Lecture 16: Chapter 7, Section 2
Binomial Random Variables

- Definition
- What if Events are Dependent?
- Center, Spread, Shape of Counts, Proportions
- Normal Approximation

Looking Back: *Review*

- 4 Stages of Statistics
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability
    - Finding Probabilities (discussed in Lectures 13-14)
    - Random Variables (introduced in Lecture 15)
      - Binomial
      - Normal
    - Sampling Distributions
  - Statistical Inference

Definition *(Review)*

- **Discrete Random Variable:** one whose possible values are finite or countably infinite (like the numbers 1, 2, 3, …)

Looking Ahead: *To perform inference about categorical variables, need to understand behavior of sample proportion. A first step is to understand behavior of sample* ***counts****. We will eventually shift from discrete counts to a normal approximation, which is continuous.*

Definition

Binomial Random Variable counts sampled individuals falling into particular category;

- Sample size n is fixed
- Each selection independent of others
- Just 2 possible values for each individual
- Each has same probability p of falling in category of interest

Example: *A Simple Binomial Random Variable*

- **Background**: The random variable *X* is the count of tails in two flips of a coin.
- **Questions:** Why is *X* binomial? What are *n* and *p*?
- **Responses:**
  - Sample size *n* fixed?
  - Each selection independent of others?
  - Just 2 possible values for each?
  - Each has same probability *p*?

Example: *A Simple Binomial Random Variable*

- **Background**: The random variable X is the count of tails in two flips of a coin.
- **Question:** How do we display X?
- **Response:**

Looking Back: *We already discussed and displayed this random variable when learning about probability distributions.*

Example: *Determining if R.V. is Binomial*

- **Background**: Consider following R.V.:
  - Pick card from deck of 52, replace, pick another. X=no. of cards picked until you get ace.
- **Question:** Is X binomial?
- **Response:**

Example: *Determining if R.V. is Binomial*

- **Background**: Consider following R.V.:
  - Pick 16 cards without replacement from deck of 52. X=no. of red cards picked.
- **Question:** Is X binomial?
- **Response:**

Practice: 7.25d p.307

Example: *Determining if R.V. is Binomial*

- **Background**: Consider following R.V.:
  - Pick 16 cards with replacement from deck of 52. W=no. of clubs, X=no. of diamonds, Y=no. of hearts, Z=no. of spades. Goal is to report how frequently each suit is picked.
- **Question:** Are W, X, Y, Z binomial?
- **Response:**

Practice: 7.25a p.307

Example: *Determining if R.V. is Binomial*

- **Background**: Consider following R.V.:
  - Pick with replacement from German deck of 32 (doesn't include numbers 2-6), then from deck of 52, back to deck of 32, etc. for 16 selections altogether. X=no. of aces picked.
- **Question:** Is X binomial?
- **Response:**

Example: *Determining if R.V. is Binomial*

- **Background**: Consider following R.V.:
  - Pick 16 cards with replacement from deck of 52. X=no. of hearts picked.
- **Question:** Is X binomial?
- **Response:**
  - fixed $n = 16$
  - selections independent (with replacement)
  - just 2 possible values (heart or not)
  - same $p = 0.25$ for all selections

→

Requirement of Independence

Snag:

- Binomial theory requires independence
- Actual sampling done *without* replacement so selections are dependent

Resolution: *When sampling without replacement, selections are approximately independent if population is at least 10n.*

Example: *A Binomial Probability Problem*

- **Background**: The proportion of Americans who are left-handed is 0.10. Of 44 presidents, 7 have been left-handed (proportion 0.16).
- **Question:** How can we establish if being left-handed predisposes someone to be president?
- **Response:** Determine if 7 out of 44 (0.16) is when sampling at random from a population where 0.10 fall in the category of interest.

