CS 1674/2074: Filters

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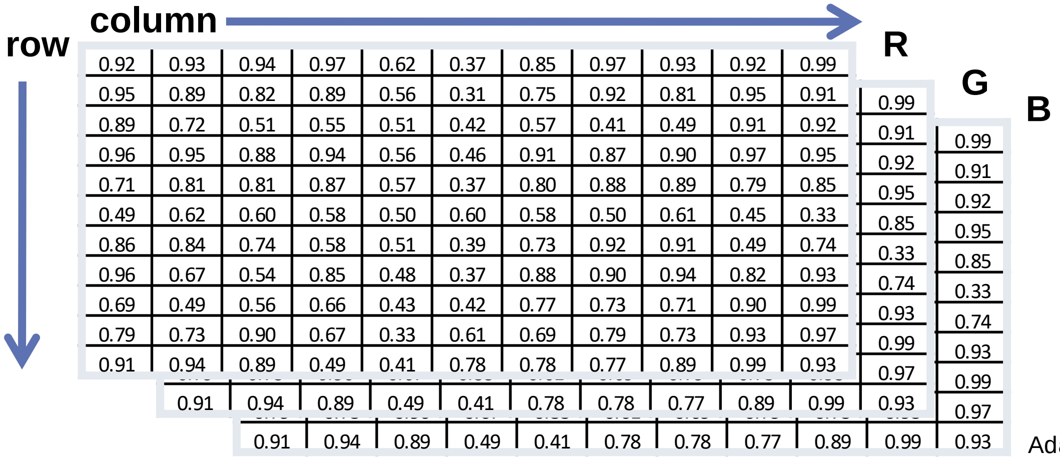
Topics

- Filters: motivation, math and properties
- Types of filters
  - Linear
    - Smoothing
    - Other
  - Non-linear
    - Median

- Applications of filters
  - Texture representation with filters
  - Anti-aliasing for image subsampling

How images are represented

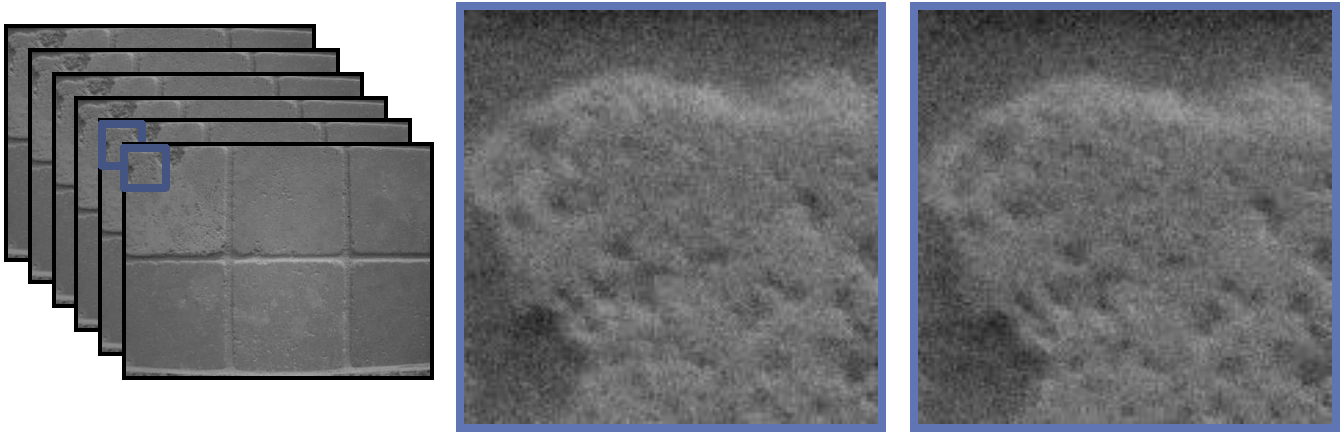
- Color images represented as a matrix with multiple channels (=1 if grayscale)
- Suppose we have a NxM RGB image called "im"
    - im(0,0,0) = top-left pixel value in R-channel
    - im(y, x, b) = y pixels **down (rows)**, x pixels **to right (cols)** in bth channel
    - im(N, M, 3) = bottom-right pixel in B-channel
- cv2.imread(filename) returns a uint8 image (values 0 to 255)



0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

Adapted from Derek Hoiem

Enter Noise



- The same object will look very different across images
- Even multiple images of same static scene won't be identical
- How could we reduce the noise, i.e., give an estimate of the true intensities?
- What if there's only one image?

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Common types of noise

- **Impulse noise**: random occurrences of white pixels
- **Salt and pepper noise**: random occurrences of black and white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Original

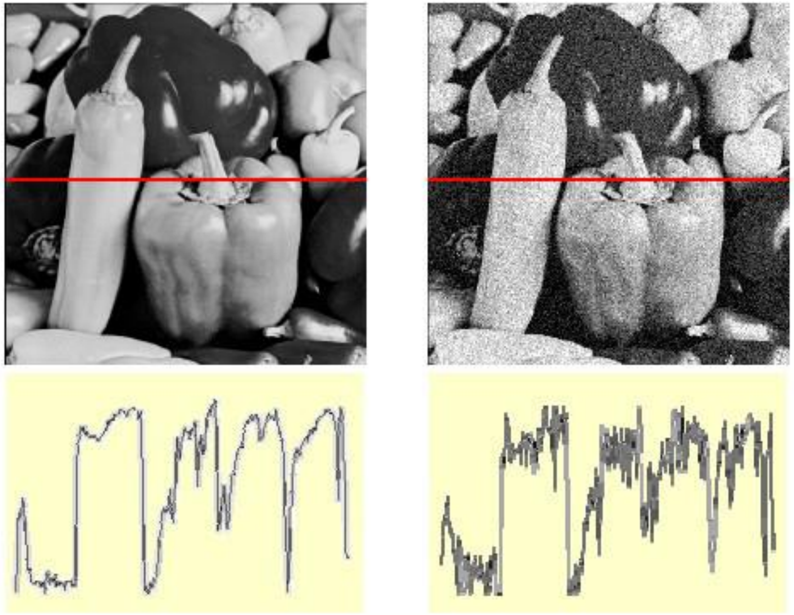
Salt and pepper noise

Impulse noise

Gaussian noise

Source: S. Seitz

Gaussian noise



Ideal Image, Noise process

$$f(x,y) = \overbrace{\bar{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

Gaussian i.i.d. ("white") noise:

$$\eta(x,y) \sim \mathcal{N}(\mu, \sigma)$$

```
>> noise = np.random.rand( *im.shape  ) * sigma
>> im_noise = im + noise
```

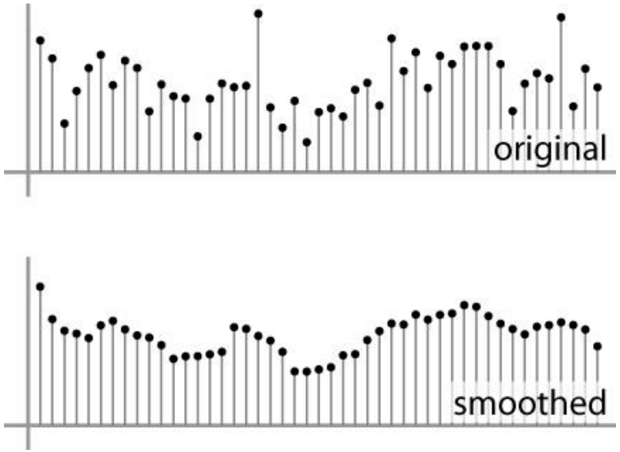
What is impact of the sigma?

[Github repo]

Fig: M. Hebert

First attempt at a solution

- Let's replace each pixel with an average of all the values in its neighborhood
- Moving average in 1D:



Source: S. Marschner

First attempt at a solution

- Let’s replace each pixel with an average of all the values in its neighborhood
- Why/when will this work?

