Lecture 30: Chapter 11, Section 3
Categorical & Quantitative Variable
Inference in Several-Sample Design

- Compare and Contrast Several- and 2-sample
- Variation Among Means or Within Groups
- *F* Statistic as Ratio of Variation
- Role of Sample Size

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability (discussed in Lectures 13-20)
  - Statistical Inference
    - 1 categorical (discussed in Lectures 21-23)
    - 1 quantitative (discussed in Lectures 24-27)
    - cat and quan: paired, 2-sample, several-sample
    - 2 categorical
    - 2 quantitative

Inference Methods for C→Q *(Review)*

- Paired: reduces to 1-sample *t*
  - Focused on mean of differences
- Two-Sample: 2-sample *t* (similar to 1-sample *t*)
  - Focused on difference between means
- Several-Sample: need new distribution (*F*)
  - Focus on difference among means

Display & Summary, Several Samples *(Review)*

- **Display: Side-by-side boxplots:**
  - One boxplot for each categorical group
  - All share same quantitative scale
- **Summarize:** Compare
  - Five Number Summaries (looking at boxplots)
  - Means and Standard Deviations

Looking Ahead: *Inference for population relationship focuses on* ***means and standard deviations****.*

Notation

	Sizes	Means	s.d.s
Sample	I =no. of groups compared $n_1, n_2, \cdots, n_I$ sum to N	$\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_I$ (overall $\bar{x}$)	$s_1, s_2, \cdots, s_I$
Population		$\mu_1, \mu_2, \cdots, \mu_I$	$\sigma_1, \sigma_2, \cdots, \sigma_I$

Two- vs. Several-Sample Inference

- **Similar:** test statistic standardizes difference among sample means, taking sample sizes and standard deviations into account
- **Different:** several-sample test statistic ***(F)*** focuses on
  - Squared differences of means in numerator
  - Squared standard deviations **(variances)** in denominator

Procedure called **ANOVA** (ANalysis Of VAriance)

Two- vs. Several-Sample Inference

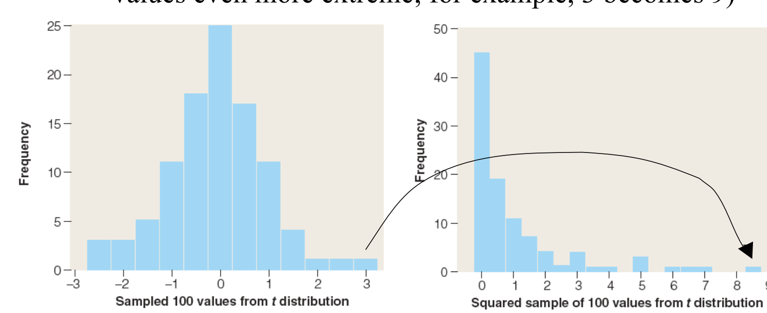
- **Similar:** test statistic standardizes difference among sample means, taking sample sizes and standard deviations into account.

For 2 groups of equal sizes and $\sigma_1 = \sigma_2, F = t^2$ and conclusions (including P-value) are the same.

t and F Distributions

- Left: sampled 100 values from a t distribution
- Right: squared the 100 values from t distribution

Squaring makes F non-negative, right-skewed (makes extreme values even more extreme; for example, 3 becomes 9)



Two- vs. Several-Sample Statistics

- **Similar:** test statistic standardizes how different sample means are, taking sample sizes and standard deviations into account

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$F = \frac{\left[n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + \cdots + n_I(\bar{x}_I - \bar{x})^2\right]/(I-1)}{\left[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \cdots + (n_I - 1)s_I^2\right]/(N-I)}$$

Two- vs. Several-Sample Statistics

- How different are sample means?
- How spread out are the distributions?
- How large are the samples?

