Lecture 22: Chapter 9, Section 2 Inference for Categorical Variable: Hypothesis Tests

- 4 steps in Hypothesis Test; Posing Hypotheses
- Details of 4 Steps, Definitions and Notation
- 3 Forms of Alternative Hypothesis
- *P*-Value
- Example with "Greater Than" Alternative

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability (discussed in Lectures 13-20)
  - Statistical Inference
    - 1 categorical: confidence intervals, hypothesis tests
    - 1 quantitative
    - categorical and quantitative
    - 2 categorical
    - 2 quantitative

Three Types of Inference Problem *(Review)*

In a sample of 446 students, 0.55 ate breakfast.

1. What is our best guess for the proportion of all students who eat breakfast?

   Point Estimate
2. What interval should contain the proportion of all students who eat breakfast?

   Confidence Interval
3. Do more than half (50%) of all students eat breakfast?

   Hypothesis Test

4 Steps in Hypothesis Test About p

(First pose question as choice between 2 opposing views about p.)

1. Check data production for bias.
2. We summarize with $\hat{p}$, standardize to z.
3. Find probability of $\hat{p}$ this extreme.
4. Perform inference, drawing conclusions about population proportion p.

These correspond to 4 Processes of Statistics.

Example: *Posing Hypothesis Test Question*

- **Background**: In a sample of 446 students, 0.55 ate breakfast. Do more than half of all students at that university eat breakfast?
- **Question:** How can we pose above question as two opposing points of view about p?
- **Response:**

4 Steps in Hypothesis Test About p

(First pose question as choice between 2 opposing views about p.)

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Example: *Considering Data Production*

- **Background**: In a sample of 446 college students, 0.55 ate breakfast. We want to draw conclusions about breakfast habits of all students at that university.
- **Question:** What data production issues should be considered?
- **Response:** (discussed with confidence intervals)
  - Sampling: ____________________ } *(for claims*
  - Study design: __________________ } *about ______)*

  Also, *(for claims about ______)* is population ≥ 10n?

  And *(for claims about ______)* is n large enough?

4 Steps in Hypothesis Test About p

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Behavior of Sample Proportion *(Review)*

For random sample of size n from population with p in category of interest, sample proportion $\hat{p}$ has

- mean p
- standard deviation $\sqrt{\frac{p(1-p)}{n}}$

Hypothesis test: assume pop. proportion p is proposed value=0.50 for breakfast example.

Looking Back: *For confidence intervals, we had to substitute sample proportion for unknown p.*

Example: *Summarizing and Standardizing*

- **Background**: In a sample of 446 students, 0.55 ate breakfast. Do more than half of all students at that university eat breakfast?
- **Question:** How do we summarize the data?
- **Response:** Summarize with ____________.
  Standardize to

So 0.55 is _____ standard deviations above 0.50: pretty unusual.

4 Steps in Hypothesis Test About p

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3. Find probability of $\hat{p}$ this extreme.
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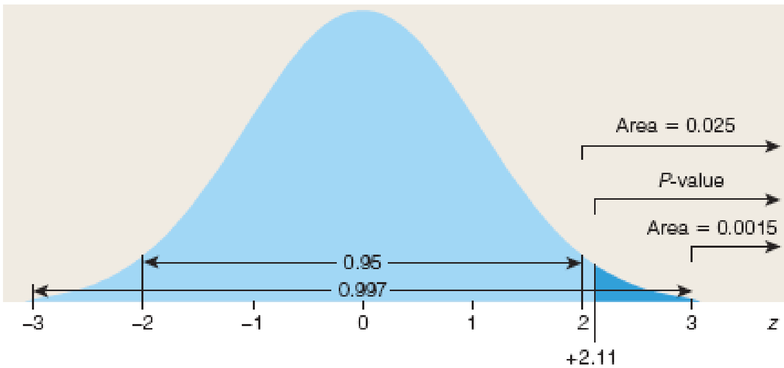
Example: *Estimating Relevant Probability*

- **Background**: In a sample of 446 students, 0.55 ate breakfast. Do more than half of all students eat breakfast? We summarized with $\hat{p} = 0.55$ and $z = \frac{0.55-0.50}{\sqrt{\frac{0.50(1-0.50)}{446}}} = +2.11$
- **Question:** If p=0.50, how unlikely is it to get $\hat{p}$ as high as 0.55 (that is, for z to be ≥ +2.11)?
- **Response:** 68-95-99.7 Rule→ since 2.11 > 2,
  $P(Z \geq +2.11)$ is ________________________
  Such a probability can be considered to be ____________.

Illustration of Relevant Probability



Looking Ahead: *The relevant probability for testing a hypothesis will be defined as the **P-value**.*

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Example: *Drawing Conclusions About p*

- **Background**: In a sample of 446 students, 0.55 ate breakfast. Do more than half of all students eat breakfast? We summarized with $\hat{p} = 0.55$ and $z = \frac{0.55-0.50}{\sqrt{\frac{0.50(1-0.50)}{446}}} = +2.11$

The probability of z being +2.11 or higher is less than (1-0.95)÷2= 0.025, (fairly unlikely).

