Lecture 14: Chapter 6, Section 2
Finding Probabilities: More General Rules

- General "And" Rule
- More about Conditional Probabilities
- Two Types of Error
- Independence

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability
    - Finding Probabilities
    - Random Variables
    - Sampling Distributions
  - Statistical Inference

Probability Rules (*Review*)

Non-Overlapping "Or" Rule: For any two ***non-overlapping*** events A and B,

P(A or B)=P(A)+P(B).

Independent "And" Rule: For any two ***independent*** events A and B,

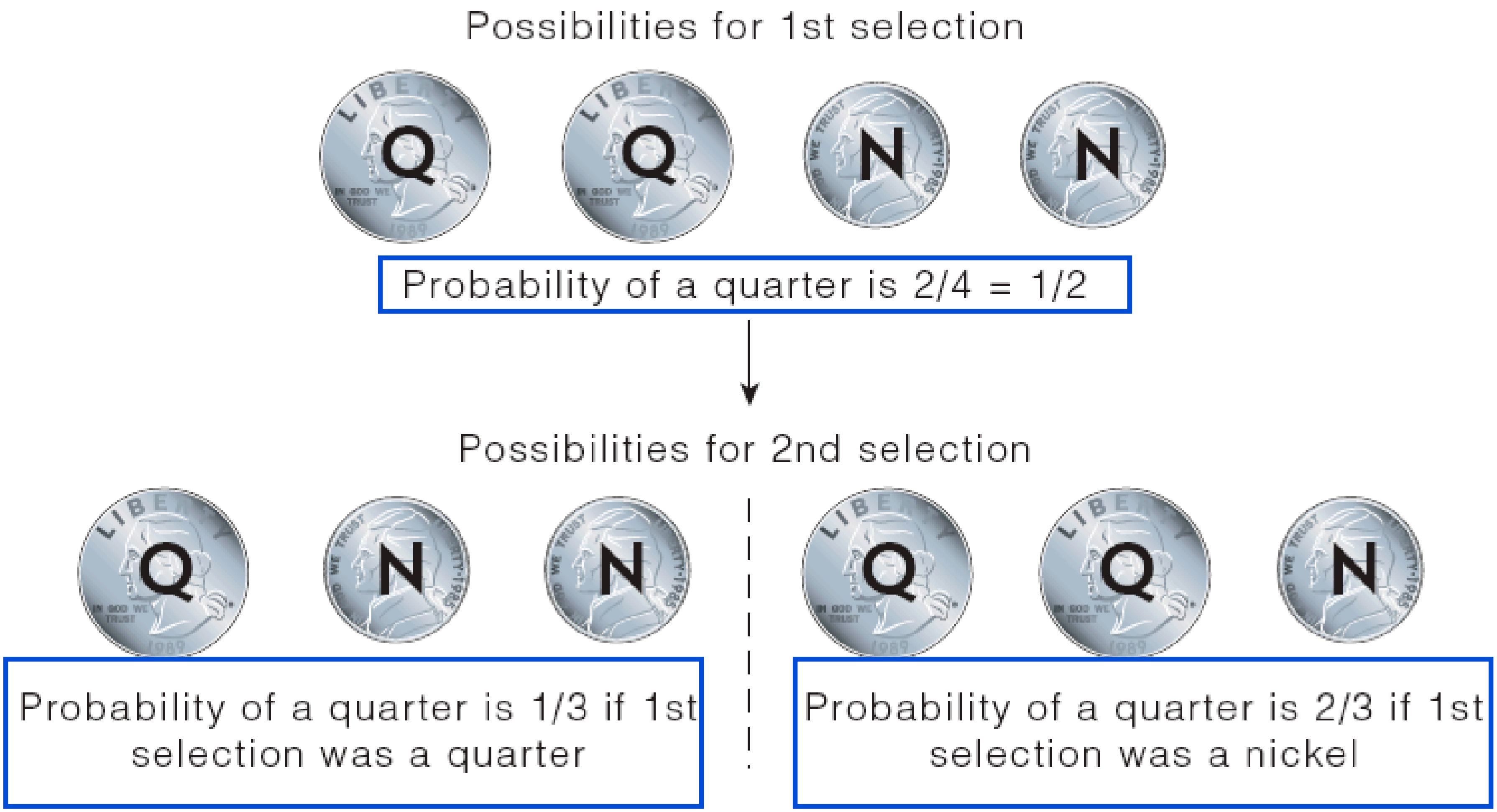
P(A and B)=P(A)×P(B).