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$F = \frac{\left[n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + \cdots + n_I(\bar{x}_I - \bar{x})^2\right]/(I-1)}{\left[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \cdots + (n_I - 1)s_I^2\right]/(N-I)}$$

What Makes *t* or *F* Statistics Large

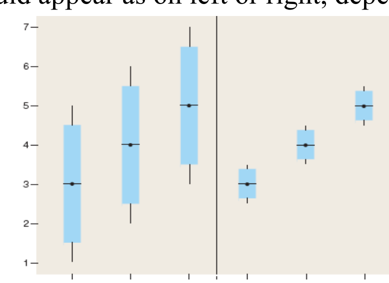
- Large diff among sample means (in numerator)
- Small spreads (in denominator)
- Large sample sizes (denominator of denominator)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$F = \frac{\left[n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + \cdots + n_I(\bar{x}_I - \bar{x})^2\right]/(I-1)}{\left[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \cdots + (n_I - 1)s_I^2\right]/(N-I)}$$

Example: *Sample S.D.s' Effect on P-Value*

- **Background**: Boxplots with $\bar{x}_1 = 3, \bar{x}_2 = 4, \bar{x}_3 = 5$ could appear as on left or right, depending on s.d.s.



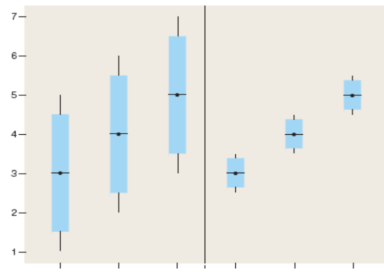
Context: *sample mean monthly pay (in \$1000s) for 3 racial/ethnic groups.*

- **Question:** For which scenario does the difference among means appear more significant?
- **Response:** Difference among means appears more significant on ______________

Practice: 11.35b p.563

Example: *Sample S.D.s' Effect on P-Value*

- **Background**: Boxplots with $\bar{x}_1 = 3, \bar{x}_2 = 4, \bar{x}_3 = 5$ could appear as on left or right, depending on s.d.s.



Context: *sample mean monthly pay (in \$1000s) for 3 racial/ethnic groups.*

- **Question:** For which scenario are we more likely to reject hypothesis of equal population means?
- **Response:** Scenario on ______: smaller s.d.s→larger F stat→ smaller P-val→likelier to reject H_0, conclude ____________

Measuring Variation Among and Within

$$F = \frac{\left[n_1(\bar{x}_1-\bar{x})^2 + n_2(\bar{x}_2-\bar{x})^2 + \cdots + n_I(\bar{x}_I-\bar{x})^2\right]/(I-1)}{\left[(n_1-1)s_1^2 + (n_2-1)s_2^2 + \cdots + (n_I-1)s_I^2\right]/(N-I)}$$

- **Numerator:** variation among groups
  - How different are $\bar{x}_1, \cdots, \bar{x}_I$ from one another?
- **Denominator:** variation within groups
  - How spread out are samples? (sds $s_1, \cdots, s_I$)

Numerator of F (Difference Among Means)

- **SSG:** Sum of Squared diffs among Groups

  $SSG = 5(3-4)^2 + 5(4-4)^2 + 5(5-4)^2 = \boxed{10}$
- **DFG:** Degrees of Freedom for Groups

  $DFG = I - 1 = 3 - 1 = \boxed{2}$
- **MSG:** Mean Squared diffs among Groups

  $MSG = \frac{SSG}{DFG} = \frac{10}{2} = \boxed{5}$

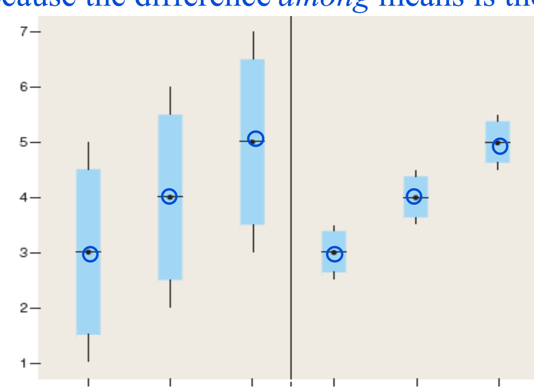
***I* = 3**

$n_1 = 5$	$\bar{x}_1 = 3$	$s_1 = 1.58$
$n_2 = 5$	$\bar{x}_2 = 4$	$s_2 = 1.58$
$n_3 = 5$	$\bar{x}_3 = 5$	$s_3 = 1.58$
$N = 15$	$\bar{x} = 4$	

monthly earnings (in \$1000s) for 3 racial/ethnic groups (hypothetical)

Numerator of F (Difference Among Means)

Note: numerator of F is the same for both scenarios because the difference *among* means is the same.



