Lecture 23/Chapter 19
Diversity of Sample Means

- Means versus Proportions
- Behavior of Sample Means: Example
- Behavior of Sample Means: Conditions
- Behavior of Sample Means: Rules

Approach to Inference

- **Step 1 (Chapter 19):** Work *forward*---if we happen to know the population mean and standard deviation, what behavior can we expect from sample means for repeated samples of a given size?
- **Step 2:** Work *backward*---if sample mean for a sample of a certain size is observed to take a specified value, what can we conclude about the value of the unknown population mean?

We covered Step 1 for proportions, now we'll cover Step 1 for means.

Proportions then Means, Probability then Inference

Today we'll establish a parallel theory for means, when the variable of interest is quantitative (number on dice instead of color on M&M). After that, we'll

- Perform inference with **confidence intervals**
  - For proportions (Chapter 20)
  - For means (Chapter 21)
- Perform inference with **hypothesis testing**
  - For proportions (Chapters 22&23)
  - For means (Chapters 22&23)

Understanding Sample Mean

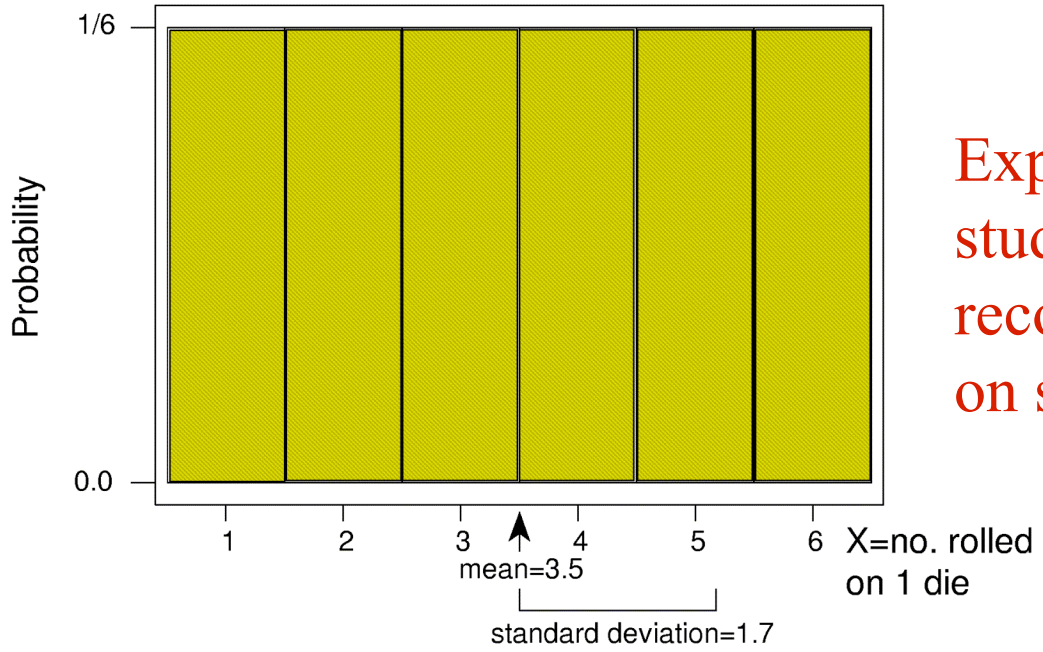
3 Approaches:

1. Intuition
2. Hands-on Experimentation
3. Theoretical Results

We'll find that our **intuition** is consistent with **experimental** results, and both are confirmed by mathematical **theory**.

Example: *Intuit Behavior of Sample Mean*

- **Background**: Population of possible dicerolls are equally likely values {1,2,3,4,5,6} with a uniform (flat) shape and mean 3.5, sd 1.7.
- **Question:** How should sample mean roll behave for repeated rolls of 2 dice?



Experiment: each student rolls 2 dice, records sample mean on sheet **and** in notes.

