Lecture 15
Chapters 12&13 Relationships between Two Categorical Variables

- Tabulating and Summarizing
- Table of Expected Counts
- Statistical Significance for Two-Way Tables

Constructing & Assessing a Two-Way Table

- Decide variables' roles, explanatory & response
- Put explanatory in rows, response in columns
- Compare conditional rates in response of interest for two (or more) explanatory groups

Example: *Constructing a Two-Way Table*

- **Background**: A study recorded heavy drinking or not for bipolar alcoholics taking Valproate or placebo.
- **Question:** What are the explanatory and response variables; what should go in the rows and columns of a two-way table for the data?
- **Response:** Explanatory is

Response is

Example: *What to Report in a Two-Way Table*

- **Background**: A study recorded incidence of heavy drinking for bipolar alcoholics taking Valproate or placebo.

	Drinking	No drinking	Total
Valproate	14	18	32
Placebo	15	7	22
Total	29	25	54

- **Question:** The numbers who drank are 14 for Valproate, 15 for placebo. Should we say the incidence of drinking was about the same for both groups?
- **Response:**

Example: *Comparisons in a Two-Way Table*

- **Background**: A study recorded incidence of heavy drinking for bipolar alcoholics taking Valproate or placebo.

	Drinking	No drinking	Total
Valproate	14	18	32
Placebo	15	7	22
Total	29	25	54

- **Question:** How do we best summarize the data?
- **Response:**

(For the *sample*, were less likely to drink).

Example: *Significance in a Two-Way Table*

- **Background**: The conditional rate of heavy drinking was 14/32=0.44 for Valproate-takers, 15/22=0.68 for placebo.

	Drinking	No drinking	Total
Valproate	14	18	32
Placebo	15	7	22
Total	29	25	54

- **Question:** Does the difference seem "significant"?
- **Response:** If the difference were 0.55 vs. 0.57, we'd say
  If it were 0.36 vs. 0.76 (more than twice as much) we'd say
  For a difference of 0.44 vs. 0.68 from a small sample, it's

