Lecture 19: Chapter 8, Section 1 Sampling Distributions: Proportions

- Typical Inference Problem
- Definition of Sampling Distribution
- 3 Approaches to Understanding Sampling Dist.
- Applying 68-95-99.7 Rule

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability
    - Finding Probabilities (discussed in Lectures 13-14)
    - Random Variables (discussed in Lectures 15-18)
    - Sampling Distributions
      - Proportions
      - Means
  - Statistical Inference

Typical Inference Problem

If sample of 100 students has 0.13 left-handed, can you believe population proportion is 0.10?

Solution Method: Assume (temporarily) that population proportion is 0.10, find probability of sample proportion as high as 0.13. If it's too improbable, we won't believe population proportion is 0.10.

Key to Solving Inference Problems

For a given population proportion p and sample size n, need to find probability of sample proportion $\hat{p}$ in a certain range:

Need to know **sampling distribution** of $\hat{p}$.

Note: $\hat{p}$ can denote a single statistic or a random variable.

Definition

Sampling distribution of sample statistic tells probability distribution of values taken by the statistic in repeated random samples of a given size.

Looking Back: *We summarize a probability distribution by reporting its* ***center, spread, shape.***

Behavior of Sample Proportion *(Review)*

For random sample of size *n* from population with *p* in category of interest, sample proportion $\hat{p} = \frac{X}{n}$ has

- mean *p*
- standard deviation $\sqrt{\frac{p(1-p)}{n}}$
- shape approximately normal for large enough *n*

Looking Back: *Can find normal probabilities using 68-95-99.7 Rule, etc.*

Rules of Thumb *(Review)*

- Population at least 10 times sample size *n*
  (formula for standard deviation of $\hat{p}$ approximately correct even if sampled without replacement)
- *np* and *n(1-p)* both at least 10
  (guarantees $\hat{p}$ approximately normal)

Understanding Dist. of Sample Proportion

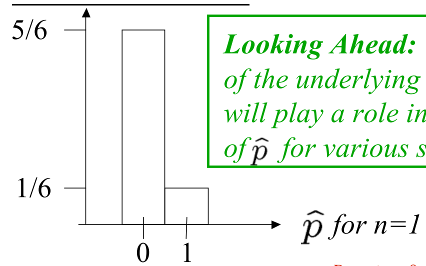
3 Approaches:

1. Intuition
2. Hands-on Experimentation
3. Theoretical Results

Looking Ahead: *We'll find that our* ***intuition*** *is consistent with* ***experimental*** *results, and both are confirmed by mathematical* ***theory****.*

Example: *Shape of Underlying Distribution (n=1)*

- **Background**: Population proportion of blue M&M's is p=1/6=0.17.
- **Question:** How does the probability histogram for sample proportions appear for samples of size 1?
- **Response:** ________________



Looking Ahead: *The shape of the underlying distribution will play a role in the shape of $\hat{p}$ for various sample sizes.*

Example: *Sample Proportion as Random Variable*

- **Background**: Population proportion of blue M&Ms is 0.17.
- **Questions:**
  - Is the underlying variable categorical or quantitative?
  - Consider the behavior of sample proportion $\hat{p}$ for repeated random samples of a given size. What type of variable is sample proportion?
  - What 3 aspects of the distribution of sample proportion should we report to summarize its behavior?
- **Responses:**
  - Underlying variable ________________________________
  - ______________________
  - Summarize with _________, _________, _________

Example: *Center, Spread of Sample Proportion*

- **Background**: Population proportion of blue M&M's is p=1/6=0.17.
- **Question:** What can we say about center and spread of $\hat{p}$ for repeated random samples of size n = 25 (a teaspoon)?
- **Response:**
  - **Center:** Some $\hat{p}$'s more than ___, others less; should balance out so mean of $\hat{p}$'s is p = _______.
  - **Spread** of $\hat{p}$'s: s.d. depends on ____.
    - For n=6, could easily get $\hat{p}$ anywhere from ___to ___.
    - For n=25, spread of $\hat{p}$ will be ____than it is for n = 6.

