

# Lecture 12: Naïve Bayes Classifier, Evaluation Methods

Ling 1330/2330 Intro to Computational Linguistics  
Na-Rae Han, 10/5/2023

# Overview

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- ▶ Text classification; Naïve Bayes classifier
  - ◆ Language and Computers: Ch.5 Classifying documents
  - ◆ NLTK book: [Ch.6 Learning to classify text](#)
  
- ▶ Evaluating the performance of a system
  - ◆ *Language and Computers*:
    - ◆ Ch.5.4 Measuring success, 5.4.1 Base rates
  - ◆ NLTK book: [Ch.6.3 Evaluation](#)
  - ◆ Cross-validation
  - ◆ Accuracy vs. precision vs. recall
  - ◆ F-measure

# Given D, chance of Spam?

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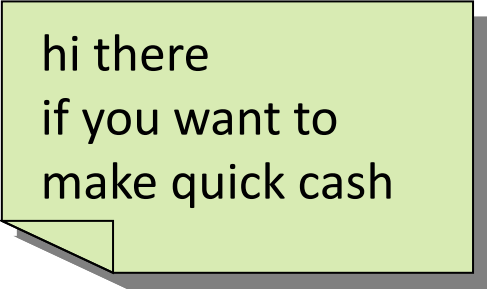
$$P(SPAM | D) = \frac{P(SPAM, D)}{P(D)} = \frac{P(SPAM, D)}{P(SPAM, D) + P(HAM, D)}$$

$P(SPAM | D)$

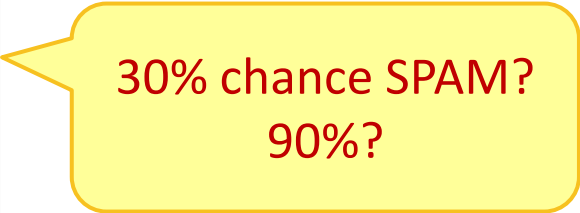
← The probability of *a given document D* being SPAM

= 1 -  $P(HAM | D)$

← Can calculate from  $P(SPAM, D)$  and  $P(HAM, D)$



hi there  
if you want to  
make quick cash



30% chance SPAM?  
90%?

# A bit of background

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▶  **$P(A)$** : the probability of A occurring

- ◆  $P(\text{SPAM})$ : the probability of having a SPAM document.

▶  **$P(A | B)$** : **Conditional probability**

the probability of A occurring, given that B has occurred

- ◆  $P(f_1 | \text{SPAM})$ : given a spam document, the probability of feature1 occurring.
- ◆  $P(\text{SPAM} | D)$ : given a specific document, the probability of it being a SPAM.

▶  **$P(A, B)$** : **Joint probability**

the probability of A occurring *and* B occurring

- ◆ Same as  $P(B, A)$ .
- ◆ If A and B are independent events, same as  $P(A) * P(B)$ .  
If not, same as  $P(A | B) * P(B)$  and also  $P(B | A) * P(A)$ .
- ◆  $P(D, \text{SPAM})$ : the probability of a specific document D occurring, and it being a SPAM.

# A bit of background

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## ▶ $P(A, B)$ : Joint probability

the probability of A occurring *and* B occurring

- ◆ Same as  $P(B, A)$ .
- ◆ If A and B are independent events, same as  $P(A)*P(B)$ .  
If not, same as  $P(A|B)*P(B)$  and also  $P(B|A)*P(A)$ .
- ◆  $P(D, SPAM)$ : the probability of a specific document D occurring, and it being a SPAM.

Throwing two dice.

A: die 1 comes up with 6.

B: die 2 comes up with an even number.

→ A and B are independent.

$$\begin{aligned}\rightarrow P(A, B) &= P(A) * P(B) \\ &= 1/6 * 1/2 = 1/12\end{aligned}$$

Throwing one die.

A: die comes up with 6.

B: die comes up with an even number.

→ A and B are NOT independent!

$$\begin{aligned}\rightarrow P(A, B) &= P(A|B) * P(B) \\ &= 1/3 * 1/2 = 1/6 \\ &= P(B|A) * P(A) \\ &= 1 * 1/6 = 1/6\end{aligned}$$

# Bayes' Theorem

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$$\textcircled{1} \quad P(B | A) = \frac{P(B, A)}{P(A)} = \frac{P(A | B) * P(B)}{P(A)}$$

- ▶ B: Pitt closing, A: snowing
  - ▶  $P(B | A)$ : probability of Pitt closing, given snowy weather
  - ▶  $P(B, A)$ : probability of Pitt closing and snowing
- ①**: the probability of Pitt closing given it's snowing is equal to the probability of Pitt closing and snowing, divided by the probability of snowing.