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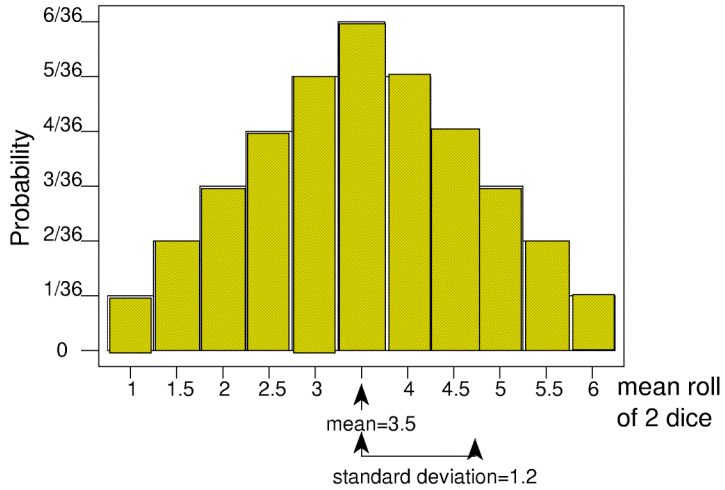
- **Question:** How should sample mean roll behave for repeated rolls of 2 dice?
- **Response:** Summarize by telling
  - **Center:** Some means less than 3.5, others more; altogether, they should average out to
  - **Spread:** Means for 2 dice easily range from
  - **Shape:** (up from 1 to 3.5, down to 6).

Example: *Intuit Behavior of Sample Mean*



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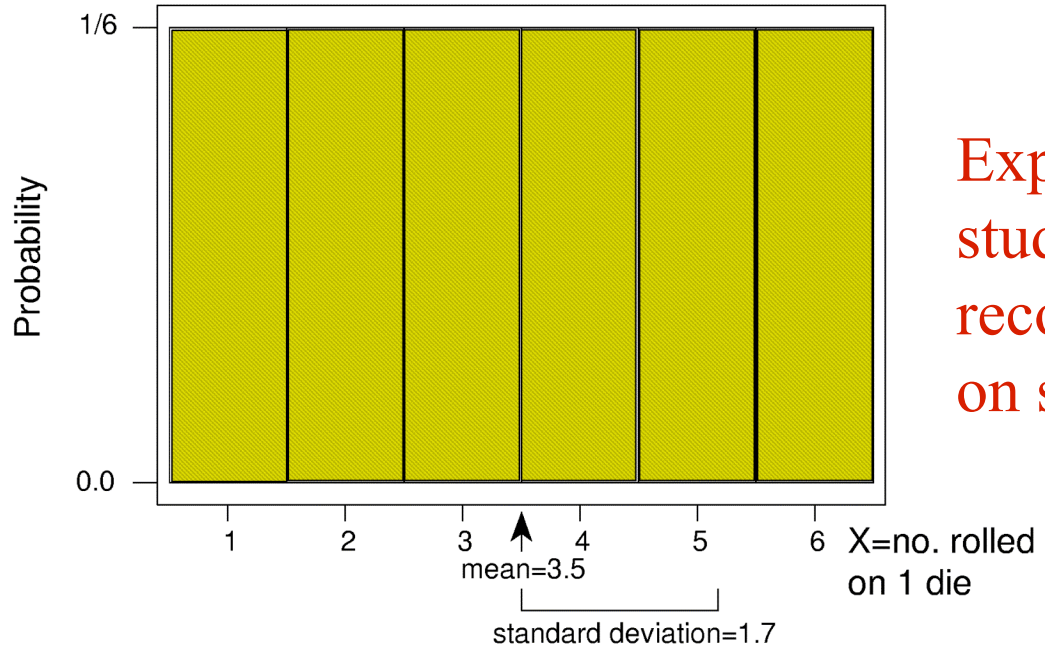
Example: *Sample Mean for Larger Samples*

- **Background**: Population of possible dicerolls are equally likely values {1,2,3,4,5,6} with a uniform (flat) shape and mean 3.5, sd 1.7.
- **Question:** How should sample mean roll behave for repeated rolls of 8 dice?