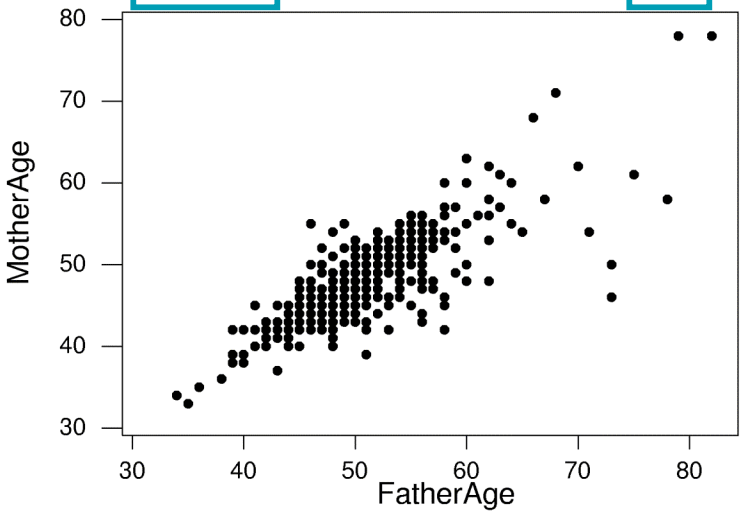
Definition *(Review)*

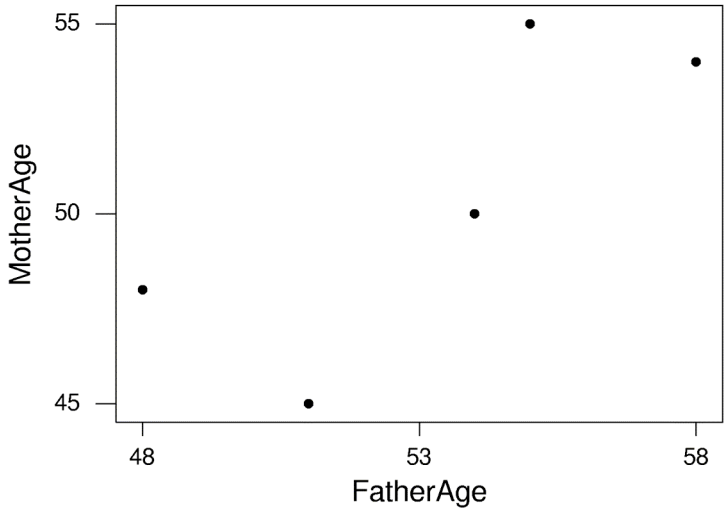
- **Statistically significant relationship:** one that cannot easily be attributed to chance. (If there were actually no relationship in the population, the chance of seeing such a relationship in a random sample would be less than 5%.)

(We'll learn to assess statistical significance in Chapters 13, 22, 23.)

Example: *Sample Size, Significance (Review)*

- **Background**: Relationship between ages of students' mothers and fathers both have r=+0.78, but sample size is over 400 (on left) or just 5 (on right):





- **Question:** Which plot shows a relationship that appears to be statistically significant?
- **Response:** The one on the left. (Relationship on right could be due to chance.)

Another Comparison in Considering Categorical Relationships

Instead of considering how different are the *proportions* in a two-way table, we may consider how different the *counts* are from what we'd expect if the "explanatory" and "response" variables were in fact unrelated.

This gives us a way to assess significance.

Example: *Expected Counts in a Two-Way Table*

- **Background**: A two-way table shows heavy drinking or not *observed* for bipolar alcoholics taking Valproate or placebo.

Observed	Drinking	No drinking	Total
Valproate	14	18	32
Placebo	15	7	22
Total	29	25	54

- **Question:** What counts would we *expect* to see, if there were no relationship whatsoever between the two variables?
- **Response:** We'd expect to see counts for which the **rate** of drinking is the same (overall) for both groups.

Example: *Expected Counts (continued)*

- **Response (continued)**: If exactly 29/54 in each group drank,

Expected	Drinking	No drinking	Total
Valproate	(29/54)×32=17.2	(25/54)×32=14.8	32
Placebo	(29/54)×22=11.8	(25/54)×22=10.2	22
Total	29	25	54

(and 25/54 in each group *didn't* drink), we'd expect…

- Valproate-takers to drink
- placebo-takers to drink
- Valproate-takers *not* to drink
- placebo-takers *not* to drink

Example: *Comparing Counts*

- **Background**: Tables of observed and expected counts in Valproate/drinking experiment:

Obs	D	ND	T
V	14	18	32
P	15	7	22
T	29	25	54

Exp	D	ND	T
V	17.2	14.8	32
P	11.8	10.2	22
T	29	25	54

- **Question:** How do the counts compare?
- **Response:**

Example: *Comparing Counts*

- **Background**: Observed and expected counts differ.

Obs	D	ND	T
V	14	18	32
P	15	7	22
T	29	25	54

Exp	D	ND	T
V	17.2	14.8	32
P	11.8	10.2	22
T	29	25	54

- **Question:** Is the difference significant?
- **Response:** We need a way of putting the four differences in perspective…

Components and Chi-Square Statistic

- **Components** to compare observed and expected counts, one table cell at a time:

$$\text{component} = \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Components are **individual** standardized squared differences.

- **Chi-square** statistic combines all components by summing them up:

$$\text{chi-square} = \text{sum of } \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Chi-square is **sum** of standardized squared differences.