- Assumptions:
  - Expect pixels to be like their neighbors
  - Expect noise processes to be independent from pixel to pixel

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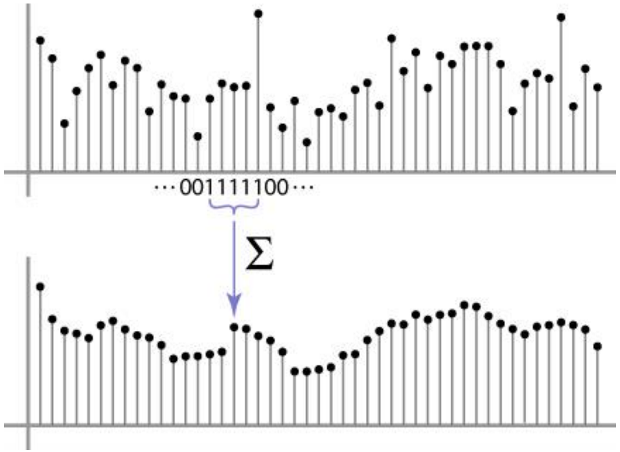
Weighted Moving Average

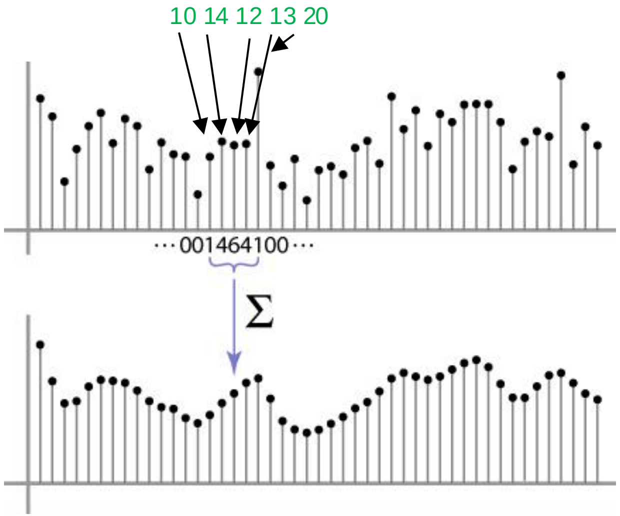
- Can add weights to our moving average
- *Weights* [1, 1, 1, 1, 1] / 5



Source: S. Marschner

Weighted Moving Average

- Non-uniform weights [1, 4, 6, 4, 1] / 16



Central pixel =
(10*1 +
14*4 +
12*6 +
13*4 +
20*1) / 16

Adapted from S. Marschner

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0								

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10							

Source: S. Seitz

Moving Average In 2D

$$F[x,y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x,y]$$

	0	10	20						

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30					

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30				

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

In what ways is this output good/expected?
In what ways is it bad (what was lost)?

Source: S. Seitz

Image Filtering

- Compute a function of the local neighborhood at each pixel in the image
  - Function specified by a "filter" or mask saying how to combine values from neighbors
  - Element-wise multiplication of filter and image patch

- Uses of filtering:
  - Enhance an image (denoise, resize, etc)
  - Extract information (texture, edges, etc)
  - Detect patterns (template matching)

Adapted from Derek Hoiem

Correlation Filtering

Non-weighted, averaging window size is 2k+1 x 2k+1:

$$G[i,j] = \underbrace{\frac{1}{(2k+1)^2}}_{\text{Attribute uniform weight to each pixel}} \underbrace{\sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]}_{\text{Loop over all pixels in neighborhood around image pixel } F[i,j]}$$

*Attribute **uniform weight** to each pixel*

Loop over all pixels in neighborhood around image pixel F[i,j]

Now generalize to allow **different weights** depending on neighboring pixel's relative position:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} \underbrace{H[u,v]}_{\text{Non-uniform weights}} F[i+u, j+v]$$

Non-uniform weights

Correlation Filtering

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

This is called **cross-correlation**, denoted $G = H \otimes F$

Filtering an image: replace each pixel with a linear combination of its neighbors.

The filter "**kernel**" or "**mask**" *H[u,v]* is the prescription for the weights in the linear combination.

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Averaging Filtering

- What values belong in the kernel *H* for the moving average example?

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$\otimes$

$H[u, v]$

$\frac{1}{9}$

1	**1**	**1**
1	**?**	**1**
1	**1**	**1**

“box filter”

$G[x, y]$

	0	10	20	30	30				

$$G = H \otimes F$$

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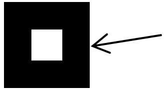
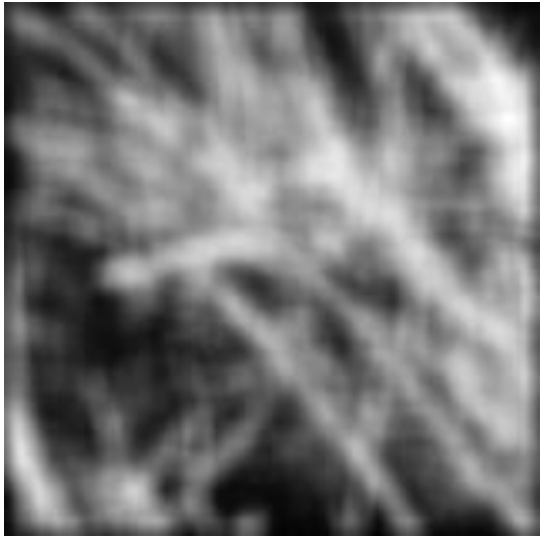
Smoothing by averaging



depicts box filter:
white = high value, black = low value

original



filtered

What if the filter size was 5 x 5 instead of 3 x 3?

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Gaussian Filter

- What if we want nearest neighboring pixels to have the most influence on the output?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

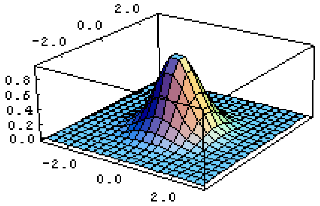
$\frac{1}{16}$

1	2	1
2	4	2
1	2	1

$H[u, v]$

This kernel is an approximation of a 2d Gaussian function:

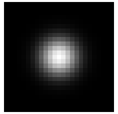
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

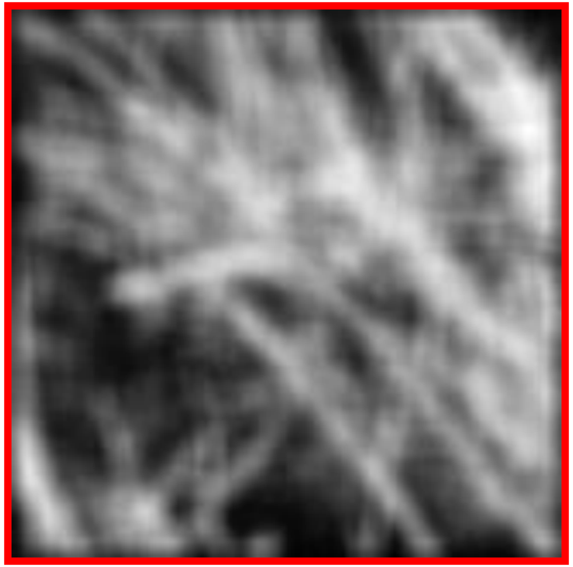


- Removes high-frequency components from the image ("low-pass filter").

Source: S. Seitz

Smoothing with a Gaussian





Vs box filter

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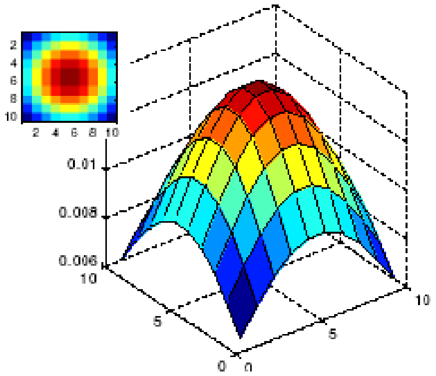
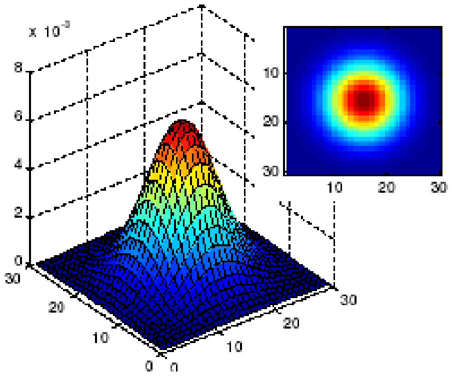
Gaussian Filters