- **Question:** What do we conclude about p?
- **Response:**

Practice: 9.59 p.446

Hypothesis Test About p *(More Details)*

First state 2 opposing views about p, called null and alternative hypotheses H_o and H_a.

1. Consider sampling and study design as for C.I.
2. Summarize with $\hat{p}$; does it tend in the suspected direction? Standardize to z, assuming $p = p_o$ (p_o is proposed value); consider if z is "large".
3. Find prob. of $\hat{p}$ this high/low/different, called '*P*-value' of the test; consider if it is "small".
4. Draw conclusions about p: choose between null and alternative hypotheses. (Statistical Inference)

Definitions

- **Null hypothesis** H_o **:** claim that parameter equals proposed value.
- **Alternative hypothesis** H_a **:** claim that parameter differs in some way from proposed value.
- ***P*-value:** probability, assuming H_o is true, of obtaining sample data at least as extreme as what has been observed.

Looking Back: *We considered the* ***probability****, assuming p=0.5 cards are red, of getting as few as 0 red cards in 4 or 5 picks.*

Notation

Proposed value of population proportion: p_o

Null and alternative hypotheses in test about unknown population proportion:

$$H_0: p = p_0 \quad \text{vs.} \quad H_a: \left\{ \begin{array}{l} p > p_0 \\ p < p_0 \\ p \neq p_0 \end{array} \right\}$$

Looking Ahead: *The form of the alternative hypothesis will affect Steps 2, 3, 4 of the test.*

Example: *What Are We Testing About?*

- **Background**: Consider 3 problems:
  - 30/400=0.075 students picked #7 "at random" from 1 to 20. Is this evidence of bias for #7?
  - Do fewer than half of commuters walk? 111/230 of surveyed commuters at a university walked.
  - % disadvantaged in Florida community colleges is 43%. Is Florida Keys College unusual with 47.5% disadvantaged?
- **Question:** In each case, are we trying to draw conclusions about a sample proportion $\hat{p}$ or a population proportion p?
- **Response:** ____________________

Looking Ahead: *We'll refer to sample proportion later, to decide which of two claims to believe about the unknown population proportion.*

Example: *Three Forms of Alternative*

- **Background**: Consider 3 problems:
  - 30/400=0.075 students picked #7 "at random" from 1 to 20. Is this evidence of bias for #7?
  - Do fewer than half of commuters walk? 111/230 of surveyed commuters at a university walked.
  - % disadvantaged in Florida community colleges is 43%. Is Florida Keys College unusual with 47.5% disadvantaged?
- **Question:** How do we write the hypotheses in each case?
- **Response:**
  - ____________________
  - ____________________
  - ____________________

Practice: 9.68a p.449

Definitions

- **One-sided alternative hypothesis** refutes equality with > or < sign
- **Two-sided alternative hypothesis** features a not-equal sign

Note: For a one-sided alternative, sometimes the accompanying null hypothesis is written as a (not strict) inequality. Either way, the same conclusions will be reached.

Assessing Merit of Data in One-Sided Test

If sample proportion does not tend in the direction claimed by alternative hypothesis in a 1-sided test, there is no need to proceed further.

Example: *When Test Can Be Cut Short*

- **Background**: The moon has four phases: new moon, first quarter, full moon, and last quarter, each in effect for 25% of the time. A neurologist whose patients claimed their seizures tended to be triggered by a full moon found 20% of 470 seizures were at full moon.
- **Question:** Do we need to carry out all 4 steps in the test?
- **Response:**

