Lecture 8: Chapter 4, Section 4 Quantitative Variables (Normal)

- 68-95-99.7 Rule
- Normal Curve
- z-Scores

Looking Back: *Review*

- **4 Stages of Statistics**
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing
    - Single variables: 1 cat. (Lecture 5), 1 quantitative
    - Relationships between 2 variables
  - Probability
  - Statistical Inference

Quantitative Variable Summaries *(Review)*

- **Shape:** tells which values tend to be more or less common
- **Center**: measure of what is typical in the distribution of a quantitative variable
- **Spread:** measure of how much the distribution's values vary

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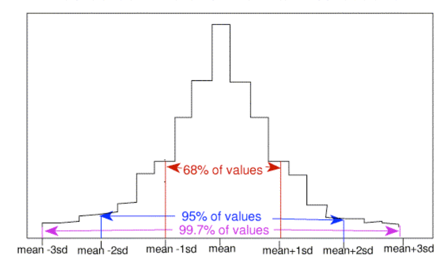
- **Mean (center):** arithmetic average of values
- **Standard deviation (spread)**: typical distance of values from their mean

68-95-99.7 Rule *(Review)*

If we know the shape is normal, then values have

- 68% within 1 standard deviation of mean
- 95% within 2 standard deviations of mean
- 99.7% within 3 standard deviations of mean

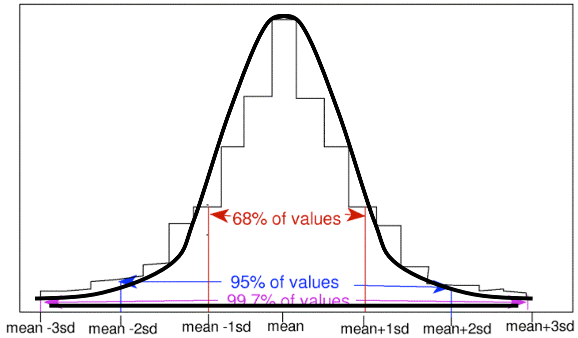
68-95-99.7 Rule for Normal Distributions



***A Closer Look:** around 2 sds above or below the mean may be considered unusual.*

From Histogram to Smooth Curve *(Review)*

- Infinitely many values over continuous range of possibilities modeled with normal curve.

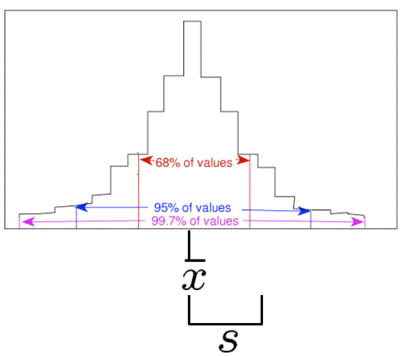


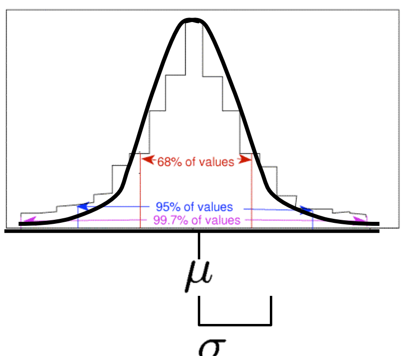
Quantitative Samples vs. Populations

- Summaries for sample of values
  - Mean $\bar{x}$
  - Standard deviation s
- Summaries for population of values
  - Mean μ (called "mu")
  - Standard deviation σ (called "sigma")

Notation: Mean and Standard Deviation

- Distinguish between sample (on the left) and population (on the right).





Example: *Notation for Sample or Population*

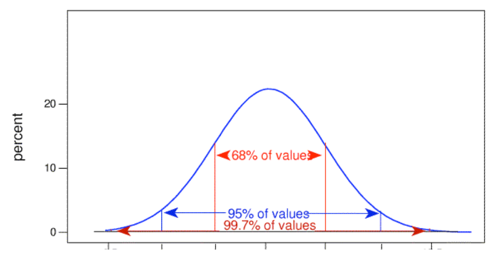
- **Background:** Adult male foot lengths are normal with mean 11, standard deviation 1.5. A sample of 9 male foot lengths had mean 11.2, standard deviation 1.7.
- **Questions:**
  - What notation applies to sample?
  - What notation applies to population?
- **Responses:**
  - If summarizing sample:
  - If summarizing population:

Practice: 4.55a-b p.121

Example: *Picturing a Normal Curve*

- **Background:** Adult male foot length normal with mean 11, standard deviation 1.5 (inches)
- **Question:** How can we display all such foot lengths?
- **Response:** Apply Rule to normal curve:

Normal curve for all adult male foot lengths



Example: *When Rule Does Not Apply*

- **Background:** Ages of all undergrads at a university have mean 20.5, standard deviation 2.9 (years).
- **Question:** How could we display the ages?
- **Response:**

Standardizing Normal Values

We count distance from the mean, in standard deviations, through a process called **standardizing**.