# Snow vs. Pitt, Bayes theorem style



$$\textcircled{1} \quad P(B | A) = \frac{P(B, A)}{P(A)} = \frac{P(A | B) * P(B)}{P(A)}$$

- ▶ B: Pitt closing, A: snowing
  - ◆ Last year, there were 15 snowy days; Pitt closed 4 days, 3 of which were snowy days.
- ▶  $P(B | A)$ : probability of Pitt closing, given snowy weather
  - =  $P(B, A) / P(A)$
  - =  $(3/365) / (15/365)$
  - =  $3/15 = 0.2$
- ▶  $P(B, A)$ : probability of Pitt closing and snowing
  - =  $3/365$
- ①: the probability of Pitt closing given it's snowing is equal to the probability of Pitt closing and snowing, divided by the probability of snowing.

# Snow vs. Pitt, Bayes theorem style



$$P(B | A) = \frac{P(B, A)}{P(A)} = \frac{P(A | B) * P(B)}{P(A)}$$

- ▶ B: Pitt closing, A: snowing
  - ▶  $P(B | A)$ : probability of Pitt closing, given snowy weather
  - ▶  $P(B, A)$ : probability of Pitt closing and snowing
- ②: the probability of Pitt closing AND it's snowing is equal to the probability of Pitt closing (=prior) multiplied by the probability of snowing given that Pitt is closed.
- ← Corollary of ①! You get this by swapping A and B and solving for  $P(B, A)$



# Bayes' Theorem & spam likelihood

$$P(SPAM | D) = \frac{P(SPAM, D)}{P(D)} = \frac{P(SPAM, D)}{P(SPAM, D) + P(HAM, D)}$$

$$P(SPAM, D)$$

$$= P(D|SPAM) * P(SPAM)$$

$$= P(SPAM) * P(D|SPAM)$$

$$= P(SPAM) * P(f_1, f_2, \dots, f_n|SPAM)$$

$$= P(SPAM) * P(f_1|SPAM) * P(f_2|SPAM) * \dots * P(f_n|SPAM) \textcircled{2}$$

A document has to be either SPAM or HAM!

- ▶ SPAM: document is spam, D: a specific document occurs
- ▶  $P(SPAM | D)$ : probability of document being SPAM, given a particular document
- ▶  $P(SPAM, D)$ : probability of D occurring and it being SPAM
- ▶ Which means: we can calculate  $P(SPAM | D)$  from  $P(SPAM, D)$  and  $P(HAM, D)$ , which are calculated as  $\textcircled{2}$ .

# Probabilities of the entire document

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$H_1$  "D is a SPAM" is closely related to  $P(D, SPAM)$ :

The probability of document D occurring *and* it being a spam

$$= P(SPAM) * P(D | SPAM)$$

$$= P(SPAM) * P(f_1, f_2, \dots, f_n | SPAM) \textcircled{1}$$

$$= P(SPAM) * P(f_1 | SPAM) * P(f_2 | SPAM) * \dots * P(f_n | SPAM) \textcircled{2}$$

- ◆ We have all the pieces to compute this.
- ◆ "Bag-of-words" assumption  $\textcircled{1}$
- ◆ "Naïve" Bayes because  $\textcircled{2}$  assumes **feature independence**.

If all we're going to do is rule between SPAM and HAM, we can simply compare  $P(D, SPAM)$  and  $P(D, HAM)$  and choose one with higher probability.

- ◆ But we may also be interested in answering:

"Given D, what are the *chances* of it being a SPAM? 70%? 5%?"

← This is  $P(SPAM | D)$ .

# Naïve Bayes Assumption

- ▶ Given a label, a set of features  $f_1, f_2, \dots, f_n$  are generated with different probabilities
- ▶ The features are **independent** of each other;  $f_x$  occurring does not affect  $f_y$  occurring, etc.

