

Lecture 11: Chapter 5, Section 3 Relationships between Two Quantitative Variables; Correlation

- Display and Summarize
- Correlation for Direction and Strength
- Properties of Correlation
- Regression Line

Looking Back: Review

- 4 Stages of Statistics
 - Data Production (discussed in Lectures 1-4)
 - Displaying and Summarizing
 - Single variables: 1 cat, 1 quan (discussed Lectures 5-8)
 - Relationships between 2 variables:
 - Categorical and quantitative (discussed in Lecture 9)
 - Two categorical (discussed in Lecture 10)
 - Two quantitative
 - Probability
 - Statistical Inference

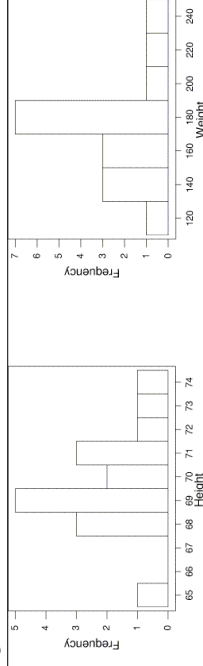
Review

- Single quantitative variables
 - Display with histogram
 - Summarize with mean and standard deviation

Example: Two Single Quantitative Variables

- Background: Data on male students' heights and weights:

Variable	N	Mean	Median	Tr-Mean	StDev	SE Mean
height	17	69.765	69.000	69.800	2.137	0.518
weight	17	170.59	175.00	169.33	28.87	7.00



- Question: What do these tell us about the relationship between male height and weight?

- Response:

Definition

- **Scatterplot** displays relationship between 2 quantitative variables:
 - Explanatory variable (x) on horizontal axis
 - Response variable (y) on vertical axis

Example: Explanatory/Response Roles

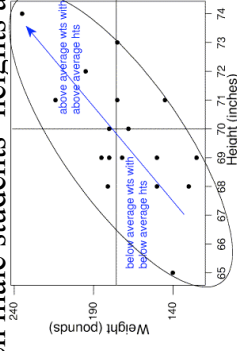
- **Background:** We're interested in the relationship between male students' heights and weights.
- **Question:** Which variable should be graphed along the horizontal axis of the scatterplot?
- **Response:**

Definitions

- **Form:** relationship is **linear** if scatterplot points cluster around some straight line
- **Direction:** relationship is
 - **positive** if points slope upward left to right
 - **negative** if points slope downward left to right

Example: Form and Direction

- **Background:** Scatterplot displays relationship between male students' heights and weights.



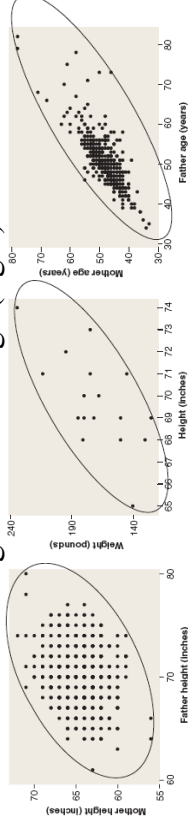
- **Question:** What are the form and direction of the relationship?
- **Response:** Form is _____ direction is _____

Strength of a Linear Relationship

- **Strong:** scatterplot points **tightly clustered** around a line
 - Explanatory value tells us **a lot** about response
- **Weak:** scatterplot points **loosely scattered** around a line
 - Explanatory value tells us **little** about response

Example: Relative Strengths

- **Background:** Scatterplots display:
 - mothers' ht. vs. fathers' ht. (left)
 - males' wt. vs. ht. (middle)
 - mothers' age vs. fathers' age (right)

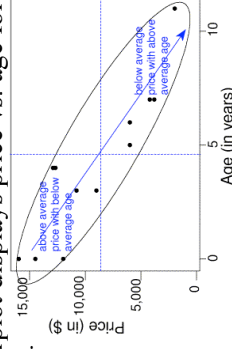


- **Question:** How do relationships' strengths compare? (Which is strongest, which is weakest?)

- **Response:** Strongest is on _____, weakest is on _____
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Example: Negative Relationship

- **Background:** Scatterplot displays price vs. age for 14 used Pontiac Grand Am's.



- **Questions:**
 - Why should we expect the relationship to be negative?
 - Does it appear linear? Is it weak or strong?