Practice: 7.74 p.338

Solving Binomial Probability Problems

- Use binomial formula or tables

  Only practical for small sample sizes
- Use software

  Won't take this approach until later
- Use normal approximation for count *X*

  Not quite: more interested in proportions
- Use normal approximation for proportion

  Need mean and standard deviation…

Example: *Mean of Binomial Count, Proportion*

- **Background**: Based on long-run observed outcomes, probability of being left-handed is approx. 0.1. Randomly sample 100 people.
- **Questions:** On average, what should be the
  - **count** of lefties?
  - **proportion** of lefties?
- **Responses:** On average, we should get
  - **count** of lefties
  - **proportion** of lefties

Practice: 7.27 p.308

Mean and S.D. of Counts, Proportions

Count X binomial with parameters n, p has:

- **Mean** np
- **Standard deviation** $\sqrt{np(1-p)}$

Sample proportion $\hat{p} = \frac{X}{n}$ has:

- **Mean** p
- **Standard deviation** $\sqrt{\frac{p(1-p)}{n}}$

Looking Back: *Formulas for s.d. require independence: population at least 10n.*

Example: *Standard Deviation of Sample Count*

- **Background**: Probability of being left-handed is approx. 0.1. Randomly sample 100 people. Sample **count** has mean 100(0.1)= 10, standard deviation $\sqrt{100(0.1)(1-0.1)} = 3$
- **Question:** How do we interpret these?
- **Response:** On average, expect

  sample count lefties.

  Counts vary; typical distance from 10 is

Example: *S.D. of Sample Proportion*

- **Background**: Probability of being left-handed is approx. 01. Randomly sample 100 people. Sample **proportion** has mean 0.1, standard deviation $\sqrt{\frac{0.1(1-0.1)}{100}} = 0.03$
- **Question:** How do we interpret these?
- **Response:** On average, expect
  sample proportion = lefties.
  Proportions vary; typical distance from 0.1 is

Practice: 7.29g-h p.308

Example: *Role of Sample Size in Spread*

- **Background**: Consider proportion of tails in various sample sizes *n* of coinflips.
- **Questions:** What is the standard deviation for
  - ***n*=1? *n*=4? *n*=16?**
- **Responses:**
  - ***n*=1:** s.d.=
  - ***n*=4:** s.d.=
  - ***n*=16:** s.d.=

A Closer Look: *Due to n in the denominator of formula for standard deviation, spread of sample proportion ______ as n increases.*

Practice: 7.34b p.309

Shape of Distribution of Count, Proportion

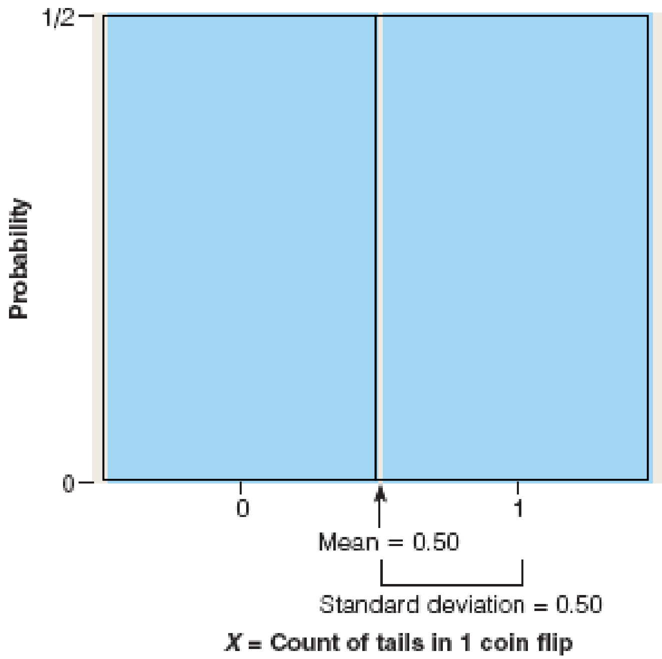
Binomial count X or proportion $\hat{p} = \frac{X}{n}$ for repeated random samples has shape approximately normal if samples are large enough to offset underlying skewness.

