Lecture 18
Part 3. Understanding Uncertainty
Ch.16: Probability

- Interpretations of Probability
- Rules of Probability

Course Divided into Four Parts *(Review)*

1. Finding Data in Life:
   scrutinizing **origin** of data
2. Finding Life in Data: **summarizing** data
   yourself or assessing another's summary
3. Understanding Uncertainty in Life:
   probability theory
4. Making Judgments from Surveys and
   Experiments: statistical **inference**

Definitions

- **Statistics:** (1) the science of principles and procedures for gaining and processing data and using the info to draw general conclusions (2) summaries of data (such as a sample average or sample proportion)
- **Probability:** (1) a science: the formal study of random behavior (2) the chance of happening

How is this "chance" determined?

Interpretations of Probability

- Proportion of equally likely outcomes in a physical circumstance
- Relative frequency in the long run, over many repetitions
- Personal probability, assessed subjectively

Example: *Three Ways to Determine a Probability*

- **Background**: Some probability statements:
  1. Probability of randomly chosen card being a heart is 0.25.
  2. Probability of randomly chosen student in a class getting A is 0.25, according to the professor.
  3. Probability of candidate being elected, according to an editorial, is 0.25.
- **Question:** Is each determined as a
  - Proportion of equally likely outcomes? Or
  - Proportion of long-run outcomes observed? Or
  - Subjective likelihood of occurring?
- **Response:** #1 (card) is _
  #2 (getting A) is
  #3 (election) is

Basic Probability Rules

We need rules for

0. What probabilities values are *permissible*
1. The probability of *not* happening
2. The probability of one *or* the other of two events occurring
3. The probability of one *and* the other of two events occurring
4. How probabilities compare if one event is the subset of another

Example: *Intuiting Rule 0*

- **Background**: A six-sided die is rolled once.
- **Questions:** What is the probability of getting a nine? What is the probability of getting a number less than nine?
- **Responses:** The probability of getting a nine is

  The probability of getting a number less than nine is

Rule 0 (Permissible Probabilities)

The probability of an impossible event is 0,
the probability of a certain event is 1, and
all probabilities must be between 0 and 1.

Example: *Applying Rule 0*

- **Background**: Consider these values: -1, -0.1, 0.1, 10.
- **Question:** Which of these are legitimate probabilities?
- **Response:**

Example: *Intuiting Rule 1*

- **Background**: Consider the roll of a six-sided die.
- **Question:** What do we get if we sum the probabilities of rolling an even number and rolling an odd number?
- **Response:**

Rule 1 (about *not* occurring)

If there are two possible outcomes in an uncertain situation, then their probabilities must add to 1.

Therefore, the probability of an event *not* happening is 1 minus the probability of happening. The probability of happening is 1 minus the probability of *not* happening.

Example: *Applying Rule 1*

- **Background**: The probability of getting an A in this course is 0.25.
- **Question:** What is the probability of not getting an A?
- **Response:**

Example: *Another Application of Rule 1*

- **Background**: The probability of a randomly chosen American owning at least one TV set is 0.98.
- **Question:** What is the probability of not owning any TV set?
- **Response:**

Example: *Intuiting Rule 2*

- **Background**: A statistics professor reports that the probability of a randomly chosen student in her class getting an A is 0.25, and the probability of getting a B is 0.30.
- **Question:** What is the probability of getting an A or a B?
- **Response:**

Example: *When Probabilities Can't Simply be Added*

- **Background**: A statistics professor reports that the probability of a randomly chosen student in her class getting an A is P(A)=0.25, and the probability of being a female is P(F)=0.60.
- **Question:** What is the probability of getting an A or being a female?
- **Response:**

Definition

For some pairs of events, if one occurs, the other cannot, and vice versa. We can say they are **mutually exclusive**, the same as **disjoint** or **non-overlapping**.

Note that "getting an A" and "getting a B" are mutually exclusive, whereas "getting an A" and "being female" are not. Next time we'll establish a rule that works even if events are not mutually exclusive.

Rule 2 (Non-overlapping "Or" Rule)

For two mutually exclusive (non-overlapping) events, the probability of one *or* the other happening is the sum of their individual probabilities.