- What parameters matter here?
- **Size** of kernel or mask
  - Note, Gaussian function has infinite support, but discrete filters use finite kernels



σ = 5 with 10 x 10 kernel



σ = 5 with 30 x 30 kernel

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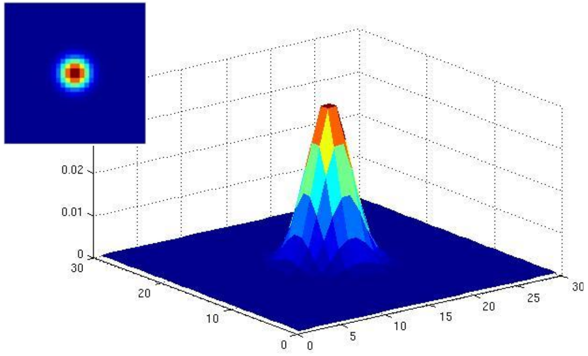
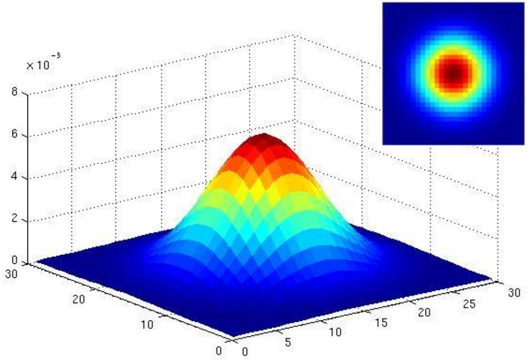
Gaussian Filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing



σ = 2 with
30 x 30
kernel



σ = 5 with
30 x 30
kernel

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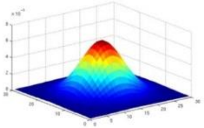
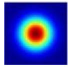
Gaussian Filters in Python

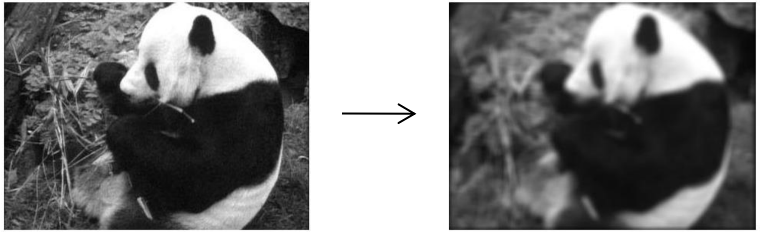
```
>> hsize = 10;
>> sigma = 5;
>> filter = fspecial_gauss(hsize, sigma)

>> plt.matshow(filter)
```





```
>> filt_im = ndimage.convolve(im, filter, mode='constant')#
correlation
>> cv2.imshow(outim);
```



outim

[Github repo]

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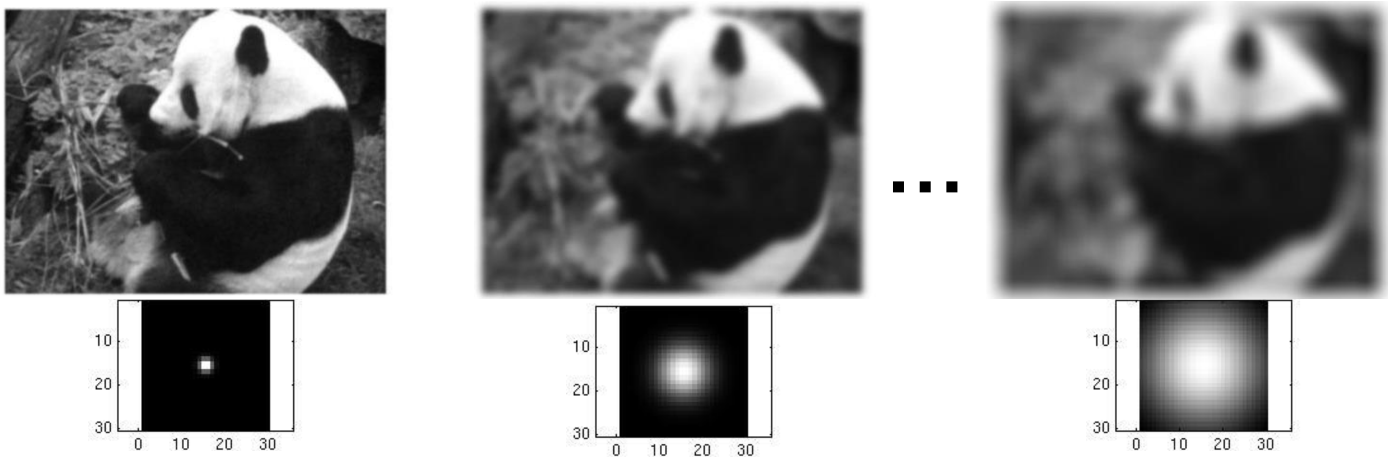
Smoothing with a Gaussian

Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.




```
for sigma=1:3:10
    h = fspecial_gaus(hsize, sigma);
    out = ndimage.convolve(im, h);
    cv2.imshow(out);
end
```

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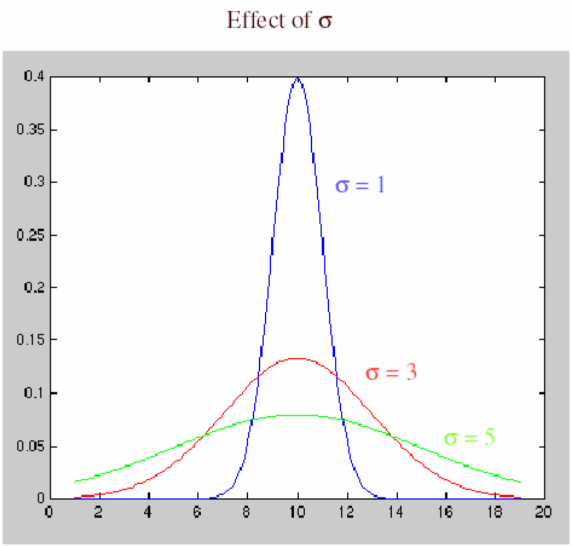
Gaussian Filters

How big should the filter be?

- Values at edges should be near zero
- Rule of thumb for Gaussian: set filter half-width (*hsize*) to 3 σ



Source: Derek Hoiem

Properties of smoothing filters

- Smoothing
  - Values positive
  - Sum to 1 → overall intensity same as input
  - Amount of smoothing proportional to mask size
  - Remove “high-frequency” components; “low-pass” filter

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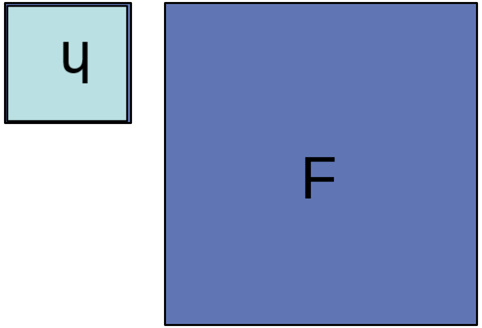
Convolution

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left)
  - Then apply cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i-u, j-v]$$

$$G = H \star F$$

↑ *Notation for convolution operator*



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Convolution vs correlation

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i-u, j-v]$$

$$G = H \star F$$

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

$$G = H \otimes F$$

For a Gaussian or box filter, how will the outputs differ?