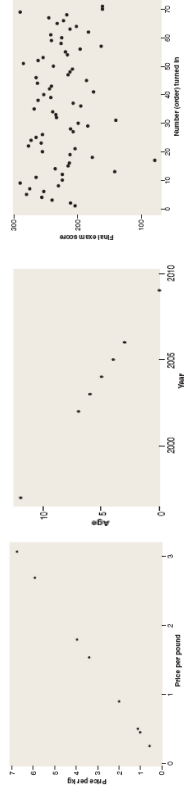
- **Responses:** _____
- _____

Definition

- **Correlation r :** tells direction and strength of linear relation between 2 quantitative variables
 - **Direction:** r is
 - positive for positive relationship
 - negative for negative relationship
 - zero for no relationship
 - **Strength:** r is between -1 and $+1$; it is
 - close to 1 in absolute value for strong relationship
 - close to 0 in absolute value for weak relationship
 - close to 0.5 in absolute value for moderate relationship

Example: Extreme Values of Correlation

- Background: Scatterplots show relationships...
 - (left) Price per kilogram vs. price per pound for groceries
 - (middle) Used cars' age vs. year made
 - (right) Students' final exam score vs. order handed in



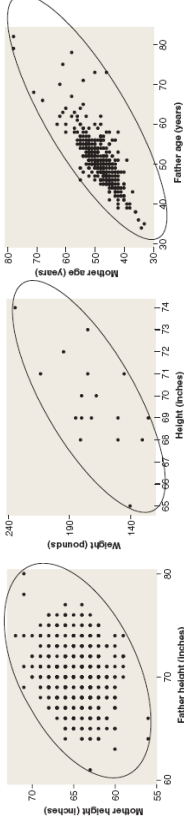
- Question: Correlations (scrambled) are -1, 0, +1. Which goes with each scatterplot?

- Response: left $r = 1$; middle $r = 0$; right $r = -1$

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Example: Relative Strengths

- Background: Scatterplots display:
 - mothers' ht. vs. fathers' ht. (left)
 - males' wt. vs. ht. (middle)
 - mothers' age vs. fathers' age (right)



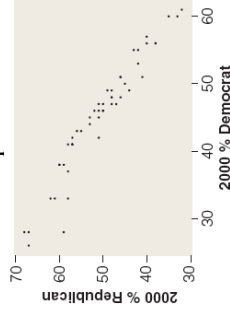
- Question: Which graphs go with which correlation: $r = 0.23$, $r = 0.78$, $r = 0.65$?

- Response: left $r = 0.23$; middle $r = 0.65$; right $r = 0.78$

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Example: Imperfect Relationships

- Background: For 50 states, % voting Republican vs. % Democrat in 2000 presidential election had $r = -0.96$.



- Questions: Why should we expect the relationship to be negative? Why is it imperfect?

- Responses:
 - Negative:
 - Imperfect:

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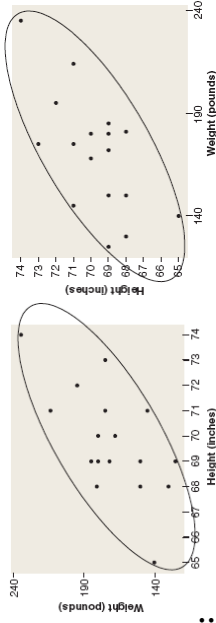
More about Correlation r

- Tells direction and strength of linear relation between 2 quantitative variables
 - A strong curved relationship may have r close to 0
 - Correlation not appropriate for categorical data
- Unaffected by roles explanatory/response
- Unaffected by change of units
- Overstates strength if based on averages

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Example: Correlation when Roles are Switched

□ **Background:** Male students' **wt vs ht (left)** or **ht vs wt (right)**:



□ **Questions:**

- How do directions and strengths compare, left vs. right?
- How do correlations r compare, left vs. right?

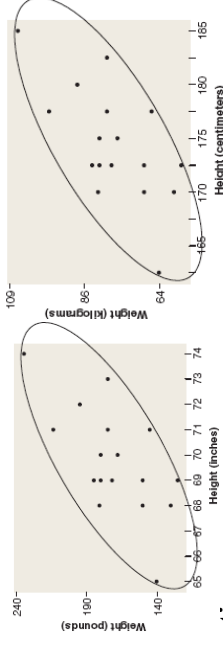
□ **Responses:**

- _____
- _____

Example: Correlation when Units are Changed

□ **Background:** For male students plot...