Example: *Chi-Square Components*

- **Background**: Observed and Expected Tables:

Obs	D	ND	T
V	14	18	32
P	15	7	22
T	29	25	54

Exp	D	ND	T
V	17.2	14.8	32
P	11.8	10.2	22
T	29	25	54

- **Question:** Find each $\text{component} = \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$
- **Response:**

Example: *Chi-Square Statistic*

- **Background**: Observed and Expected Tables:

Obs	D	ND	T
V	14	18	32
P	15	7	22
T	29	25	54

Exp	D	ND	T
V	17.2	14.8	32
P	11.8	10.2	22
T	29	25	54

- **Question:** Find chi-square = sum of $\frac{(\text{observed - expected})^2}{\text{expected}}$
- **Response:**

$$\frac{(14-17.2)^2}{17.2} + \frac{(18-14.8)^2}{14.8} + \frac{(15-11.8)^2}{11.8} + \frac{(7-10.2)^2}{10.2}$$

Example: *Assessing Significance*

- **Background**: Chi-square=0.6+0.7+0.9+1.0=3.2.

Obs	D	ND	T
V	14	18	32
P	15	7	22
T	29	25	54

Exp	D	ND	T
V	17.2	14.8	32
P	11.8	10.2	22
T	29	25	54

- **Question:** Is the relationship significant?
- **Response:** Need to assess the relative size of 3.2.

Statistical Significance in a 2×2 Table

It can be shown that for a 2×2 table, a chi-square statistic larger than 3.84 indicates a large enough difference between observed and expected values that there's almost certainly a relationship.

Note: 1.96 is the "magic" z value for which the chance of being at least that extreme is 0.05. In fact, chi-square for a 2×2 table corresponds to the square of z: $1.96^2 = 3.84$.

Example: *Assessing Chi-Square Statistic*

- **Background**: Chi-square=0.6+0.7+0.9+1.0=3.2.

Obs	D	ND	T
V	14	18	32
P	15	7	22
T	29	25	54

Exp	D	ND	T
V	17.2	14.8	32
P	11.8	10.2	22
T	29	25	54

- **Question:** Is the difference between observed and expected counts significant?
- **Response:** Since 3.2 is not as large as 3.84, the difference is

(A larger sample would help, but not easy to get here…)

Are Variables in a 2×2 Table Related?

1. Compute each expected count $= \dfrac{\textit{Column total} \times \textit{Row total}}{\textit{Table total}}$

2. Calculate each component $= \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

3. Find chi-square = sum of $\dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

4. If chi-square > 3.84, there is a statistically significant relationship. Otherwise, we don't have evidence of a relationship.

Example: *Smoking and Alcohol Related?*

- **Background**: Overall proportion alcoholic is $\frac{40}{1000} = 0.04$

	Alcoholic	Not Alcoholic	Total
Smoker	30	200	230
Non-smoker	10	760	770
Total	40	960	1000

- **Questions:** If proportions were same for smokers and non-smokers, what counts do we expect?
- **Response:** Expect…
  - [redacted] smokers to be alcoholic
  - [redacted] non-smokers to be alcoholic; also
  - [redacted] smokers not alcoholic
  - [redacted] non-smokers not alcoholic

Example: *Smoking & Alcohol (continued)*

- **Background**: Observed and Expected Tables:

Obs	A	NA	Total
S	30	200	230
NS	10	760	770
Total	40	960	1000

Exp	A	NA	Total
S	9.2	220.8	230
NS	30.8	739.2	770
Total	40	960	1000

- **Question:** Find components & chi-square; conclude?
- **Response:** chi-square=

$$\frac{(30-9.2)^2}{9.2}+\frac{(10-30.8)^2}{30.8}+\frac{(200-220.8)^2}{220.8}+\frac{(760-739.2)^2}{739.2}$$

The relationship is

EXTRA CREDIT (Max. 5 pts.) Choose two categorical variables included in the survey data 800surveyf06.txt at www.pitt.edu/~nancyp/stat-0800/index.html (see instructions to highlight, copy, and paste into MINITAB). Follow steps 1 through 4 outlined above to determine if there is a statistically significant relationship between them.

Bring a calculator to Lecture 16!