HP-VERSION DGFEM FOR HAMILTON–JACOBI–BELLMAN EQUATIONS WITH CORDES COEFFICIENTS

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Abstract. It is well-known that viscosity solutions of uniformly elliptic fully nonlinear PDE with convex/concave nonlinearities posses $C^{2,\alpha}$ interior estimates. However, the development of high-order numerical methods for fully nonlinear PDE that achieve the key properties of consistency, stability and high-order accuracy remains a significant challenge for many problems. In this talk, we will present theoretical and computational aspects of the hp-version DGFEM for fully nonlinear second-order elliptic and parabolic Hamilton–Jacobi–Bellman equations with Cordes coefficients, which is joint work with Endre Süli. The discretisation of the PDE is motivated by a continuous analysis based on the Cordes condition, which originated in the PDE literature in the analysis of well-posedness for nondivergence form equations with discontinuous coefficients. Our continuous analysis is based on a variational strong monotonicity argument which establishes well-posedness of the fully nonlinear HJB PDE in the class of strong solutions. We will see that the numerical method is consistent and stable, with error bounds that are optimal in the mesh size, and suboptimal in the polynomial degrees, as standard for hp-version DGFEM. For parabolic problems, the discretisation is extended by a high-order DG time-stepping method, permitting high-order approximation in both time and space. The time-discretisation further allows a priori error bounds for solutions with very low temporal regularity, showing optimal convergence rates with respect to the time-step size and temporal polynomial degrees. Numerical experiments demonstrate the accuracy and efficiency of the numerical scheme on problems featuring strongly anisotropic diffusion coefficients and singular solutions, including exponential convergence rates under hp-refinement.