Example: *Intuit Shape of Sample Proportion*

- **Background**: Population proportion of blue M&M's is p=1/6=0.17.
- **Question:** What can we say about the shape of $\hat{p}$ for repeated random samples of size n = 25 (a teaspoon)?
- **Response:**

$\hat{p}$ close to____ most common, far from _____ in either direction increasingly less likely→

____

Example: *Sample Proportion for Larger n*

- **Background**: Population proportion of blue M&M's is p=1/6=0.17.
- **Question:** What can we say about center, spread, shape of $\hat{p}$ for repeated random samples of size n = 75 (a Tablespoon)?
- **Response:**
  - **Center:** mean of $\hat{p}$'s should be p = _____ (for any n).
  - **Spread** of $\hat{p}$'s: compared to n=25, spread for n=75 is ____
  - **Shape:** $\hat{p}$'s clumped near 0.17, taper at tails→ __________

***Looking Ahead:** Sample size does **not** affect center but plays an important role in spread and shape of the distribution of sample proportion (also of sample mean).*

 Practice: 8.5 p.354

Understanding Sample Proportion

3 Approaches:

1. Intuition
2. Hands-on Experimentation
3. Theoretical Results

***Looking Ahead:** We'll find that our **intuition** is consistent with **experimental** results, and both are confirmed by mathematical **theory**.*

Central Limit Theorem

Approximate normality of sample statistic for repeated random samples of a large enough size is cornerstone of inference theory.

- Makes intuitive sense.
- Can be verified with experimentation.
- Proof requires higher-level mathematics; result called Central Limit Theorem.

Center of Sample Proportion *(Implications)*

For random sample of size n from population with p in category of interest, sample proportion $\hat{p} = \frac{X}{n}$ has

- mean p

→ $\hat{p}$ is *unbiased estimator* of p

(sample must be random)

Spread of Sample Proportion *(Implications)*

For random sample of size n from population with p in category of interest, sample proportion $\hat{p} = \frac{X}{n}$ has

- mean p
- standard deviation $\sqrt{\frac{p(1-p)}{n}}$ ← n in denominator

→ $\hat{p}$ has less spread for larger samples

(population size must be at least 10n)

Shape of Sample Proportion *(Implications)*

For random sample of size n from population with p in category of interest, sample proportion $\hat{p} = \frac{X}{n}$ has

- mean p
- standard deviation $\sqrt{\frac{p(1-p)}{n}}$
- shape approx. normal for large enough n

→can find probability that sample proportion takes value in given interval

Example: *Behavior of Sample Proportion*

- **Background**: Population proportion of blue M&M's is p=0.17.
- **Question:** For repeated random samples of n=**25**, how does $\hat{p}$ behave?
- **Response:** For n=25, $\hat{p}$ has
  - **Center:** mean ________
  - **Spread:** standard deviation __________________
  - **Shape:** not really normal because

    ____

Practice: 8.53c-e p.375

Example: *Sample Proportion for Larger n*

- **Background**: Population proportion of blue M&M's is p=0.17.
- **Question:** For repeated random samples of n=**75**, how does $\hat{p}$ behave?
- **Response:** For n=75, $\hat{p}$ has
  - **Center:** mean ________
  - **Spread:** standard deviation __________________
  - **Shape:** approximately normal because

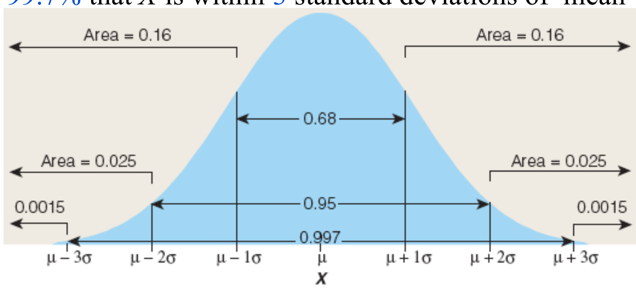
    ____

Practice: 8.53f-h p.375

68-95-99.7 Rule for Normal R.V. *(Review)*

Sample at random from normal population; for sampled value X (a R.V.), probability is