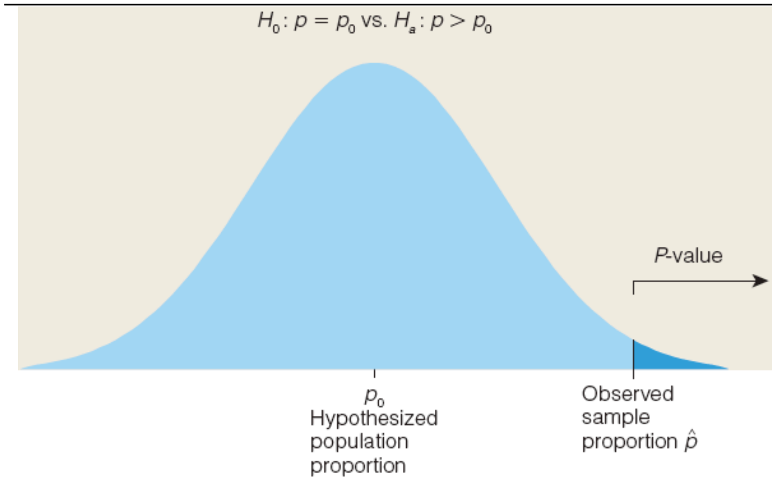
Practice: 9.40a p.441

How to Assess *P*-Value

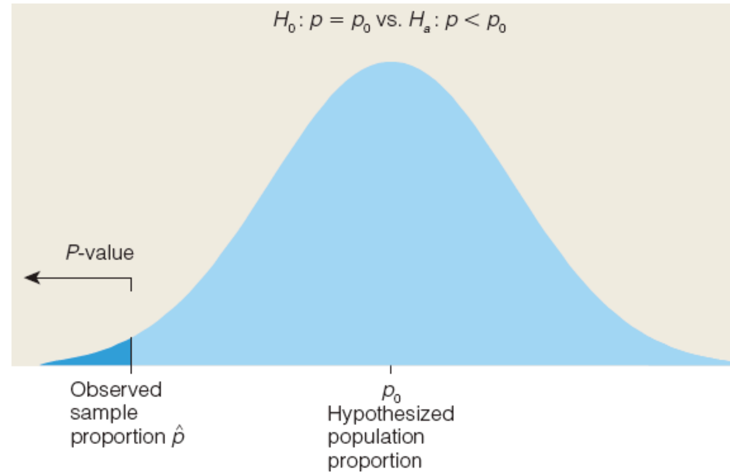
***P*-value:** probability, assuming H_o is true, of obtaining sample data at least as extreme as what has been observed. How to find *P*-value depends on form of alternative hypothesis:

- Right-tailed probability for $H_a : p > p_o$
- Left-tailed probability for $H_a : p < p_o$
- Two-tailed probability for $H_a : p \neq p_o$

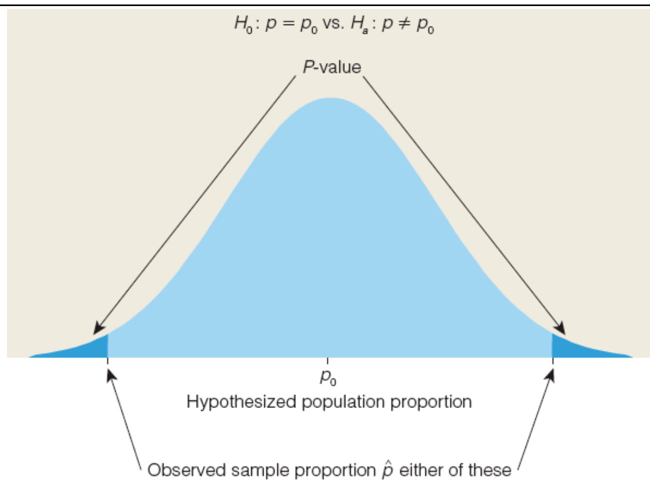
P-Value for $H_a : p > p_o$ is Right-tailed Probability



P-Value for $H_a : p < p_o$ is Left-tailed Probability



P-Value for $H_a : p \neq p_o$ is Two-tailed Probability



Drawing Correct Conclusions

Two possible conclusions:

- P-value small→reject H_o→conclude H_a. State we have evidence in favor of H_a.
  (not same as **proving** H_a true and H_o false).
- P-value not small→don't reject H_o→ conclude H_o may be true.
  (not same as **proving** H_o true and H_a false)

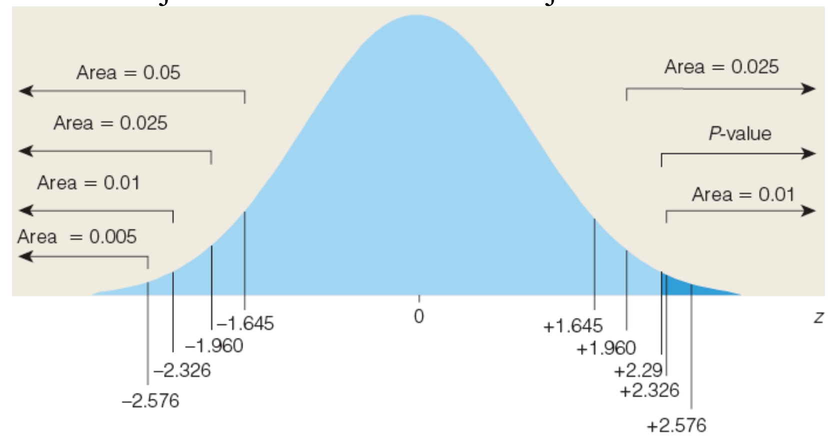
Example: *Test with "Greater Than" Alternative*

- **Background**: 30/400=0.075 students picked #7 "at random" from 1 to 20.
- **Question:** In general, is p>0.05? (evidence of bias?)
- **Response:** First write H_o:_________ vs. H_a: _________

1. Students are "typical" humans; bias is issue at hand.
2. 0.075 > 0.05 so the *sample* did favor #7. If p = 0.05, $\hat{p}$ standardizes to z =
3. *P*-value = ______________________________
4. Reject H_o? _____ Conclude? ____________________

Assessing a *P*-value with 90-95-98-99 Rule

2.29 just under 2.326 ➔ *P*-value just over 0.01



Lecture Summary
(Inference for Proportions: Hypothesis Test)

- 4 steps in hypothesis test
  - Checking data production
  - Summarizing and standardizing
  - Finding a probability (*P*-value)
  - Conclusions as inference
- Posing null and alternative hypotheses
- Definitions and notation
- 3 forms of alternative hypothesis
- Assessing *P*-value
- Example with "greater than" alternative