Left: wt (lbs) vs. ht (in) or **Right:** wt (kg) vs. ht (cm)



□ **Question:**

- How do directions, strengths, and r compare, left vs. right?

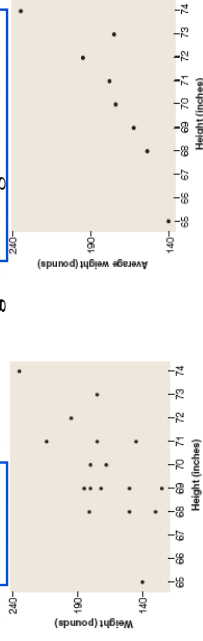
□ **Response:**

Example: Correlation Based on Averages

□ **Background:** For male students plot...

Ht	65	68	69	70	71	72	73	74					
Wt	140	130	150	172	180	168	180	145	175	214	195	175	235
AvWt	140	153.7	162.4	174.0	178.0	195	175	235					

Left: wt. vs. ht. or **Right:** average wt. vs. ht.



□ **Question:** Which one has $r = +0.87$? (other $r = +0.65$)

□ **Response:** Plot on _____ has $r = +0.87$ (stronger).

Least Squares Regression Line

If form appears **linear**, then we picture points clustered around a straight line.

■ **Questions (Rhetorical):**

1. Is there only one “best” line?
2. If so, how can we find it?
3. If found, how can we use it?

■ **Responses:** (in reverse order)

3. If found, can use line to make predictions.

Least Squares Regression Line

- **Response:**
3. If found, can use line to **make predictions**.
Write equation of line $\hat{y} = b_0 + b_1x$:
 - Explanatory value is x
 - Predicted response is \hat{y}
 - y-intercept is b_0
 - Slope is b_1and use the line to **predict** a response for any given explanatory value.

Least Squares Regression Line

If form appears linear, then we picture points clustered around a straight line.

- **Questions:**
1. Is there only one “best” line?
 2. If so, how can we find it?
 3. If found, how can we use it? *Predictions*
- **Response:**
2. Find line that **makes best predictions**.

Least Squares Regression Line

- **Response:**
2. Find line that **makes best predictions**:
Minimize sum of squared *residuals* (prediction errors). Resulting line called **least squares line** or **regression line**.

A Closer Look: *The mathematician Sir Francis Galton called it the “regression” line because of the “regression to mediocrity” seen in any imperfect relationship: besides responding to x , we see y tending towards its average value.*

Least Squares Regression Line

If form appears linear, then we picture points clustered around a straight line.

- **Questions:**
1. Is there only one “best” line?
 2. If so, how can we find it? *Minimize errors*
 3. If found, how can we use it? *Predictions*
- **Response:**
1. Methods of calculus → unique “best” line

Least Squares Regression Line

If form appears linear, then we picture points clustered around a straight line.

- **Questions:**

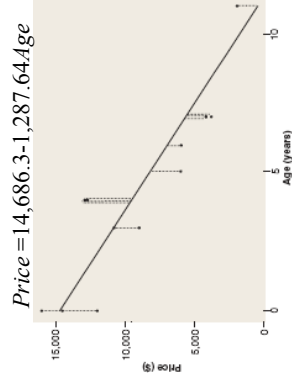
1. Is there only one “best” line?
2. If so, how can we find it?
3. If found, how can we use it?

- **Response:**

1. “Best” line has $b_1 = r \frac{s_y}{s_x}$ $b_0 = \bar{y} - b_1 \bar{x}$

Example: Least Squares Regression Line

- **Background:** Car-buyer wants to know if \$4,000 is a fair price for an 8-yr-old Grand Am; uses software to regress price on age for 14 used Grand Am’s:



- **Question:** How can she use the line?

- **Response:** Predict for $x=8$, \hat{y}

Lecture Summary

(Quantitative Relationships; Correlation)

- Display with scatterplot
- Summarize with form, direction, strength
- Correlation r tells direction and strength
- Properties of r
 - Unaffected by explanatory/response roles
 - Unaffected by change of units
 - Overstates strength if based on averages
- Least squares regression line for predictions