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Example: *Sample Mean for Larger Samples*

- **Background**: Population of possible dicerolls are equally likely values {1,2,3,4,5,6} with a uniform (flat) shape and mean 3.5, sd 1.7.
- **Question:** How should sample mean roll behave for repeated rolls of 8 dice?
- **Response:** Summarize by telling
  - **Center:** Altogether they should average out to
  - **Spread:** Means for 8 dice rarely as low as 1 or as high as 6: spread than for 2 dice.
  - **Shape:** Bulges more near 3.5, tapers more at extremes 1 and 6→shape close to

Conditions for Rule of Sample Means

- Randomness [affects center]
- Independence [affects spread]
  - If sampling without replacement, sample should be less than 1/10 population size
- Large enough sample size [affects shape]
  - If population shape is normal, any sample size is OK
  - If population if not normal, a larger sample is needed.

Example: *Checking Conditions for 2 Dice*

- **Background**: Population of possible dicerolls are equally likely values {1,2,3,4,5,6} with a uniform (flat) shape and mean 3.5, sd 1.7. Repeatedly roll 2 dice and calculate the sample mean roll.
- **Question:** Are the 3 Conditions met?
- **Response:**
  - **Random?**
  - **Independent?**
  - **Sample large enough?**

Example: *Checking Conditions for 8 Dice*

- **Background**: Population of possible dicerolls are equally likely values {1,2,3,4,5,6} with a uniform (flat) shape and mean 3.5, sd 1.7. Repeatedly roll 8 dice and calculate the sample mean roll.
- **Question:** Are the 3 Conditions met?
- **Response:**
  - **Random?**
  - **Independent?**
  - **Sample large enough?**

Rule for Sample Means (if conditions hold)

- **Center:** The mean of sample means equals the true population mean.
- **Spread:** The standard deviation of sample means is standard error =

$$\frac{\text{population standard deviation}}{\sqrt{\text{sample size}}}$$

- **Shape:** (Central Limit Theorem) The frequency curve will be approximately normal, depending on how well 3rd condition is met.

Example: *Behavior of Sample Mean, 2 Dice*

- **Background**: Population of dice rolls has mean 3.5, sd 1.7. Repeatedly roll 2 dice.
- **Question:** How must sample means behave?
- **Response:** For repeated random samples of size 2, sample mean roll has…
  - **Center:** mean of sample means is
  - **Spread:** standard error is
  - **Shape:**

Example: *Behavior of Sample Mean, 8 Dice*

- **Background**: Population of dice rolls has mean 3.5, sd 1.7. Repeatedly roll 8 dice.
- **Question:** How must sample means behave?
- **Response:** For repeated random samples of size 8, sample mean roll has…
  - **Center:** mean of sample means is
  - **Spread:** standard error is
  - **Shape:**

Empirical Rule *(Review)*

For any normal curve, approximately

- 68% of values are within 1 sd of mean
- 95% of values are within 2 sds of mean
- 99.7% of values are within 3 sds of mean

Example: *68-95-99.7 Rule for 8 Dice*

- **Background**: Sample mean roll for 8 dice has mean 3.5, sd 0.6, and shape fairly normal.
- **Question:** What does 68-95-99.7 Rule tell us about behavior of sample mean?
- **Response:** The probability is approximately
  - 0.68 that sample mean is within : in (2.9, 4.1)
  - 0.95 that sample mean is within : in (2.3, 4.7)
  - 0.997 that sample mean is within : in (1.7, 5.3)

Activity: check how class dice rolls conform.