General "Or" Rule: For any two events A and B,

P(A or B)=P(A)+P(B)-P(A and B).

Need "And" Rule that applies even if events are ***dependent***.

Example: *When Probabilities Can't Simply be Multiplied*



Definition and Notation

Conditional Probability of a second event, given a first event, is the probability of the second event occurring, assuming that the first event has occurred.

P(B given A) denotes the conditional probability of event B occurring, given that event A has occurred.

Looking Ahead: *Conditional probabilities help us handle dependent events.*

Example: *Intuiting the General "And" Rule*

- **Background**: In a child's pocket are 2 quarters and 2 nickels. He randomly picks a coin, does *not* replace it, and picks another.
- **Question:** What is the probability that the first *and* the second coin are quarters?
- **Response:** probability of first a quarter ( ), times (conditional) probability that second is a quarter, **given** first was a quarter ( ):

Practice: 6.16a p.253

Example: *Intuiting General "And" Rule with Two-Way Table*

- **Background**: Surveyed students classified by sex and ears pierced or not.

	Ears Pierced	Ears Not Pierced	Total
Female	270	30	300
Male	20	180	200
Total	290	210	500

- **Question:** What are the following probabilities?
  - Probability of being male
  - Probability of having ears pierced, given a student is male
  - Probability of being male and having ears pierced
- **Response:**
  - P(M) =
  - P(E given M) =
  - P(M and E)=

 Practice: 6.14g,i,b p.252

General "And" Rule (General Multiplication Rule)

For *any* two events A and B,

P(A and B)=P(A) × P(B given A)

=P(B) if A and B are independent

A Closer Look: *In general, the word "and" in probability entails multiplication.*

Example: *Applying General "And" Rule*

- **Background**: Studies suggest lie detector tests are "well below perfection", 80% of the time concluding someone is a spy when he actually is, 16% of the time concluding someone is a spy when he isn't. Assume 10 of 10,000 govt. employees are spies.
- **Question:** What are the following probabilities?
  - Probability of being a spy and being detected as one
  - Probability of *not* being a spy but "detected" as one
  - Overall probability of a positive lie detector test
- **Response:** First "translate" to probability notation:

  P(D given S)=____ P(D given not S)=____ P(S)=____ P(not S)=____
  - P(S and D) = ____
  - P(not S and D) = ____
  - P(D) = P(S and D or not S and D) = ____

Example: *"Or" Probability as Weighted Average of Conditional Probabilities*

- **Background**: Studies suggest lie detector tests are "well below perfection", 80% of the time concluding someone is a spy when he actually is, 16% of the time concluding someone is a spy when he isn't. Assume 10 of 10,000 govt. employees are spies.
- **Question:** Should we expect the overall probability of being "detected" as a spy, P(D), to be closer to P(D given S)=0.80 or to P(D given not S)=0.16?
- **Response:** Expect P(D) closer to
  because

  (In fact, P(D) =0.16064.)

Practice: 6.16c p.253

General "And" Rule Leads to Rule of Conditional Probability

Recall: For *any* two events A and B,

P(A and B)=P(A)×P(B given A)

Rearrange to form ***Rule of Conditional Probability:***

$$P(\text{B given A}) = \frac{P(\text{A and B})}{P(\text{A})}$$

Example: *Applying Rule of Conditional Probability*

- **Background**: For the lie detector problem, we have
  - Probability of being a spy: P(S)=0.001
  - Probability of spies being detected: P(D given S)=0.80
  - Probability of non-spies detected: P(D given not S)=0.16
  - Probability of being a spy and detected: P(D and S)=0.0008
  - Overall probability of positive lie detector: P(D)=0.16064
- **Question:** If the lie-detector indicates an employee is a spy, what is the probability that he actually is one?
- **Response:** P(S given D) =

Note: *P(S given D) is very different from P(D given S).*

A Closer Look: *Bayes Theorem uses conditional probabilities to find probability of earlier event, given later event is known to occur.*

 Practice: 6.16e,f p.253

Two Types of Error in Lie Detector Test

1st **Type of Error:** Conclude employee is a spy when he/she actually is not.