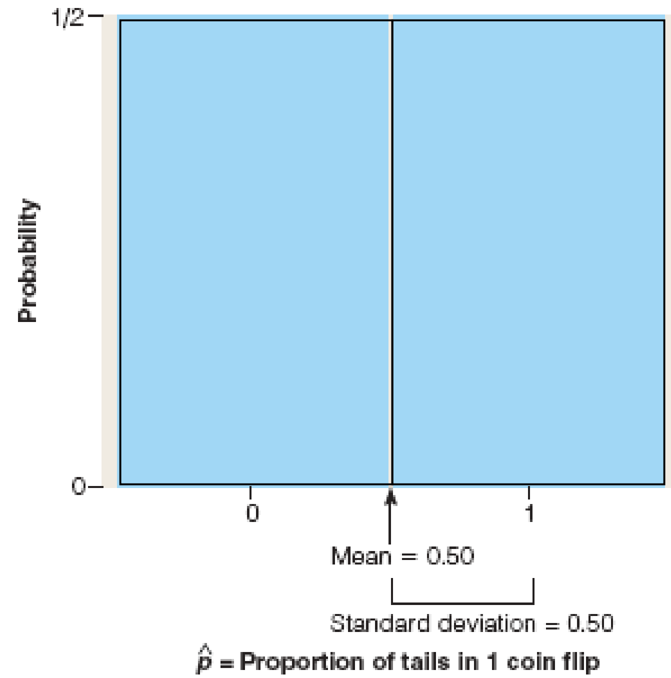
(Central Limit Theorem)

For a given sample size n, shapes are identical for count and proportion.

Example: *Underlying Coinflip Distribution*

- **Background**: Distribution of count or proportion of tails in $n=1$ coinflip ($p=0.5$):