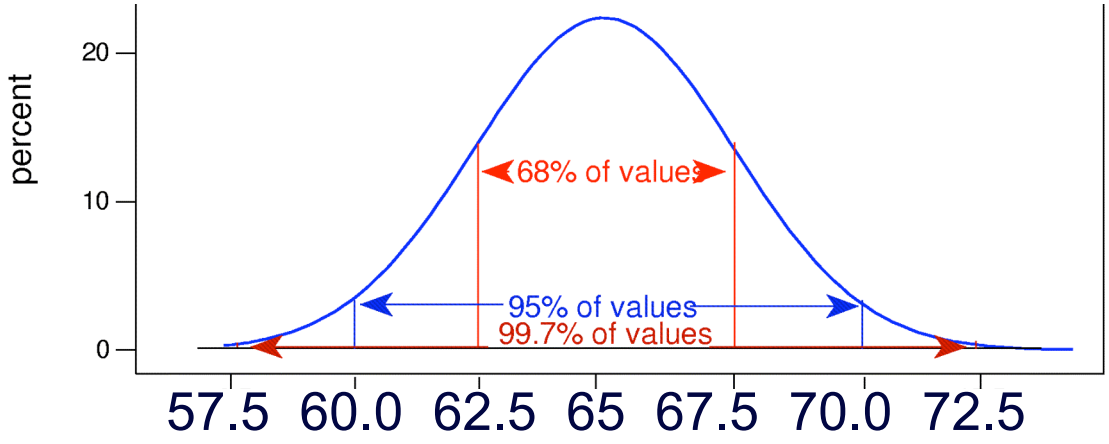
Intuiting Behavior of Individual vs. Mean

Imagine 1 woman is picked at random from the university. We're pretty sure her height is in what range?

Now imagine 64 women are picked at random from the university. We're pretty sure their sample mean height is in what range?

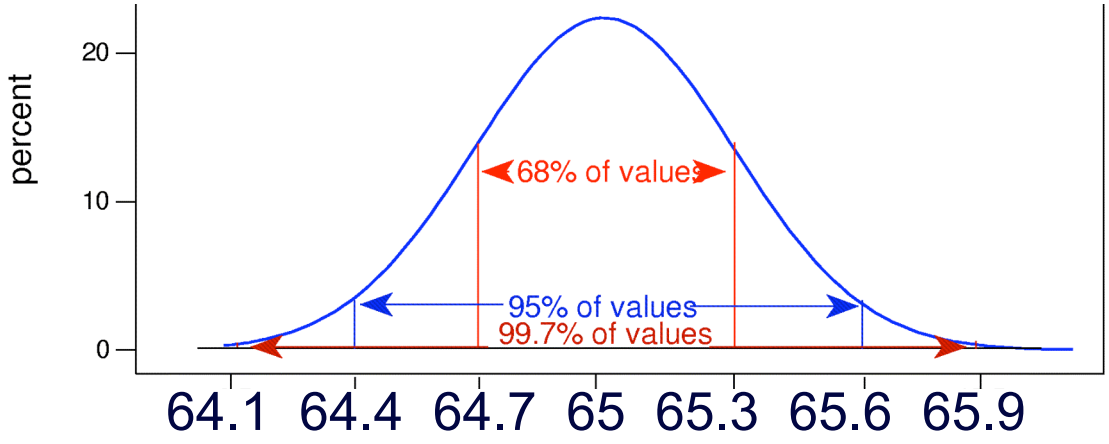
Example: *68-95-99.7 Rule for Single Hts*

- **Background**: Women's hts normal; mean 65, sd 2.5.
- **Question:** What does 68-95-99.7 Rule tell us about the height of a randomly chosen woman?
- **Response:** The probability is
  - 0.68 that her height is within : in (62.5, 67.5)
  - 0.95 that her height is within : in (60.0, 70.0)
  - 0.997 that her height is within : in (57.5, 72.5)