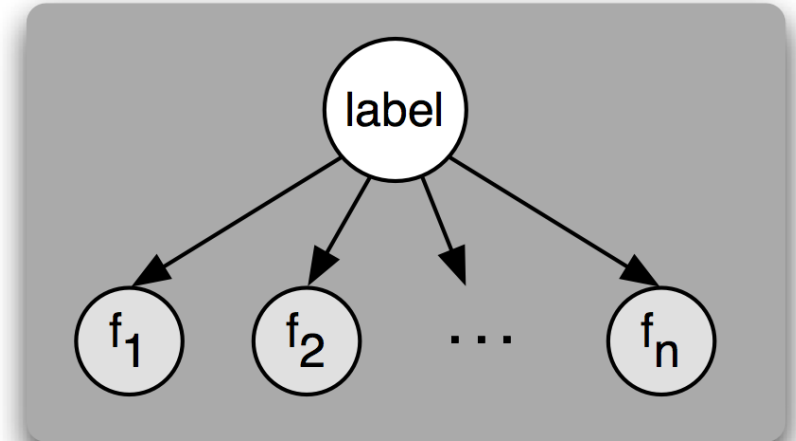
## → Naïve Bayes Assumption

- This **feature independence assumption** simplifies combining contributions of features; you just **multiply** their probabilities:

$$P(f_1, f_2, \dots, f_n | L) = P(f_1 | L) * P(f_2 | L) * \dots * P(f_n | L)$$

← "Naïve" because features are often inter-dependent.

←  $f_1$ : 'contains-*Linguistics*:YES' and  $f_2$ : 'contains-*syntax*:YES' are not independent.



# Homework 4: Who Said It?



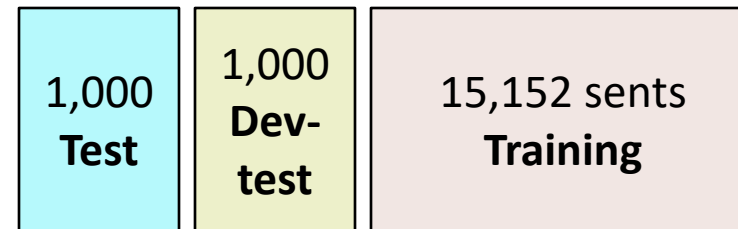
## ▶ Jane Austen or Herman Melville?

- ◆ *I never met with a disposition more truly amiable.*
- ◆ *But Queequeg, do you see, was a creature in the transition stage -- neither caterpillar nor butterfly.*
- ◆ *Oh, my sweet cardinals!*

## ▶ Task: build a Naïve Bayes classifier and explore it

## ▶ Do three-way partition of data:

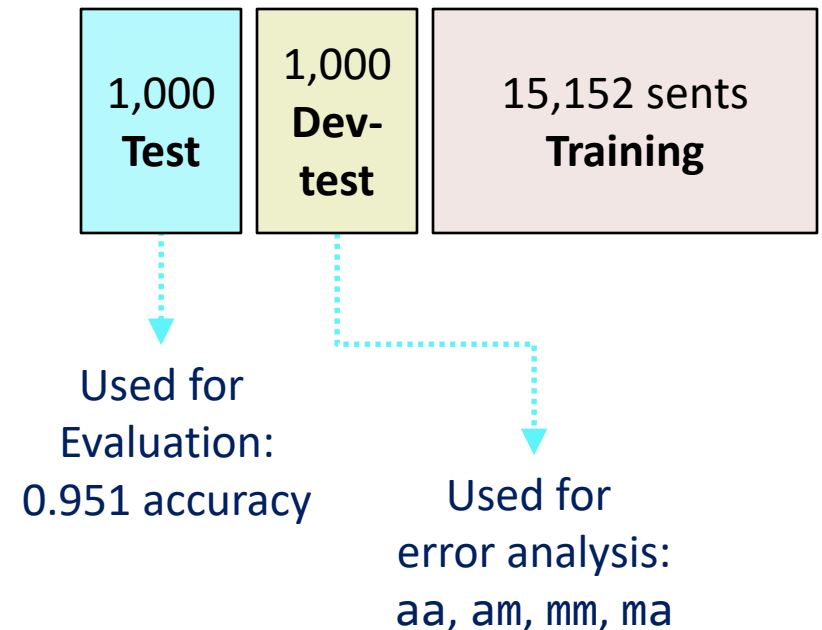
- ◆ test data
- ◆ development-test data
- ◆ training data



# whosaid: a Naïve Bayes classifier

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- ▶ How did the classifier do?
  - ◆ **0.951 accuracy** on the test data, using a fixed random data split.
- ▶ Probably outperformed your expectation.
- ▶ What's behind this high accuracy? How does the NB classifier work?
  - ➔ HW4 PART [B]
- ▶ How good is 0.951?