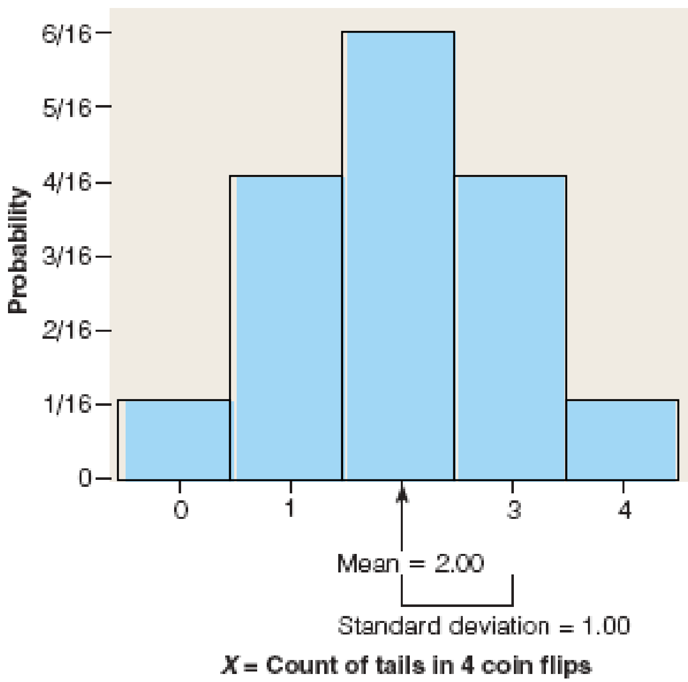


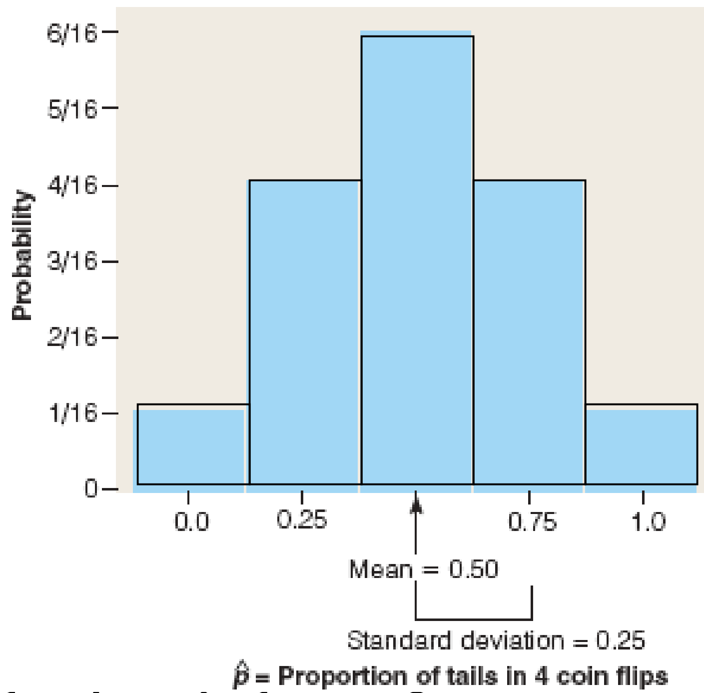


- **Question:** What are the distributions' shapes?
- **Response:**

Example: *Distribution for 4 Coinflips*

- **Background**: Distribution of count or proportion of tails in $n=4$ coinflips ($p=0.5$):





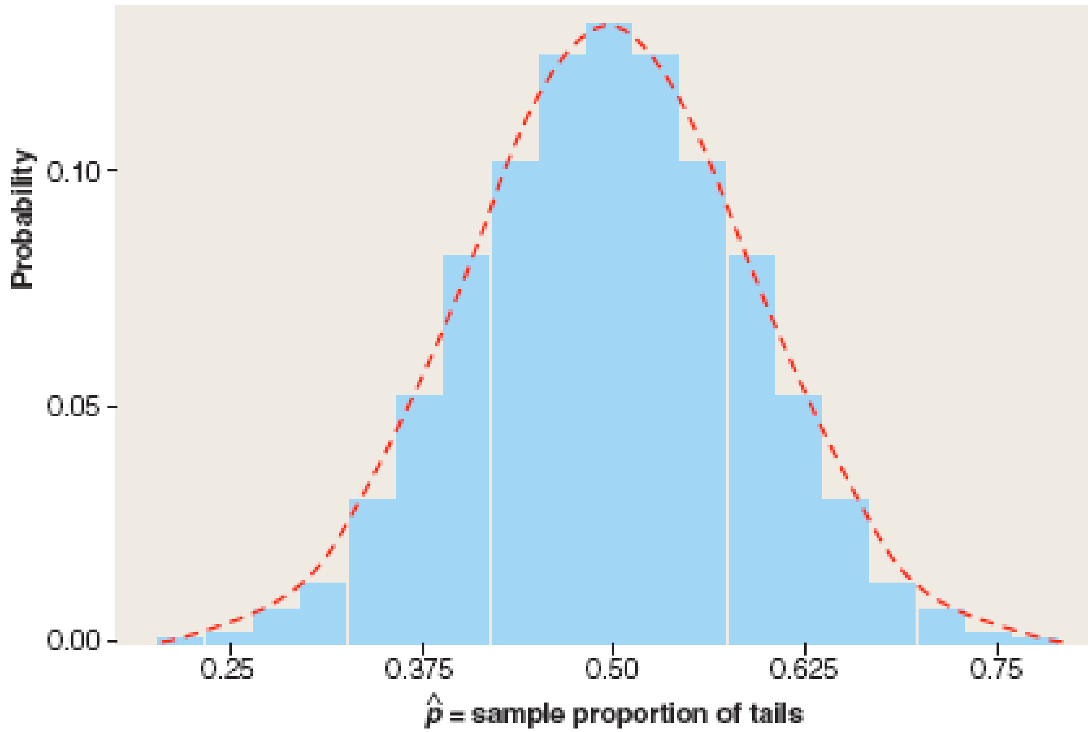
- **Question:** What are the distributions' shapes?
- **Response:**