Note: *The word "or" entails addition.*

Example: *Applying Rule 2*

- **Background**: Assuming adult male foot lengths have mean 11 and standard deviation 1.5, if we randomly sample 100 adult males, the probability of their sample mean being less than 10.7 is 0.025; probability of being greater than 11.3 is also 0.025.
- **Question:** What is the probability of sample mean foot length being less than 10.7 or greater than 11.3?
- **Response:**

Example: *Intuiting Rule 3*

- **Background**: A balanced coin is tossed twice.
- **Question:** What is the probability of both the first and the second toss resulting in tails?
- **Response:**

Alternatively, since there are 4 equally likely outcomes HH, HT, TH, TT, we know each has probability

Example: *Another Application of Rule 3*

- **Background**: In a child's pocket are 2 quarters and 2 nickels. He randomly picks a coin, replaces it, and picks another.
- **Question:** What is the probability of the first and the second coins both being quarters?
- **Response:**

Example: *When Probabilities Can't Simply Be Multiplied*

- **Background**: In a child's pocket are 2 quarters and 2 nickels. He randomly picks a coin, does *not* replace it, and picks another.
- **Question:** What is the probability of the first and the second coins both being quarters?
- **Response:**

Definitions

For some pairs of events, whether or not one occurs impacts the probability of the other occurring, and vice versa: the events are said to be **dependent**.

If two events are **independent**, they do not influence each other; whether or not one occurs has no effect on the probability of the other occurring.

Rule 3 (Independent "And" Rule)

For any two independent events, the probability of one *and* the other happening is the *product* of their individual probabilities.

Note: The word "and" entails multiplication.

Sampling With or Without Replacement

- Sampling *with replacement* is associated with events being *independent*.
- Sampling *without replacement* is associated with events being *dependent*.

Example: *Applying Rule 3*

- **Background**: We pick 2 cards from 4 (1 club, 1 diamond, 1 heart, 1 spade) and want to know the probability of both the first *and* second card a spade…
- **Questions:**
  - if we *do* replace the first before picking the second?
  - if we *don't* replace the first before picking the second?
- **Responses:**
  - Selections are
  - Selections are

Example: *Intuiting Rule 4*

- **Background**: A card is picked from a deck.
- **Question:** Which is higher: the probability of being an ace, or the probability of being a black ace?
- **Response:**

Rule 4 (Comparing probability of a subset)

If the ways in which one event can occur are a subset of the ways in which another can occur, then the probability of the first can’t be more than the probability of the second.

Example: *Applying Rule 4*

- **Background**: Consider these two events:

  a. The world will come to an end.

  b. The world will end by either a meteorite or nuclear war.
- **Question:** Which is more likely to occur in the next 50 years?
- **Response:**

Probability of Occurring At Least Once

To find the probability of occurring at least once in a certain number of trials, we can either

- apply Rule 2 to find the probability of occurring on the 1st or 2nd or 3rd…trial; or
- apply Rule 1 (the "Not" Rule): subtract from 1 the probability of not occurring at all.

Example: *Probability of Occurring At Least Once*

- **Background**:Keep rolling a die until you get a 2.
- **Question:** What is probability of getting a 2 by the 4th roll?
- **Response:** same as probability of getting first 2 on 1st roll *or* 2nd roll *or* 3rd roll *or* 4th roll

= prob of 1st 2 on 1st roll +…+ prob of 1st 2 on 4th roll

$$= \frac{1}{6} + \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$= \frac{1}{6} + \frac{5}{36} + \frac{25}{216} + \frac{125}{1296} = \frac{671}{1296} = 0.52$$

Example: *Probability of Occurring At Least Once*

- **Background**:Keep rolling a die until you get a 2.
- **Question:** What is probability of getting a 2 by the 4th roll?
- **Response:** same as

Example: *Probability of Occurring At Least Once*

- **Background**:Probability of heads in coin toss is 0.5.
- **Question:** What is the probability of getting heads by the 10th toss (same as at least one head in 10 tosses)
- **Response:**

We could also have used the "Or" Rule, adding the probabilities of all the ways to get at least one head. However, there are over 1,000 ways altogether!