# Common evaluation setups

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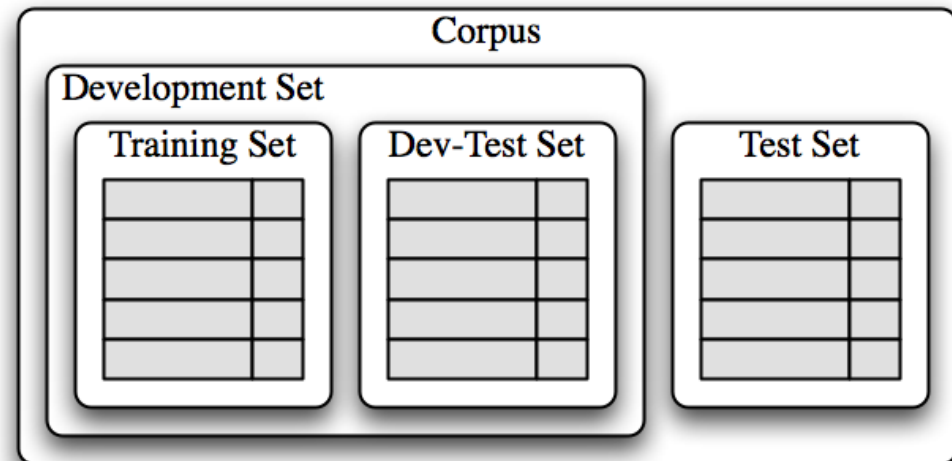
## ▶ **Training** vs. **testing** partitions

1. Training data ← classifier is trained on this section
2. Testing data ← classifier's performance is measured

## ▶ Training, testing, + **development-testing**

+ 3. Development testing data

← In feature engineering, researcher can error-analyze the data to improve performance



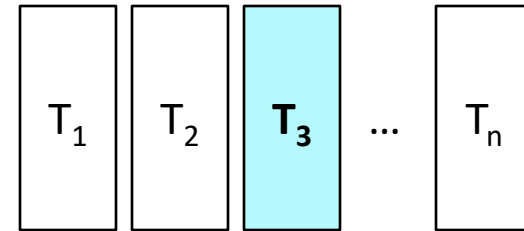
# Cross validation

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- ▶ But what if our training/testing split is somehow biased?

- ➔ We could randomize

- ➔ or use cross-validation.



- ▶  **$n$ -fold cross validation method**

- ◆ Partition the data set into equally sized  $n$  sets
- ◆ Conduct  $n$  rounds of training-testing, each using 1 partition as testing and the rest  $n-1$  partitions for training
- ◆ And then take an average of the  $n$  accuracy figures
- ← More reliable accuracy score. Performance evaluation is less dependent on a particular training-testing split
- ← We can see how widely performance varies across different training sets

# Confusion matrices

- ▶ When classifying among 3+ labels, **confusion matrices** can be informative
- ▶ L1 classification of ESL essays:

	ARA	DEU	FRA	HIN	ITA	JPN	KOR	SPA	TEL	TUR	ZHO
ARA	57	0	3	9	1	8	2	9	6	10	2
DEU	6	79	5	2	7	4	2	5	0	1	3
FRA	2	7	60	3	8	0	3	5	1	1	3
HIN	5	1	1	46	3	1	2	7	19	6	4
ITA	5	4	10	2	67	2	3	14	0	4	3
JPN	2	1	4	0	5	72	20	0	0	2	6
KOR	1	0	0	0	1	2	51	6	1	8	6
SPA	6	4	8	12	1	3	2	45	11	6	1
TEL	10	1	0	17	3	2	3	1	53	2	1
TUR	5	2	6	7	1	6	5	5	7	53	8
ZHO	1	1	3	2	3	0	7	3	2	7	63
	ARA	DEU	FRA	HIN	ITA	JPN	KOR	SPA	TEL	TUR	ZHO



# Accuracy as a measure

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- ▶ **Accuracy:** of all labeling decisions that a classifier made, how many of them are *correct*?
  - ◆ POS tagger
  - ◆ Name gender identifier
  - ◆ whosaid: Austen/Melville author classifier
  - ◆ Document topic identifier
  - ◆ Movie review classifier: positive/neg. ("sentiment classifier")

# Accuracy as a measure

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- ▶ **Accuracy:** of all labeling decisions that a classifier made, how many of them are *correct*?
- ▶ Interpreting accuracy numbers
  - ◆ A movie review sentiment classifier tests 85% accurate. Is this good or bad?
    - ◆ What if it turns out 80% movie reviews are positive?
    - ◆ How about 60%?
  - ◆ A document topic identifier tests 60% accurate. Good or bad?
    - ◆ What if 55% of documents are on "Politics"?
    - ◆ What if there are as many as 20 different topics, and the largest category only accounts for 10% of the data?
- ← These questions cannot be answered without considering **base probabilities (priors)**.

