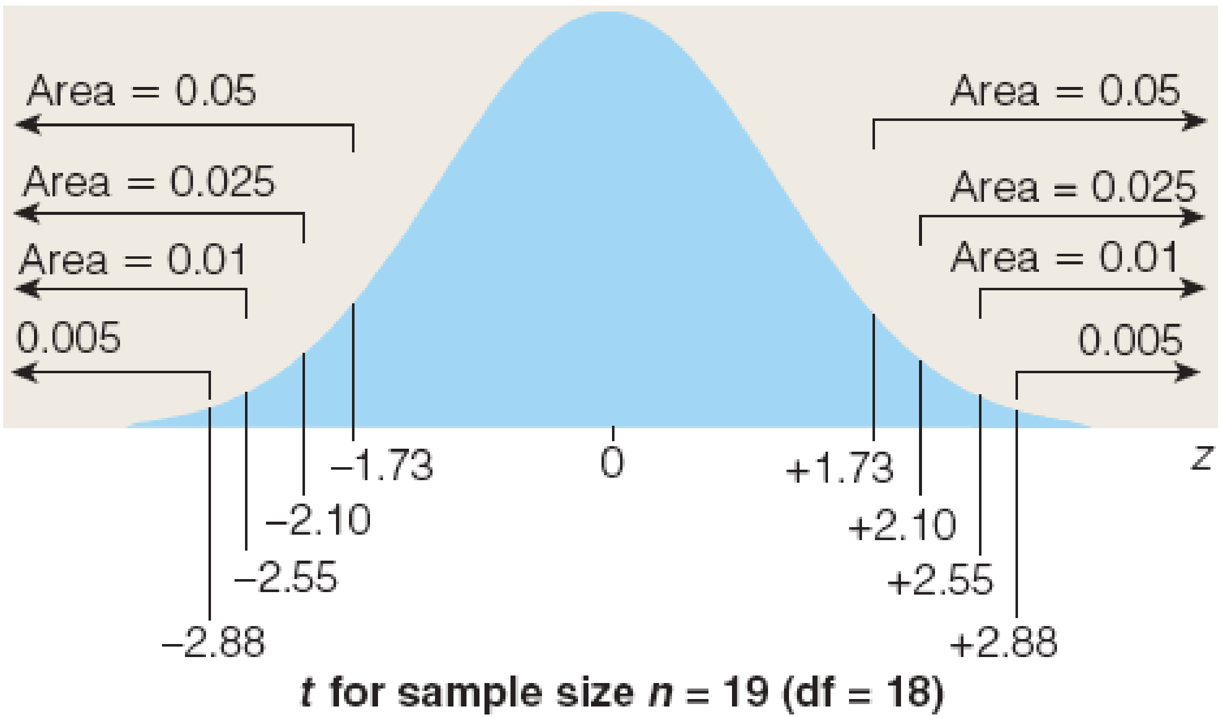
- **Question:** How do *P*-values compare for z and t? (Assume alternative is "greater than".)
- **Response:** 90-95-98-99 Rule→z *P*-value
  t curve for 18 df→t *P*-value

Comparing Critical Values, *z* with ***t* for 18 df**



Example: *Hypothesis Test: t vs. z*

- **Background**: Consider *t* curve for 18 df.



- **Question:** Would a value of $t = 3$ be considered extreme?
- **Response:** ______; $|t|$ for 18 df almost never exceeds ______

Example: *t Test (by Hand)*

- **Background**: Wts. of 19 female college students:

110 110 112 120 120 120 125 125 130 130 132 133 134 135 135 135 145 148 159

- **Question:** Is pop. mean 141.7 reported by NCHS plausible, or is there evidence that we've sampled from pop. with lower mean (or that there is bias due to under-reporting)?
- **Response:**

1. Pop.≥10(19); shape of weights close to normal→n=19 OK
2. $\bar{x} = 129.36, s = 12.82, t =$
3. P-value = small because |t| more extreme than 3 can be considered unusual for most n; in particular, for 18 df, P(t<-2.88) is less than 0.005.
4. Reject H_0? Conclude?

Practice: 10.68 p.590

Lecture Summary
(Inference for Means: t Confidence Intervals)

- t confidence interval for population mean
  - Multiplier from t distribution with n-1 df
  - When to perform inference with z or t
  - Constructing t CI by hand or with software
- Comparing z and t confidence intervals
- When neither z nor t applies
- Other levels of confidence
- from confidence interval to hypothesis test
- t test by hand