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Convolution vs correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

F

5	2	5	4	4
5	200	3	200	4
1	5	5	4	4
5	5	1	1	2
200	1	3	5	200
1	200	200	200	1

(i, j)

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06
.12	.25	.12
.06	.12	.06

(0, 0)

Convolution vs correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

F

5	2	5	4	4
5	200	3	200	4
1	5	5	4	4
5	5	1	1	2
200	1	3	5	200
1	200	200	200	1

(i, j)

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06
.12	.25	.12
.06	.12	.06

(0, 0)

Convolution vs correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

v = +1

F

5	2	5	4	4
5	200	3	200	4
1	5	5	4	4
5	5	1	1	2
200	1	3	5	200
1	200	200	200	1

(i, j)

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06
.12	.25	.12
.06	.12	.06

(0, 0)

Convolution vs correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1
v = 0
v = +1
u = 0, v = -1

F

5	2	5	4	4
5	200	3	200	4
1	5	5	4	4
5	5	1	1	2
200	1	3	5	200
1	200	200	200	1

(i, j)

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06
.12	.25	.12
.06	.12	.06

(0, 0)

Convolution vs correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

F

5	2	5	4	4
5	200	3	200	4
1	5	5	4	4
5	5	1	1	2
200	1	3	5	200
1	200	200	200	1

(i, j)

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H \star F$$

H

.06	.12	.06
.12	.25	.12
.06	.12	.06

(0, 0)

Convolution vs correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

F

5	2	5	4	4
5	200	3	200	4
1	5	5	4	4
5	5	1	1	2
200	1	3	5	200
1	200	200	200	1

(i, j)

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06
.12	.25	.12
.06	.12	.06

(0, 0)

Convolution vs correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

v = +1

F

5	2	5	4	4
5	200	3	200	4
1	5	5	4	4
5	5	1	1	2
200	1	3	5	200
1	200	200	200	1

(i, j)

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06
.12	.25	.12
.06	.12	.06

(0, 0)

Convolution vs correlation

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

$$G = H \otimes F$$

u = -1, v = -1

v = 0

v = +1

u = 0, v = -1

F

5	2	5	4	4
5	200	3	200	4
1	5	5	4	4
5	5	1	1	2
200	1	3	5	200
1	200	200	200	1

(i, j)

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

$$G = H \star F$$

H

.06	.12	.06
.12	.25	.12
.06	.12	.06

(0, 0)

Properties of Convolution

- Commutative:

  $f * g = g * f$
- Associative:

  $(f * g) * h = f * (g * h)$
- Distributes over addition:

  $f * (g + h) = (f * g) + (f * h)$
- Scalars factor out:

  $kf * g = f * kg = k(f * g)$
- Identity:

  unit impulse e = [$\ldots$, 0, 0, 1, 0, 0, $\ldots$]. $f * e = f$

Kristen Grauman

Predict the outputs using correlation filtering

$*$

0	0	0
0	1	0
0	0	0

= ?

$*$

0	0	0
0	0	1
0	0	0

= ?

$*$

0	0	0
0	2	0
0	0	0

$- \frac{1}{9}$

1	1	1
1	1	1
1	1	1

= ?

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

Filtered
(no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

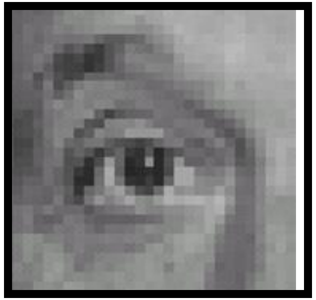
?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0



Shifted left by 1 pixel with correlation

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

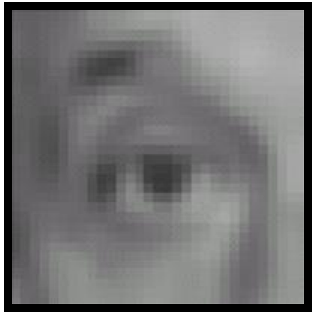
?

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$



Blur (with a box filter)

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

?

Source: D. Lowe

Practice with linear filters

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{Original image}} + \underbrace{\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right)}_{\text{Image minus blur = details}}$$

Practice with linear filters

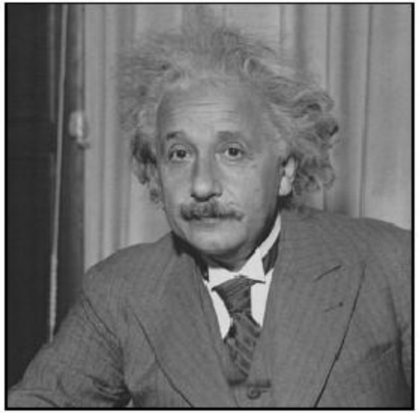
Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

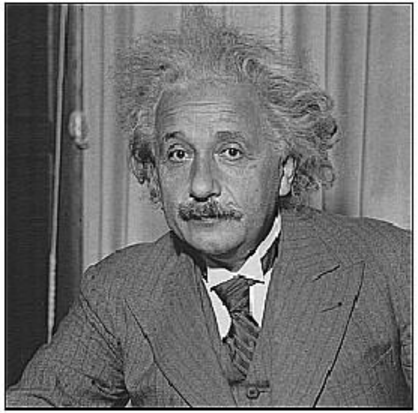
Sharpening filter:
accentuates differences with local average