Example: *68-95-99.7 Rule for Mean Ht*

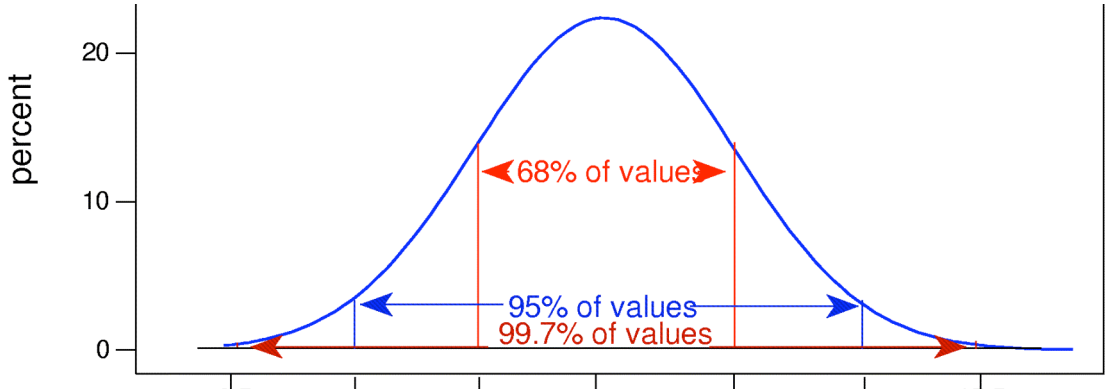
- **Background**: Women's hts normal; mean 65, sd 2.5.
- **Question:** What does 68-95-99.7 Rule tell us about sample mean ht for random samples of 64 women?
- **Response:** Sample means have mean 65, sd and shape normal because population is normal. Probability is
  - 0.68 that sample mean is within : in (64.7, 65.3)
  - 0.95 that sample mean is within : in (64.4, 65.6)
  - 0.997 that sample mean is within in (64.1,65.9)



Mean of 64 females in class is 64.9.

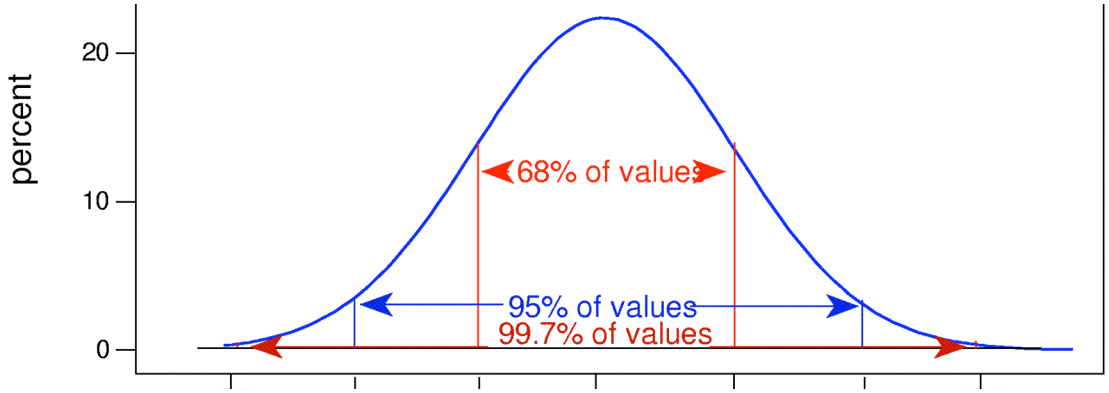
Example: *68-95-99.7 Rule for Male Hts*

- **Background**: Men's hts normal; mean 70, sd 3.
- **Question:** What does 68-95-99.7 Rule tell us about the height of a randomly chosen man?
- **Response:** The probability is
  - 0.68 that his height is within 1(3) of 70: in _____
  - 0.95 that his height is within 2(3) of 70: in _____
  - 0.997 that his height is within 3(3) of 70: in _____



Example: *68-95-99.7 Rule: Mean Male Ht*

- **Background**: Men's hts normal; mean 70, sd 3.
- **Question:** What does 68-95-99.7 Rule tell us about sample mean ht for random samples of 25 men?
- **Response:** Sample means have mean 70, sd and shape normal because population is normal. Probability is
  - 0.68 that sample mean is within 1(0.6) of 70: in
  - 0.95 that sample mean is within 2(0.6) of 70: in
  - 0.997 that sample mean is within 3(0.6) of 70: in



Mean of 25 males in class is 70.5.