2nd **Type of Error:** Conclude employee is not a spy when he/she actually is.

Example: *Two Types of Error in Lie Detector Test*

- **Background**: For the lie detector problem, we have
  - Probability of spies being detected: P(D given S)= 0.80
  - Probability of non-spies detected: P(D given not S)= 0.16
- **Questions:**
  - What is probability of 1^{st} type of error (conclude employee is spy when he/she actually is not)?
  - What is probability of 2^{nd} type of error (conclude employee is not a spy when he/she actually is)?
- **Responses:**
  - 1^{st} type:
  - 2^{nd} type:

Testing for Independence

The concept of independence is tied in with conditional probabilities.

Looking Ahead: *Much of statistics concerns itself with whether or not two events, or two variables, are dependent (related).*

Example: *Intuiting Conditional Probabilities When Events Are Dependent*

- **Background**: Students are classified according to gender, M or F, and ears pierced or not, E or not E .

	Ears Pierced	Ears Not Pierced	Total
Female	270	30	300
Male	20	180	200
Total	290	210	500

- **Questions:**
  - Should gender and ears pierced be dependent or independent? If dependent, which should be less, P(E) or P(E given M)?
  - What are the above probabilities, and which is less?
- **Responses:**
  - Expect P(E given M) P(E) because fewer have pierced ears.
  - P(E given M) = P(E) =

Example: *Intuiting Conditional Probabilities When Events Are Independent*

- **Background**: Students are classified according to gender, M or F, and whether they get an A in stats.

	A	Not A	Total
Female	0.15	0.45	0.60
Male	0.10	0.30	0.40
Total	0.25	0.75	1.00

- **Questions:**
  - Should gender and getting an A or not be dependent or independent? How should P(A) and P(A given F) compare?
  - What are the above probabilities, and how do they compare?
- **Responses:**
  - Expect P(A given F) P(A) because knowing a student's gender doesn't impact probability of getting an A.
  - P(A)= ; P(A given F)=

Practice: 6.19a,b p.254

Independence and Conditional Probability

Rule:

A and B independent → $P(B)=P(B \text{ given } A)$

Test:

$P(B)=P(B \text{ given } A)$ → A and B are independent

$P(B) \neq P(B \text{ given } A)$ → A and B are dependent

Independent ←→ regular and conditional probabilities are equal (occurrence of A doesn't affect probability of B)

Independence and Product of Probabilities

Rule:

Independent→$P(A \text{ and } B)=P(A)\times P(B)$

Test:

$P(A \text{ and } B) = P(A)\times P(B)$→independent

$P(A \text{ and } B) \neq P(A)\times P(B)$→dependent

Independent←→probability of both equals product of individual probabilities

Table of Counts Expected if Independent

- For A, B independent,
  P(A and B)=P(A)×P(B).
- This Rule dictates what counts would appear in two-way table if the variable A or not A is independent of the variable B or not B:
- If independent, count in category-combination A and B must equal total in A times total in B, divided by overall total in table.

Example: *Counts Expected if Independent*

- **Background**: Students are classified according to gender and ears pierced or not. A table of expected counts ($174 = \frac{290 \times 300}{500}$, etc.) has been produced.

Counts expected if gender and pierced ears were independent

	E	not E	Total
not M	174	126	300
M	116	84	200
Total	290	210	500

Counts actually observed

	E	not E	Total
not M	270	30	300
M	20	180	200
Total	290	210	500

- **Question:** How different are the observed and expected counts?
- **Response:** Observed and expected counts are very different (270 vs. 174, 20 vs. 116, etc.) because

Practice: 6.18e-j p.254

Example: *Counts Expected if Independent*

- **Background**: Students are classified according to gender and grade (A or not). A table of expected counts ($15 = \frac{25 \times 60}{100}$, etc.) has been produced.

Exp	A	not A	Total
F	15	45	60
M	10	30	40
Total	25	75	100

Obs	A	not A	Total
F	15	45	60
M	10	30	40
Total	25	75	100

- **Question:** How different are the observed and expected counts?
- **Response:** Counts are identical because

Lecture Summary
(Finding Probabilities; More General Rules)

- General "And" Rule
- More about Conditional Probabilities
- Two Types of Error
- Independence
  - Testing for independence
  - Rule for independent events
  - Counts expected if independent