Example: *Standardizing a Normal Value*

- **Background:** Ages of mothers when giving birth is approximately normal with mean 27, standard deviation 6 (years).
- **Question:** Are these mothers unusually old to be giving birth? (a) Age 35 (b) Age 43
- **Response:**

(a) Age 35 is ________________ sds above mean:
Unusually old? ____

(b) Age 43 is ________________ sds above mean:
Unusually old? ____

Definition

- **z-score**, or **standardized value**, tells how many standard deviations below or above the mean the original value x is:

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$

- Notation:
  - Sample: $z = \frac{x - \bar{x}}{s}$
  - Population: $z = \frac{x - \mu}{\sigma}$
- **Unstandardizing** z-scores:

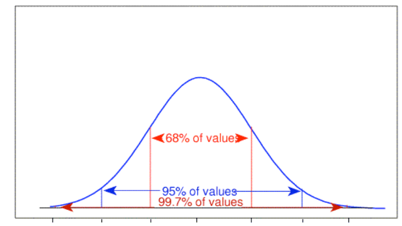
Original value x can be computed from z-score.

Take the mean and add z standard deviations:

$$x = \mu + z\sigma$$

Example: *68-95-99.7 Rule for z*

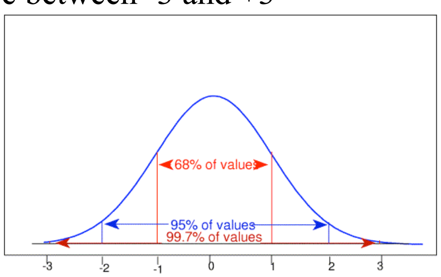
- **Background:** The 68-95-99.7 Rule applies to any normal distribution.
- **Question:** What does the Rule tell us about the distribution of standardized normal scores z?
- **Response:** Sketch a curve with mean__, standard deviation __:



68-95-99.7 Rule for z-scores

For distribution of standardized normal values z,

- 68% are between -1 and +1
- 95% are between -2 and +2
- 99.7% are between -3 and +3



Example: *What z-scores Tell Us*

- **Background:** On an exam (normal), two students' z-scores are -0.4 and +1.5.
- **Question:** How should they interpret these?
- **Response:**
  - -0.4: ______________________________
  - +1.5: ______________________________

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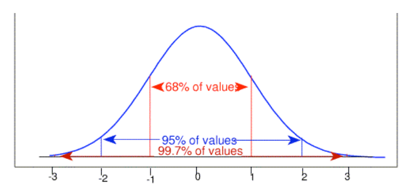

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Interpreting z-scores

This table classifies ranges of z-scores informally, in terms of being unusual or not.

Size of z	Unusual?
$\|z\|$ greater than 3	extremely unusual
$\|z\|$ between 2 and 3	very unusual
$\|z\|$ between 1.75 and 2	unusual
$\|z\|$ between 1.5 and 1.75	maybe unusual (depends on circumstances)
$\|z\|$ between 1 and 1.5	somewhat low/high, but not unusual
$\|z\|$ less than 1	quite common

Example: *Calculating and Interpreting z*

- **Background:** Adult heights are normal:
  - Females: mean 65, standard deviation 3
  - Males: mean 70, standard deviation 3
- **Question:** Calculate your own z score; do standardized heights conform well to the 68-95-99.7 Rule for females and for males in the class?
- **Response:** *Females and then males should calculate their z-score; acknowledge if it's*
  - *between -1 and +1?*
  - *between -2 and +2? beyond -2 or +2?*
  - *between -3 and +3? beyond -3 or +3?*

Practice: 4.48b p.119

Example: *z Score in Life-or-Death Decision*

- **Background:** IQs are normal; mean=100, sd=15. In 2002, Supreme Court ruled that execution of mentally retarded is cruel and unusual punishment, violating Constitution's 8th Amendment.
- **Questions:** A convicted criminal's IQ is 59. Is he borderline or well below the cut-off (z=-2)for mental retardation? Is the death penalty appropriate?
- **Response:** His z-score is ____________________

____

Practice: 4.55f p.121

Example: *From z-score to Original Value*

- **Background:** IQ's have mean 100, sd. 15.
- **Question:** What is a student's IQ, if z=+1.2?
- **Response:**

Practice: 4.48d p.119

Example: *Negative z-score*

- **Background:** Exams have mean 79, standard deviation 5. A student's z score on the exam is -0.4.
- **Question:** What is the student's score?
- **Response:**

If z is negative, then the value x is below average.

Example: *Unstandardizing a z-score*

- **Background:** Adult heights are normal:
  - Females: mean 65, standard deviation 3
  - Males: mean 70, standard deviation 3
- **Question:** Have a student report his or her z-score; what is his/her actual height value?
- **Response:**
  - Females: take $65+z(3)=$___
  - Males: take $70+z(3)=$___

Example: *When Rule Does Not Apply*

- **Background**: Students' computer times had mean 97.9 and standard deviation 109.7.
- **Question:** How do we know the distribution of times is not normal?
- **Response:**

Lecture Summary *(Normal Distributions)*

- Notation: sample vs. population
- Standardizing: z=(value-mean)/sd
- 68-95-99.7 Rule: applied to standard scores z
- Interpreting Standard Score z
- Unstandardizing: x=mean+z(sd)