Source: D. Lowe

Sharpening



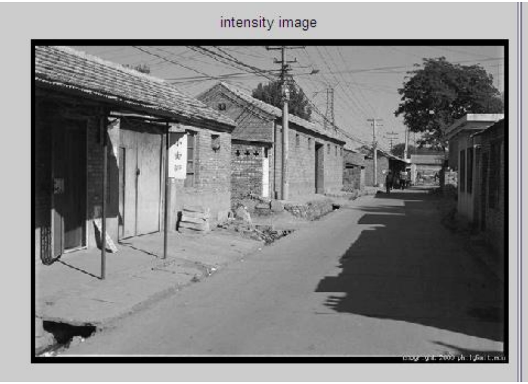
before



after

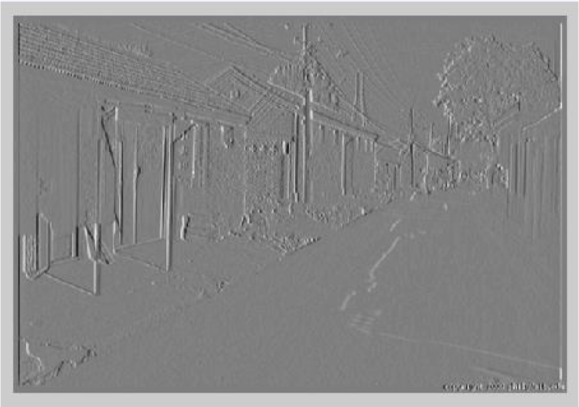
Filters for computing *gradients*

1	0	-1
2	0	-2
1	0	-1



*

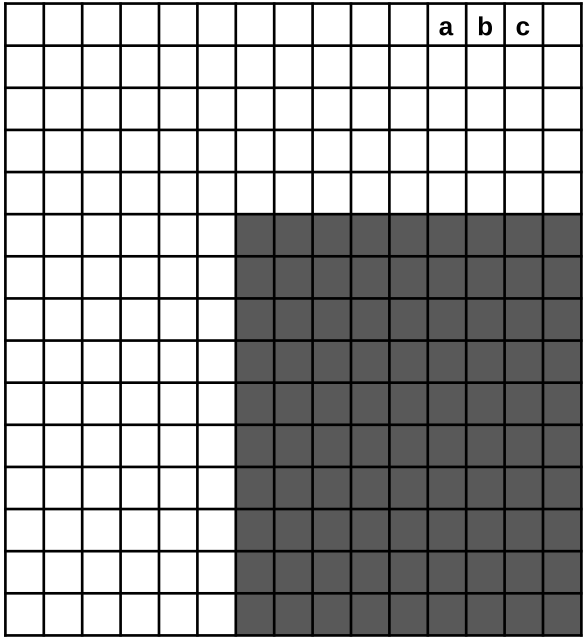
=

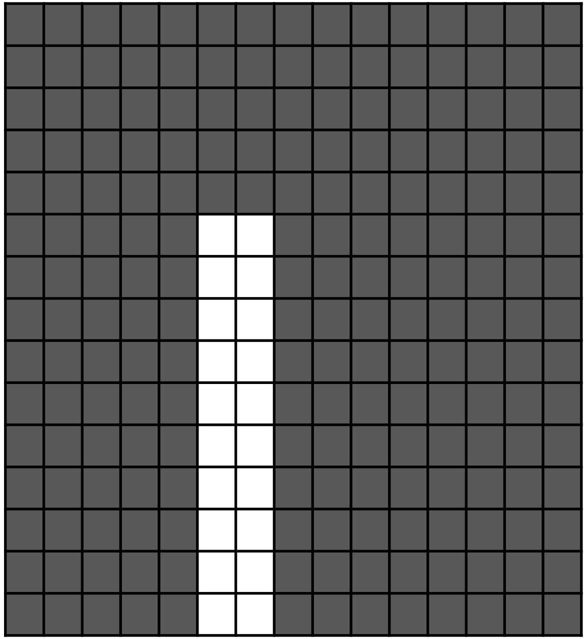


Adapted from Derek Hoiem

Filters for computing *gradients*

+1*a + 0*b + (-1)*c = a - c



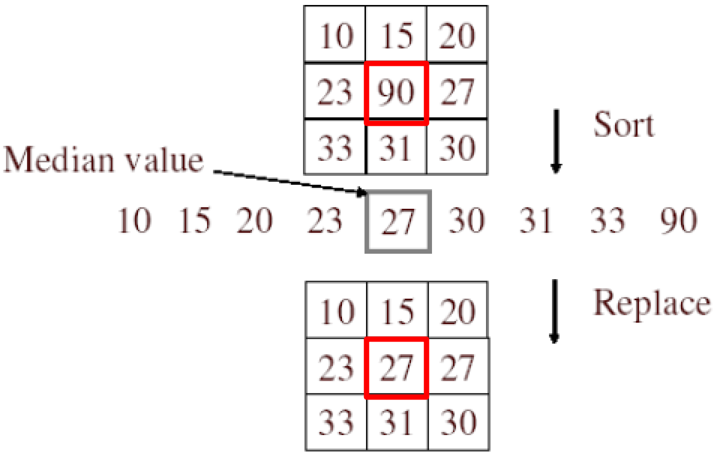


Input image
White = 1, black = 0

Filter: [+1 0 -1]

Output image

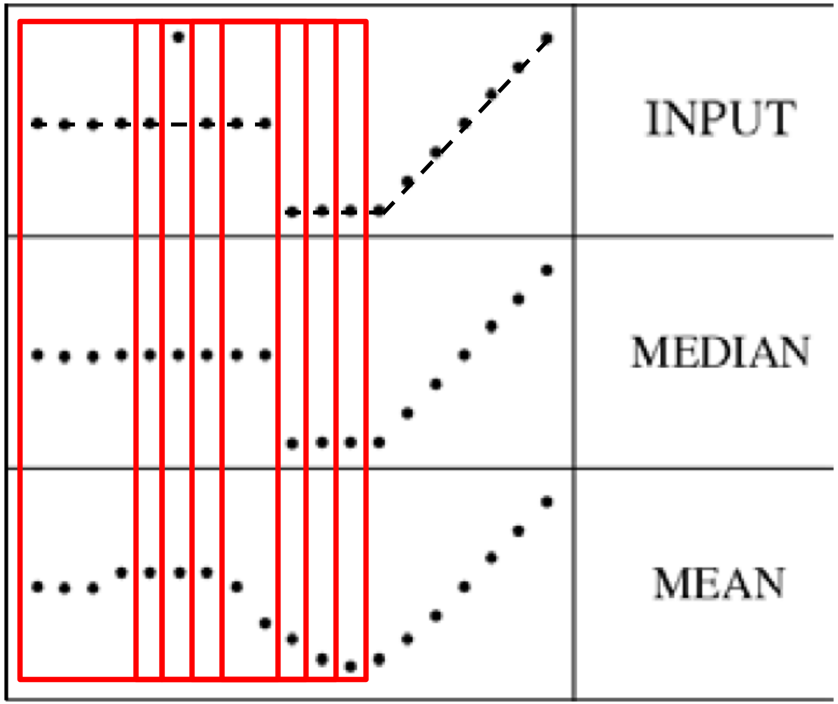
Median Filter



- No new pixel values introduced
- Removes spikes: good for impulse, salt & pepper noise
- Non-linear filter

Median Filter

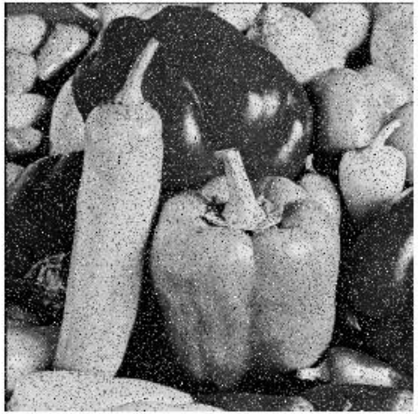
- Median filter is edge preserving



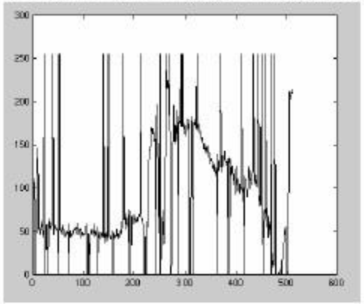
Adapted from Kristen Grauman

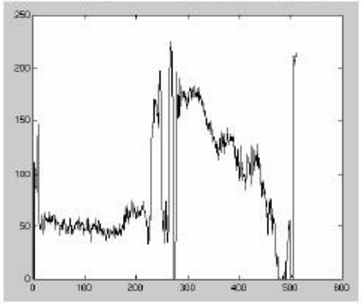
Median Filter

Salt and pepper noise



Median filtered





Plots of a row of the image

https://docs.scipy.org/doc/scipy/reference/generated/scipy.ndimage.median_filter.html#scipy.ndimage.median_filter

Source: M. Hebert

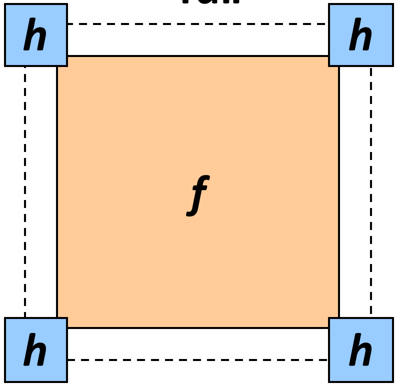
Boundary Issues

- What is the size of the output?
  - 'full': output size is larger than the size of ***f***
  - 'same': output size is same as ***f (Default for Python)***

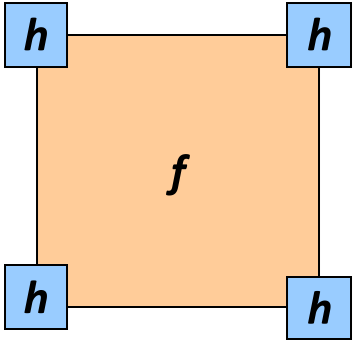
f = image
h = filter

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

full



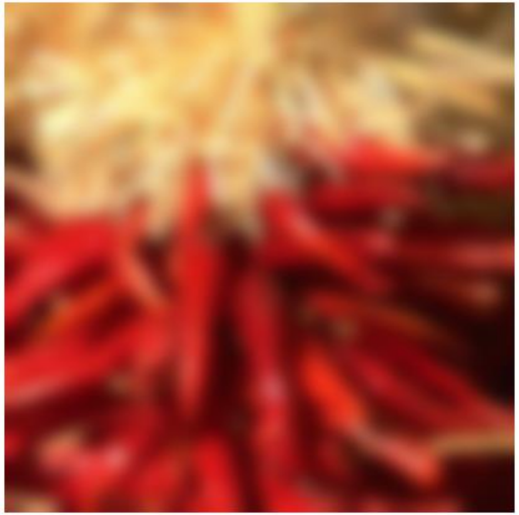
same



Adapted from S. Lazebnik

Boundary Issues

- What about near the edge?
  - the filter window might fall off the edge of the image (in ‘same’ or ‘full’)
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge



Source: S. Marschner

Separability

- In some cases, filter is separable, and we can factor into two steps:
  - Convolve all rows
  - Convolve all columns

Kristen Grauman

Separability Example

2D filtering
(center location only)

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix}$$

filter

The filter factors
into an *outer* product
of 1D filters:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Perform filtering
along rows:

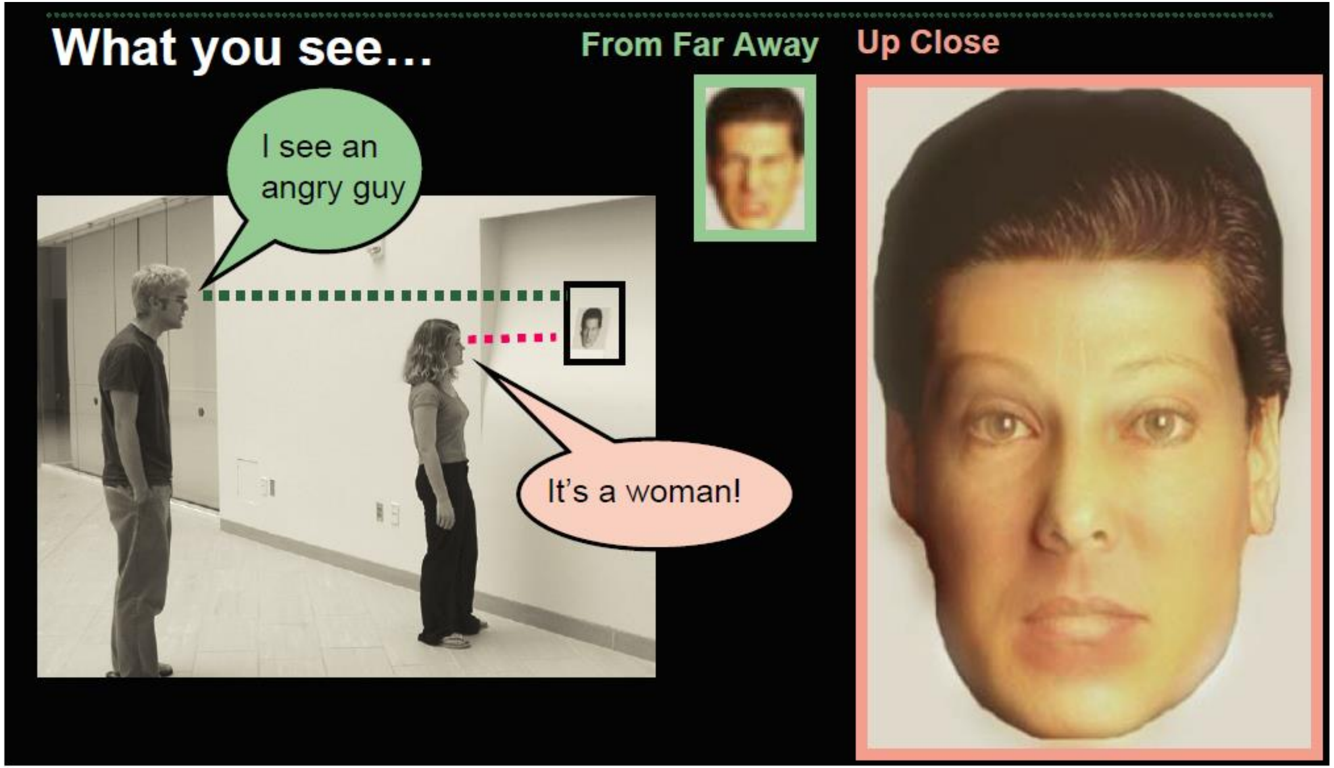
$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 11 & 18 & 18 \end{bmatrix}$$

Followed by filtering
along the remaining column:

Asymptotic cost for 2D vs 1D filtering? Let image be PxP, filter be NxN

Kristen Grauman

Application: Hybrid Images

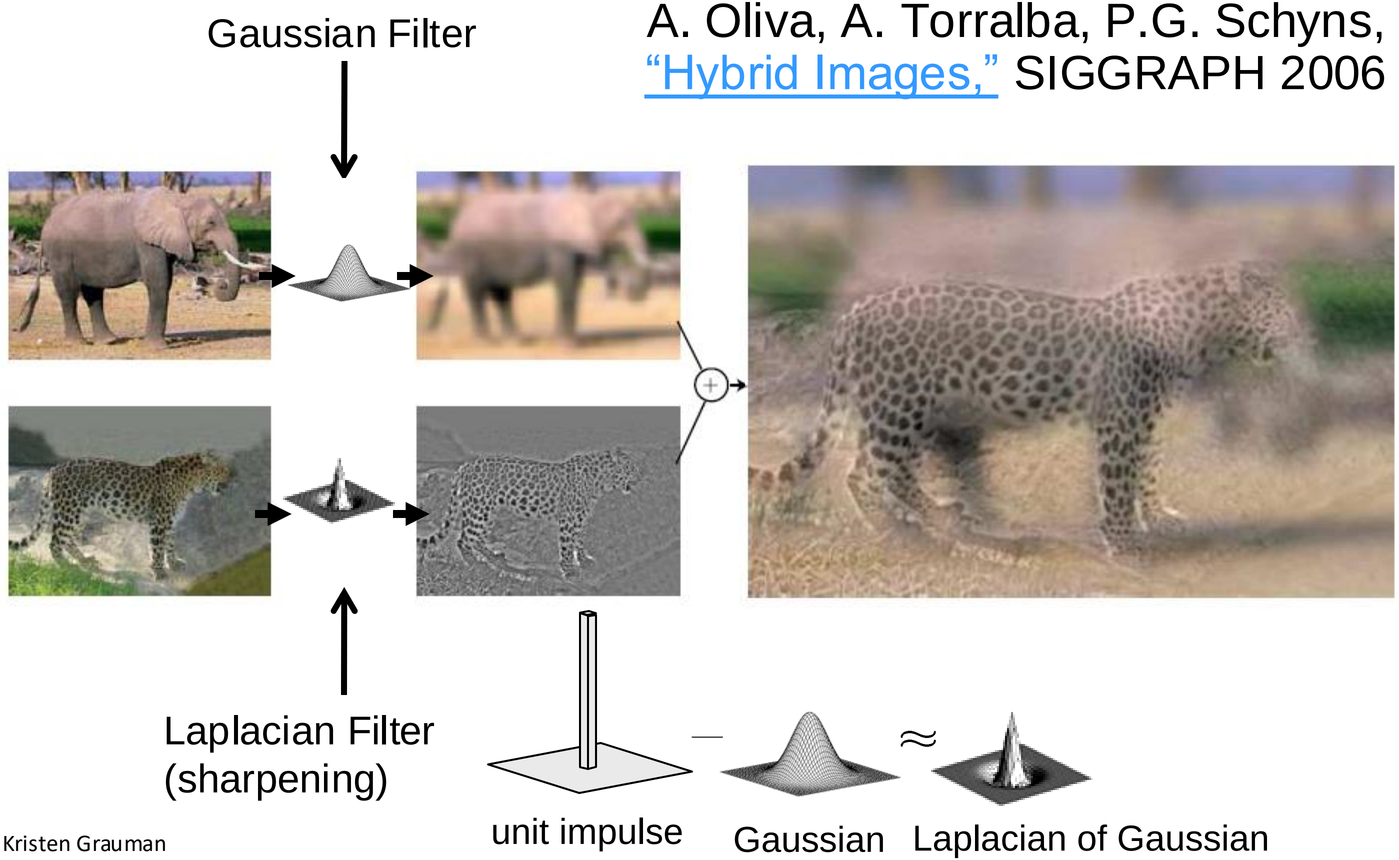


Kristen Grauman

Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006

Application: Hybrid Images

A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006



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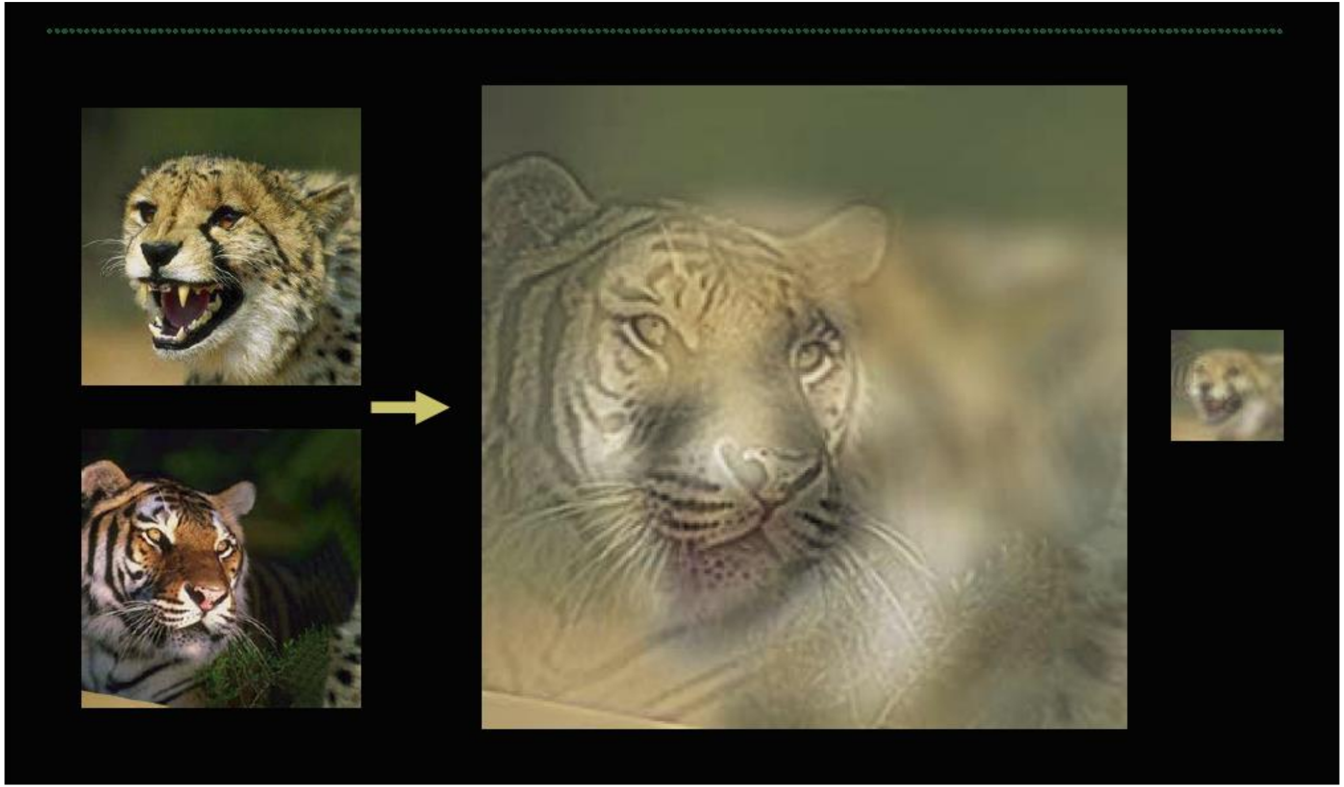
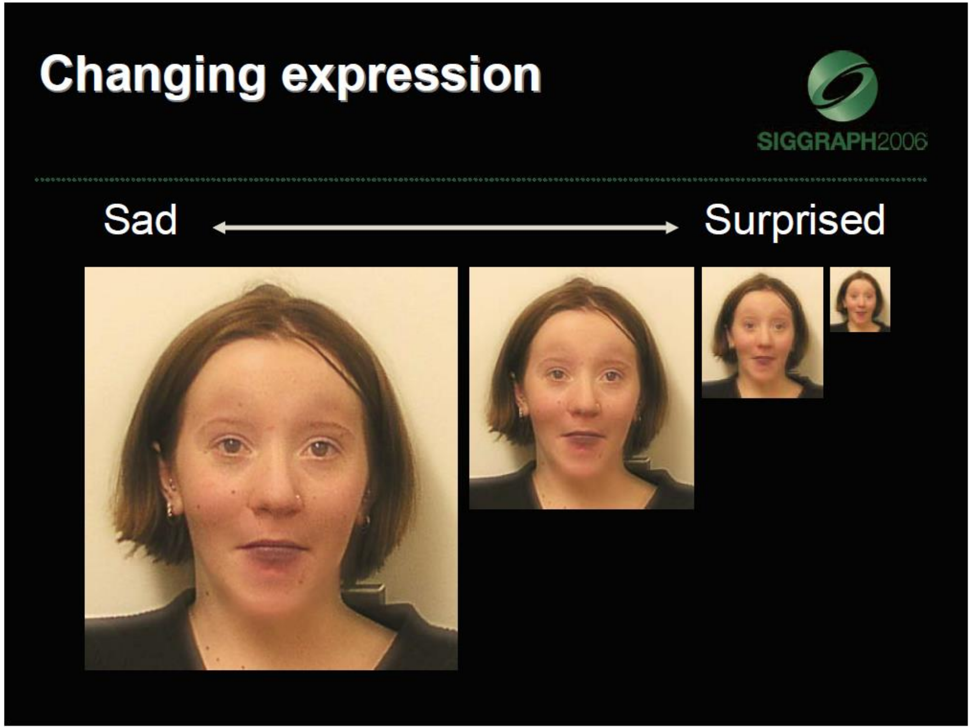
Application: Hybrid Images



Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006

Application: Hybrid Images

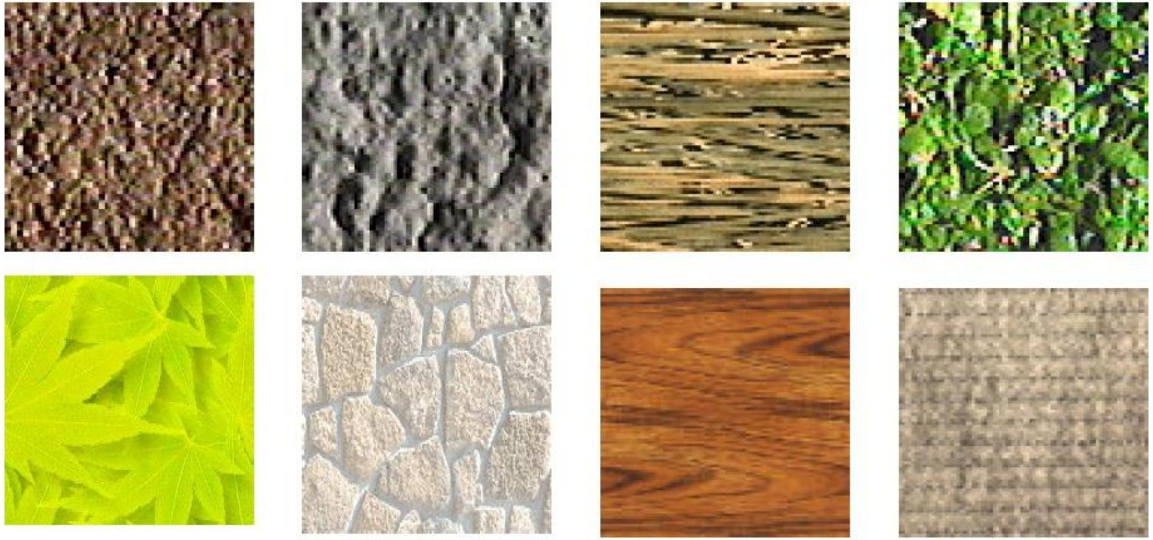


Aude Oliva & Antonio Torralba & Philippe G Schyns, SIGGRAPH 2006

Plan for next two lectures

- Filters: math and properties
- Types of filters
  - Linear
    - Smoothing
    - Other
  - Non-linear
    - Median
- Applications
  - Texture representation with filters
  - Anti-aliasing for image subsampling

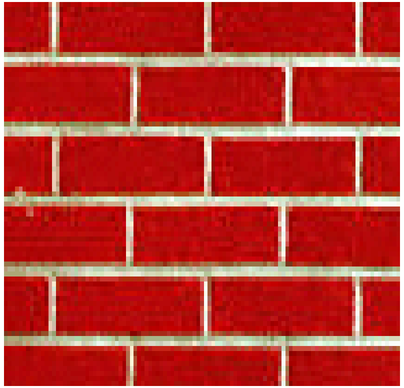
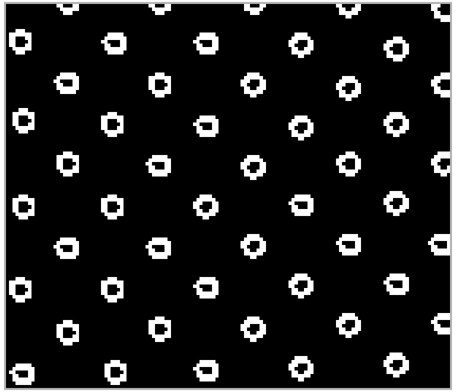
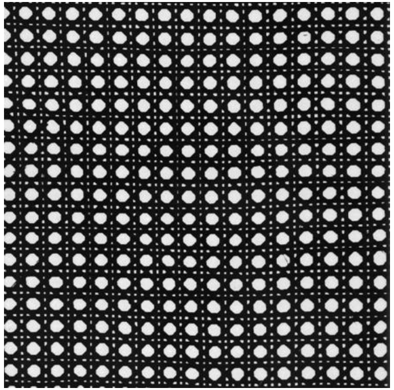
Texture



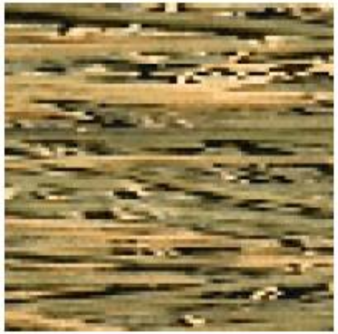
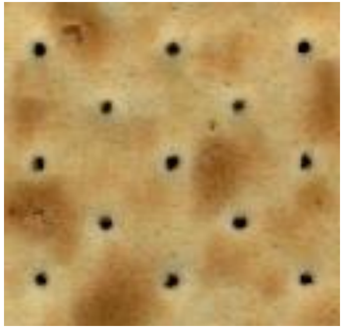
Due to:

Patterns, marks, etches, blobs, holes, relief, etc.