- 68% that X is within 1 standard deviation of mean
- 95% that X is within 2 standard deviations of mean
- 99.7% that X is within 3 standard deviations of mean



68-95-99.7 Rule for Sample Proportion

For sample proportions $\hat{p}$ taken at random from a large population with underlying p, probability is

- 68% that $\hat{p}$ is within $1\sqrt{\frac{p(1-p)}{n}}$ of p
- 95% that $\hat{p}$ is within $2\sqrt{\frac{p(1-p)}{n}}$ of p
- 99.7% that $\hat{p}$ is within $3\sqrt{\frac{p(1-p)}{n}}$ of p

Example: *Sample Proportion for n=75, p=0.17*

- **Background**: Population proportion of blue M&Ms is p=0.17. For random samples of n=75, $\hat{p}$ approx. normal with mean 0.17, s.d. $\sqrt{\frac{0.17(1-0.17)}{75}} = 0.043$
- **Question:**

What does 68-95-99.7 Rule tell us about behavior of $\hat{p}$?

- **Response:** The probability is approximately
  - 0.68 that $\hat{p}$ is within _______ of ____: in (0.13, 0.21)
  - 0.95 that $\hat{p}$ is within _______ of ____: in (0.08, 0.26)
  - 0.997 that $\hat{p}$ is within _______ of ____: in (0.04, 0.30)

Looking Back: *We don't use the Rule for n=25 because ________________*

 Practice: 8.57 p.376

90-95-98-99 Rule *(Review)*

For standard normal Z, the probability is

- 0.90 that Z takes a value in interval (-1.645, +1.645)
- 0.95 that Z takes a value in interval (-1.960, +1.960)
- 0.98 that Z takes a value in interval (-2.326, +2.326)
- 0.99 that Z takes a value in interval (-2.576, +2.576)

Example: *Sample Proportion for n=75, p=0.17*

- **Background**: Population proportion of blue M&Ms is $p=0.17$. For random samples of $n=75$, $\hat{p}$ approx. normal with mean 0.17, s.d. $\sqrt{\frac{0.17(1-0.17)}{75}} = 0.043$
- **Question:**

What does 90-95-98-99 Rule tell about behavior of $\hat{p}$?

- **Response:** The probability is approximately
  - 0.90 that $\hat{p}$ is within _____(0.043) of 0.17: in (0.10,0.24)
  - 0.95 that $\hat{p}$ is within _____(0.043) of 0.17: in (0.09,0.25)
  - 0.98 that $\hat{p}$ is within _____(0.043) of 0.17: in (0.07,0.27)
  - 0.99 that $\hat{p}$ is within _____(0.043) of 0.17: in (0.06,0.28)

Typical Inference Problem *(Review)*

If sample of 100 students has 0.13 left-handed, can you believe population proportion is 0.10?

Solution Method: Assume (temporarily) that population proportion is 0.10, find probability of sample proportion as high as 0.13. If it's too improbable, we won't believe population proportion is 0.10.

Example: *Testing Assumption About p*

- **Background**: We asked, "*If sample of 100 students has 0.13 left-handed, can you believe population proportion is 0.10?*"
- **Questions:**
  - What are the mean, standard deviation, and shape of $\hat{p}$?
  - Is 0.13 improbably high under the circumstances?
  - Can we believe $p = 0.10$?
- **Response:**
  - For $p=0.10$ and $n=100$, $\hat{p}$ has mean ____, s.d. _______________ ; shape approx. normal since _______________________________.
  - According to Rule, the probability is ______________ that $\hat{p}$ would take a value of 0.13 (1 s.d. above mean) or more.
  - Since this isn't so improbable, ____________________________.

Lecture Summary
(Distribution of Sample Proportion)

- Typical inference problem
- Sampling distribution; definition
- 3 approaches to understanding sampling dist.
  - Intuition
  - Hands-on experiment
  - Theory
- Center, spread, shape of sampling distribution
  - Central Limit Theorem
- Role of sample size
- Applying 68-95-99.7 Rule