

Lecture 27/Chapter 24

Significance vs. Importance

Undetected Differences

- Review Decision in z Test
- Factors Impacting Decision in z Test
- Examples

Hypothesis Test for Means: Details

1. null hypothesis: pop mean = proposed value
alt hyp: pop mean < or > or ≠ proposed value
 2. Find sample mean (and sd) and standardize to z.
 3. Find the P-value = prob of z this far from 0
 4. If the P-value is small, conclude alt hyp is true.
- Final conclusion hinges on size of P-value (is it small?) which hinges on size of z (is it large?).

Standardized Sample Mean

- To test a hypothesis about an unknown population mean, find sample mean (and standard deviation) and standardize to

$$z = \frac{\text{sample mean} - \text{pop mean}}{\frac{\text{standard deviation}}{\sqrt{\text{sample size}}}} = \frac{(\text{sample mean} - \text{pop mean})\sqrt{\text{sample size}}}{\text{standard deviation}}$$

- z is called the **test statistic**.

Note that “sample mean” is what we’ve observed, “population mean” is the value proposed in the null hypothesis, and “standard deviation” is from population (preferred) or sample (OK if sample size ≥ 30).

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What makes z large?

- Large difference between observed sample mean and hypothesized population mean
- Large sample size
- Small standard deviation (recall discussion of natural variability in Chapter 3 p. 49)

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Conversely, when is z not large?

- Observed sample mean close to hypothesized population mean
- Small sample size
- Large standard deviation

Making a Decision Based on a P -value

If the P -value in our hypothesis test is **small**, our sample mean is probably low/high/different, assuming the null hypothesis to be true. We conclude it is **not** true: we reject the null hypothesis and believe the alternative.

If the P -value is **not small**, our sample mean is believable, assuming the null hypothesis to be true. We are willing to believe the null hypothesis.

P -value **small** \rightarrow **reject** null hypothesis

P -value **not small** \rightarrow **don't reject** null hypothesis

Example: Hypothesis Test with Large Sample

- Background:** Number of credits taken by a sample of 400 students has mean 15.3, sd 2.
- Question:** Can we believe population mean is 15?
- Response:**
 1. Null: _____ Alt: _____
 2. Sample mean _____, sd=2, $z =$ _____
 3. P -value=prob of z this far from 0: _____
 4. Because the P -value is very small, we _____ null hypothesis.

Example: Hypothesis Test with Large Sample

- Background:** Number of credits taken by a sample of 400 students has mean 15.3, sd 2. A test to see if the population mean is 15 has very small P -value (0.0026).
- Question:** Does this mean... (a) The true population mean is very different from 15? Or (b) We have very strong evidence that the true population mean is not 15?
- Response:** _____ because other things factor into a small P -value besides how far what we observed is from what the null hypothesis claims. In fact, 15.3 seems quite close to 15. How close?

Example: Confidence Interval after Test

- **Background:** Number of credits taken by a sample of 400 students has mean 15.3, sd 2.
- **Question:** What is a 95% confidence interval for the population mean?
- **Response:** _____ so the population mean is apparently quite close to 15.

Example: Asbestos & Lung Cancer?

- **Background:** M. Kanarek found a “strong relationship” between the rate of lung cancer among white males and the concentration of asbestos fibers in the drinking water: $P\text{-value} < 0.001$. An increase of 100 times the asbestos concentration went with an increase of 1.05 per 1000 in the lung cancer rate: 1 more case per year per 20,000 people... In tests of 200+ relationships, the P -value for lung cancer in white males was the smallest... They adjusted for age & other demographic variables, but not smoking.
- **Question:** Is there really a strong relationship between asbestos in drinking water and lung cancer in white males?
- **Response:** _____

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- **Question:** Is there really a strong relationship between asbestos in drinking water and lung cancer in white males?
- **Response:** (1) The evidence might be _____ (small P -value thanks to large samples) but the relationship is _____. 1 more case per 20,000 people, when asbestos increases $\times 100$, is minimal.

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- **Question:** Is there really a strong relationship between asbestos in drinking water and lung cancer in white males?
- **Response:** (2) Beware of _____! If we reject the null for every $P\text{-value} < 0.05$, then _____% of the time, in the long run, we make a Type I Error, rejecting the null even though it's true. For every 100 tests of a true null hyp, about _____ incorrectly reject it.

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- **Question:** Is there really a strong relationship between asbestos in drinking water and lung cancer in white males?
- **Response:** (3) Principles learned in Part One shouldn't be forgotten: they failed to control for an obvious confounding variable, _____. Perhaps there were more smokers in areas that had higher asbestos concentrations.

Example: Hypothesis Test with Small Sample

- **Background:** A manufacturer bragged: “Tests comparing our product to the more expensive competitor’s product showed no statistically significant difference in quality.”
- **Question:** How impressed should we be?
- **Response:** _____ If they only sampled a few products, a very small sample size would tend to produce a small z , which in turn yields a large P -value, failing to show a statistically significant difference.

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Conversely, when is z *not* large?

- Observed sample mean close to hypothesized population mean
- Small sample size
- Large standard deviation

Example: Another Test with a Small Sample

- **Background:** An experiment compared decrease in blood pressure over a 12-wk period for 10 men taking calcium vs. 11 taking placebo. The two-sample t was 1.634, with $P\text{-value} = 0.06$. Using 0.05 as the cut-off, the test has failed to produce statistically significant evidence of the benefits of calcium for blood pressure.
- **Question:** Can we be sure calcium doesn't help b.p.?
- **Response:** _____; a P -value of 0.06 is still on the small side. Perhaps larger samples would yield significant results.

Example: *Small vs. Large Samples*

- **Background:** An experiment compared decrease in blood pressure over a 12-wk period for 10 men taking calcium vs. 11 taking placebo. The two-sample t was 1.634, with P -value=0.06. Using 0.05 as the cut-off, the test has failed to produce statistically significant evidence of the benefits of calcium for blood pressure.
- **Question:** Why didn't the study use more subjects?
- **Response:**