# Base probabilities

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## ▶ **Base probabilities (priors)**

The probability of a randomly drawn sample to have a label  $x$

- ◆ whosaid? POS tagger? Disease test?
- ◆ whosaid: 'melville' has a higher prior than 'austen'
- ◆ POS tagger: 'Noun' may have the highest prior than other tags
- ◆ Disease test: 'Negative' is typically much higher than 'Positive'

## ▶ **Base rate neglect**

- ◆ A cognitive bias humans have
- ◆ We tend to assume that base probabilities are equal

## ▶ **Base performance**

- ◆ The "absolute bottom line" for system performances  
= the highest base probability

ex. POS tagger: if 20% of all words are 'Noun', then the worst-performing system can be constructed which blindly assigns 'Noun' to every word, whose accuracy is 20%.

# When accuracy isn't a good measure

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- ▶ A **medical test for a disease** is 96% accurate. Good or bad?
  - ◆ What if 95% of population is free of the disease?
- ▶ A **grammatical error detector** is 96% accurate. Good or bad?
  - ◆ Suppose 95% of all sentences are error-free.
    - ← Accuracy alone doesn't tell the whole story.
- ▶ We are interested in:
  - ◆ Of all "ungrammatical" flags the system raises, what % is correct?
    - ← This is the **precision** rate.
  - ◆ Of all actual ungrammatical sentences, what % does the system correctly capture as such?
    - ← This is the **recall** rate.

# Outcome of a diagnostic test

## ▶ A grammatical error detector as a diagnostic test

- ◆ Positive: has grammatical error
- ◆ Negative: is error-free

		Real	
		Has grammatical error	Is error-free
Test	positive	<b>True positives</b>	<b>False positives</b>
	negative	<b>False negatives</b>	<b>True negatives</b>

### ◆ **Accuracy:**

$$(Tp + Tn) / (Tp + Tn + Fp + Fn)$$

← When the data is predominantly error-free (high base rate), this is not a meaningful measure of system performance.

# Outcome of a diagnostic test

## ▶ A grammatical error detector as a diagnostic test

- ◆ Positive: has grammatical error
- ◆ Negative: is error-free

		Real	
		Has grammatical error	Is error-free
Test	positive	① True positives	False positives
	negative	False negatives	True negatives

### ◆ Precision:

Rate of "True positives" out of all positive rulings (①)

$$= T_p / (T_p + F_p)$$

# Outcome of a diagnostic test

## ▶ A grammatical error detector as a diagnostic test

- ◆ Positive: has grammatical error
- ◆ Negative: is error-free

		Real	
		Has grammatical error	Is error-free
Test	positive	② True positives	False positives
	negative	False negatives	True negatives

### ◆ Recall:

Rate of "True positives" out of all actual positive cases (②)

$$= T_p / (T_p + F_n)$$

# Precision vs. recall

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- ▶ **Precision and recall** are in a trade-off relationship.
  - ◆ Highly precise grammatical error detector:  
Ignores many lower-confidence cases → drop in recall
  - ◆ High recall (captures as many errors as possible):  
many non-errors will also be flagged → drop in precision
- ▶ In developing a real-world application, picking the right trade-off point between the two is an important usability issue.
  - ◆ A **grammar checker** for general audience (MS-Word, etc)
    - ◆ Higher precision or higher recall?
  - ◆ Same, but for English learners.
    - ◆ Higher precision or higher recall?



# F-measure

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- ▶ **Precision** and **recall** are in a trade-off relationship.

← Both measures should be taken into consideration when evaluating performance

- ▶ **F-measure**

- ◆ Also called F-score,  $F_1$  score
- ◆ An overall measure of a test's accuracy:  
Combines *precision* (P) and *recall* (R) into a single measure
- ◆ Harmonic mean →

- ◆ Best value: 1,  
worst value: 0
- ◆ = average if P=R,  
< average if P and R different

$$F_1 = \frac{2PR}{P + R}$$

# Wrapping up

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- ▶ HW 4 Part A, B due on Tue
  - ◆ **Don't procrastinate!** Part B is more complex.
  
- ▶ Next class (Tue)
  - ◆ HW4 review
  - ◆ Midterm review
  
- ▶ Midterm exam on Thursday → NEXT SLIDE

# Midterm exam: what to expect

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- ▶ 10/12 (Thursday)
  - ◆ 75 minutes.
  - ◆ At LMC's PC Lab (**G17 CL**)
- ▶ Exam format:
  - ◆ Closed book. All pencil-and-paper.
  - ◆ Topical questions: "what is/discuss/analyze/find out/calculate..."
  - ◆ **Bring your calculator! →**
- ▶ A letter-sized **cheat sheet** allowed.
  - ◆ Front and back.
  - ◆ Hand-written only.