Shift from Counts to Proportions

- Binomial Theory begins with counts
- Inference will be about proportions

Example: *Distribution of $\hat{p}$ for 16 Coinflips*

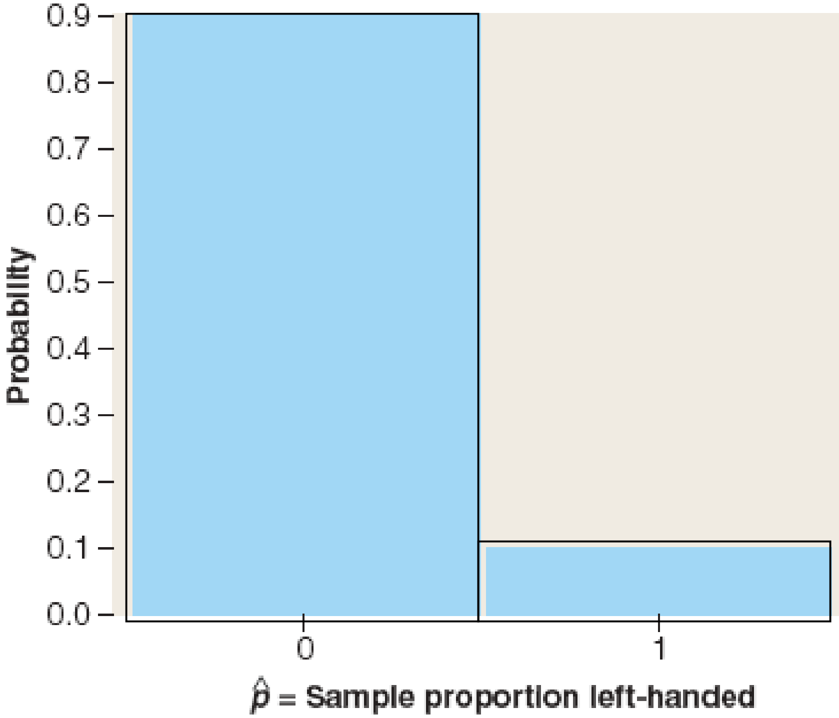
- **Background**: Distribution of **proportion** of tails in n=16 coinflips (p=0.5):



- **Question:** What is the shape?
- **Response:**

Example: *Underlying Distribution of Lefties*

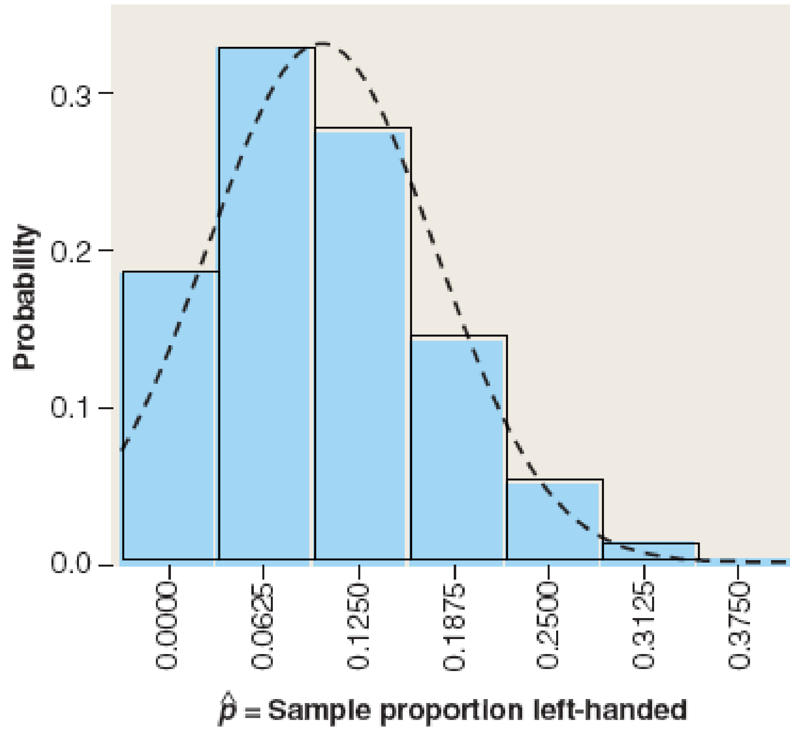
- **Background**: Distribution of **proportion** of lefties (p=0.1) for samples of n=1:



- **Question:** What is the shape?
- **Response:**

Example: *Dist of* $\hat{p}$ *of Lefties for n=* *16*

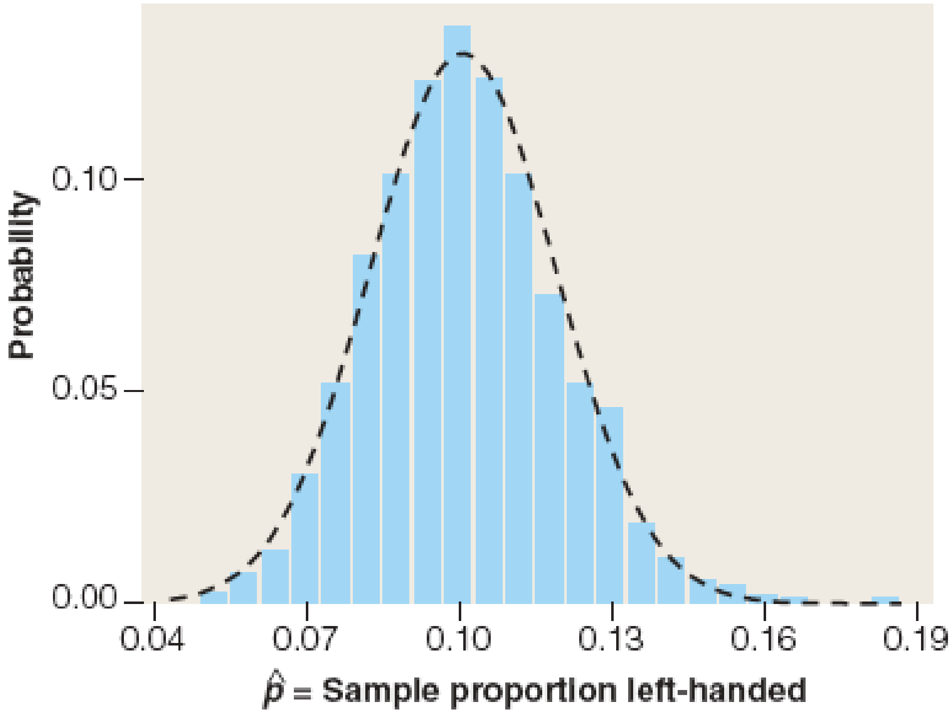
- **Background**: Distribution of proportion of lefties (p=0.1) for n=16:



- **Question:** What is the shape?
- **Response:**

Example: *Dist of* $\hat{p}$ *of Lefties for n=100*

- **Background**: Distribution of proportion of lefties (p=0.1) for n=100:



- **Question:** What is the shape?
- **Response:**

Rule of Thumb:
Sample Proportion Approximately Normal

Distribution of $\hat{p}$ is approximately normal if sample size *n* is large enough relative to shape, determined by population proportion *p*.

Require $np \geq 10$ and $n(1-p) \geq 10$

Together, these require us to have larger *n* for *p* close to 0 or 1 (underlying distribution skewed right or left).

Example: *Applying Rule of Thumb*

- **Background**: Consider distribution of sample proportion for various n and p:

n=4, p=0.5; n=20, p=0.5; n=20, p=0.1; n=20, p=0.9; n=100, p=0.