Denominator of F (Spread Within Groups)

- **SSE:** Sum of Squared Error within Groups

$SSE = (5-1)1.58^2 + (5-1)1.58^2 + (5-1)1.58^2 = \boxed{30}$

- **DFE:** Degrees of Freedom for Error

$DFE = N - I = 15 - 3 = \boxed{12}$

- **MSE:** Mean Squared Error within Groups

$MSE = \frac{SSE}{DFE} = \frac{30}{12} = \boxed{2.5}$

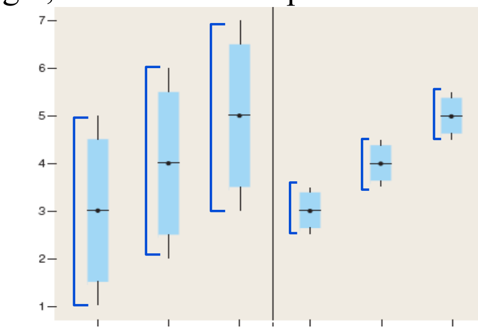
I = 3

$n_1 = 5$	$\bar{x}_1 = 3$	$s_1 = 1.58$
$n_2 = 5$	$\bar{x}_2 = 4$	$s_2 = 1.58$
$n_3 = 5$	$\bar{x}_3 = 5$	$s_3 = 1.58$
$N = 15$	$\bar{x} = 4$	

monthly earnings (in \$1000s) for 3 racial/ethnic groups (hypothetical)

Denominator of F (Spread Within Groups)

- Note: denominator of F is smaller for the scenario on the right, because of less spread.



- Because the numerators are the same, F (the quotient) is considerably larger on the right.

The F Statistic

$$F = \frac{\left[n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + \cdots + n_I(\bar{x}_I - \bar{x})^2\right]/(I-1)}{\left[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \cdots + (n_I - 1)s_I^2\right]/(N-I)}$$

$$= \frac{MSG}{MSE} = \frac{5}{2.5} = 2$$ ***Is 2 large???***

measures difference among sample means (relative to spreads and sample sizes)

If F is large, reject $H_o: \mu_1 = \mu_2 = \mu_3$

Conclude population means differ.

Example: *Size of Standardized Statistics*

- **Background:** Say standardized statistic is 2.
- **Question:** Is 2 large…
  - For z?
  - For t?
  - For F?
- **Response:**
  - z=2 large?__________ (combined tail probs 0.05)
  - t=2 large? depends on ____________________
  - F=2 large?

  depends on ______________________________

  (based on total sample size N **and** number of groups I)

F and its Degrees of Freedom

Family of F curves all non-neg, right-skewed.

Spreads vary, depending on DFG = I - 1 in numerator, DFE = N - I in denominator.

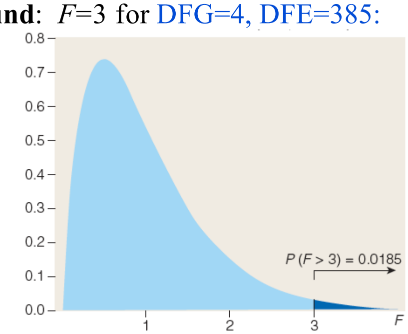
Example: *Degrees of Freedom for F*

- **Background**: Consider these F distributions
  - F with I=5, N=390
  - F with DFG=2, DFE=12 [written $F(2, 12)$]
- **Questions:**
  - What are degrees of freedom if I=5, N=390?
  - What are I and N if DFG=2, DFE=12?
- **Responses:**
  - I = 5, N = 390→
    DFG = ____________, DFE = ________________
  - DFG = 2, DFE = 12→

Practice: 11.29d-e p.561

Example: *Assessing Size of F Statistic*

- **Background**: F=3 for DFG=4, DFE=385:

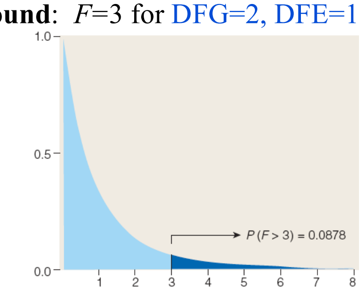


- **Questions:** Is F=3 large? Will we reject a claim that the 5 population means are equal?
- **Responses:** P-val= 0.0185 →Very little area past F=3 → F is _____Reject claim that 5 population means are equal?___