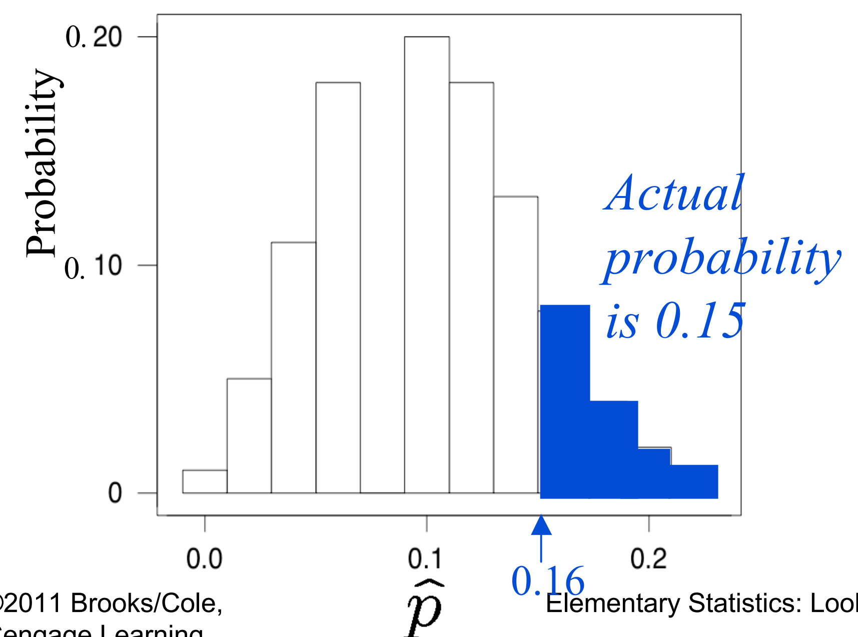
- **Question:** Is shape approximately normal?
- **Response:** Normal?
  - n=4, p=0.5 [np=4(0.5)=2<10]
  - n=20, p=0.5 [np=20(0.5)=10=n(1-p)]
  - n=20, p=0.1 No [ ]
  - n=20, p=0.9 No [ ]
  - n=100, p=0.1

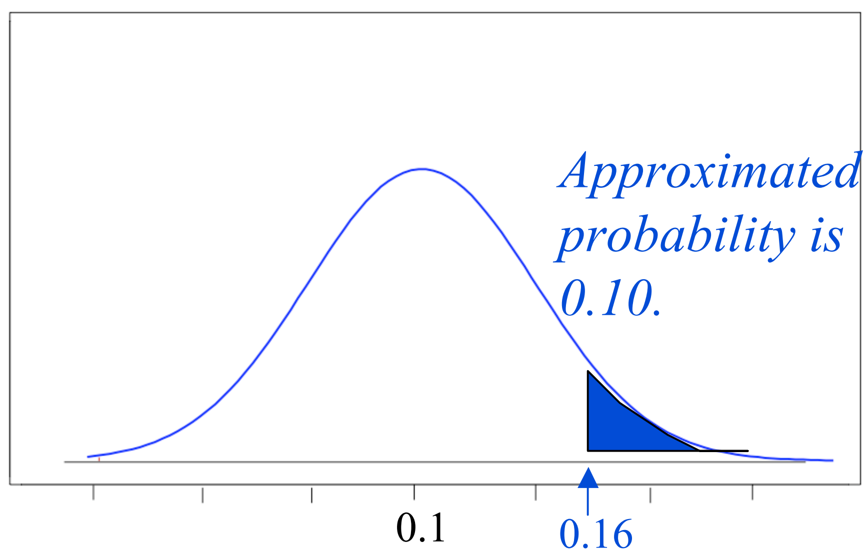
[np=100(0.1)=10, n(1-p)=100(0.9)=90 both ≥ 10]

Practice: 7.38 p.310

Example: *Solving the Left-handed Problem*

- **Background**: The proportion of Americans who are lefties is 0.1. Consider P($\hat{p} \geq 7/44 = 0.16$) for a sample of 44 presidents.
- **Question:** Can we use a normal approximation to find the probability that at least 7 of 44 (0.16) are left-handed?
- **Response:**





Practice: 7.35b p.309

Example: *From Count to Proportion and Vice Versa*

- **Background**: Consider these reports:
  - In a sample of 87 assaults on police, 23 used weapons.
  - 0.44 in sample of 25 bankruptcies were due to med. bills
- **Question:** In each case, what are n, X, and $\hat{p}$?
- **Response:**
  - First has $n =$ $X =$ ____ $\hat{p} =$
  - Second has $n =$ $\hat{p} =$ $X =$

Lecture Summary

(Binomial Random Variables)

- Definition; 4 requirements for binomial
- R.V.s that do or don't conform to requirements
- Relaxing requirement of independence
- Binomial counts, proportions
  - Mean
  - Standard deviation
  - Shape
- Normal approximation to binomial