Practice: 11.77a p.582

Example: *Assessing F for Different DF*

- **Background**: F=3 for DFG=2, DFE=12



- **Questions:** Is F=3 large?
  What would we conclude if F=2 for DFG=2, DFE=12?
- **Responses:** P-val=0.0878→F=3 is ___________________
  P-val for F=2 must be _______________ Reject H_0?
  ____ Conclude population means may be equal? ____

Practice: 11.29h p.561

The F Statistic *(Review)*

$$F = \frac{\left[n_1(\bar{x}_1-\bar{x})^2 + n_2(\bar{x}_2-\bar{x})^2 + \cdots + n_I(\bar{x}_I-\bar{x})^2\right]/(I-1)}{\left[(n_1-1)s_1^2 + (n_2-1)s_2^2 + \cdots + (n_I-1)s_I^2\right]/(N-I)}$$

$$= \frac{MSG}{MSE} = \frac{5}{2.5} = 2$$ ***Is 2 large for DFG=2, DFE=12?*** ***NO***

measures difference among sample means
(relative to spreads and sample sizes)

If F is large, reject $H_o: \mu_1 = \mu_2 = \mu_3$

Conclude population means differ.

Example: *Drawing Conclusions Based on F*

- **Background**: Earnings for 5 sampled individuals from three racial/ethnic groups had means 3, 4, 5 (in thousands of dollars). ANOVA procedure resulted in F=2, which in this case is **not** large.
- **Question:** What do we conclude about mean earnings for populations in the three racial/ethnic groups?
- **Response:** Since F is not large, sample means ________ differ significantly from one another. Conclude population mean earnings ____________

Practice: 11.24j-k p.560

Example: *Role of n in ANOVA Test*

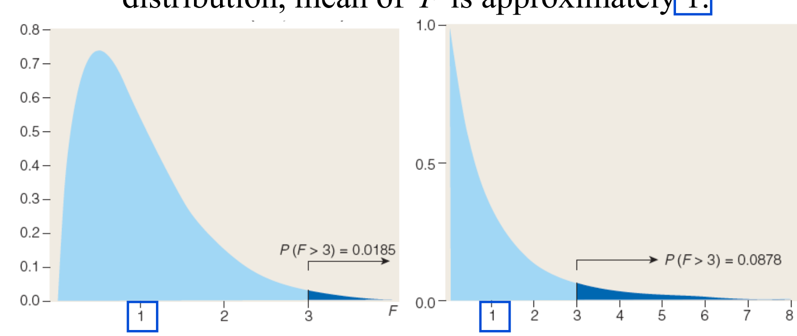
- **Background**: Earnings for **12** (instead of 5) sampled individuals from three racial/ethnic groups had means 3, 4, 5 (in thousands of dollars). ANOVA procedure resulted in ***F*=4.8**, and a P-value of 0.015.
- **Question:** What do we conclude about mean earnings for populations in the three racial/ethnic groups?
- **Response:** Conclude population mean earnings for the three groups are ____________

______samples help provide more evidence against Ho.

Practice: 11.29i p.561

Mean of F

Since t has s.d.= typical distance of values from 0 = approximately 1, and F is similar to squaring t distribution, mean of F is approximately 1.



Example: *Test Relationship/Parameters (Review)*

- **Background**: Research question: "For all students at a university, are Math SATs related to what year they're in?"
- **Question:** How can the question be reformulated in terms of relevant parameters (means) instead of in terms of whether or not the variables are related?
- **Response:**

Example: *Testing Relationship or Parameters*

- **Background**: Research question: "Do mean earnings differ significantly for three racial/ethnic groups?"
- **Question:** How can the question be reformulated in terms of relevant variables instead of in terms of whether or not the means are equal?
- **Response:**

Practice: 11.30j p.562

Lecture Summary
(Inference for Cat & Quan: ANOVA)

- Several-sample vs. 2-sample design
  - Notation
  - Compare and contrast t and F statistics
  - What makes t or F large?
- Variation among means or within groups; F as ratio of variations
- How large is "large" F?
  - F degrees of freedom
  - F distribution
- Role of sample size