Regular (top), random (bottom) patterns



Kristen Grauman

Why analyze texture?

- Important for how we perceive objects

- Can be an important appearance cue that allows us to distinguish objects, especially if shape is similar across objects

Adapted from Kristen Grauman

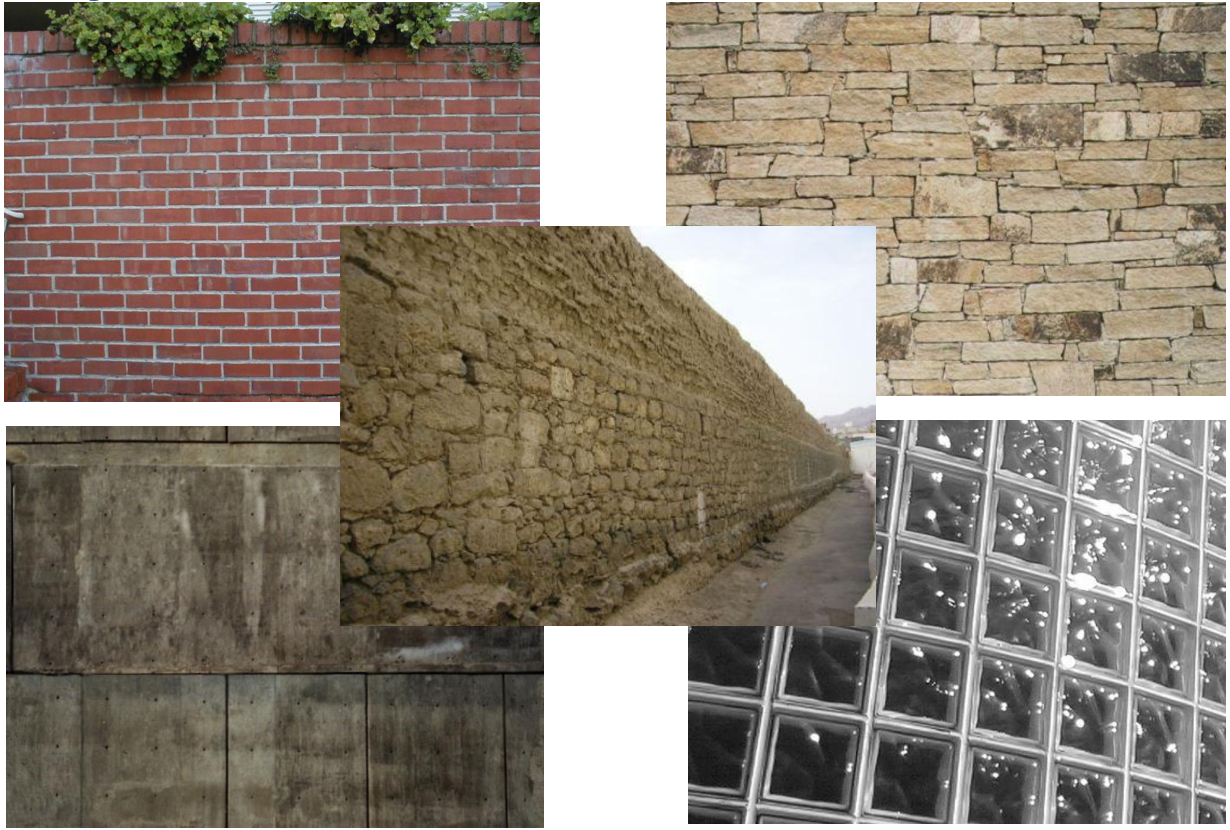
Same shape, different texture/object




Kristen Grauman

http://animals.nationalgeographic.com/

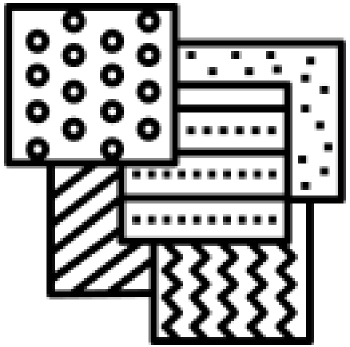
Same object, different texture



Kristen Grauman

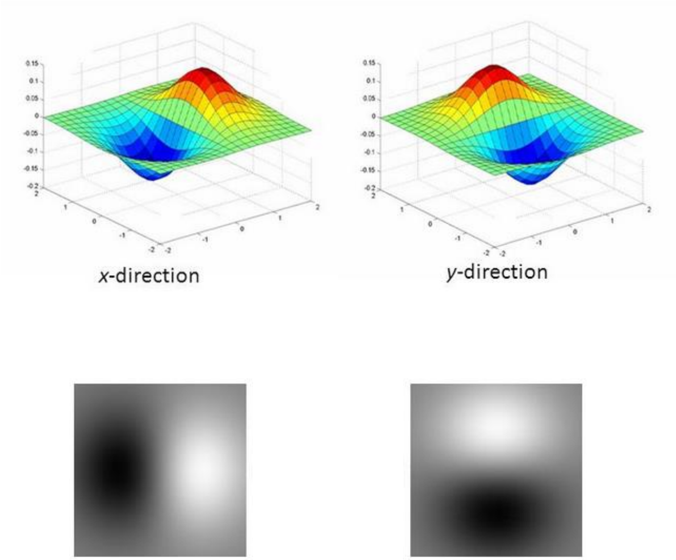
Texture Representation

- Textures are made up of repeated local patterns, so:
  - Find the patterns
    - Use filters that look like patterns (spots, bars, raw patches…)
    - Consider magnitude of response
  - Describe their statistics within each local window
    - E.g. mean, standard deviation, histogram



Kristen Grauman

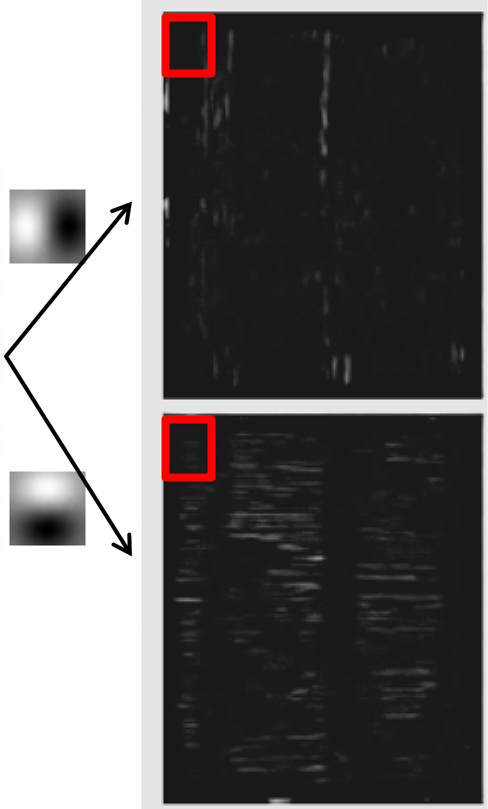
Derivative of Gaussian filter



Figures from Noah Snavely

Texture representation: Example

original image



derivative filter responses, squared

	mean d/dx value	mean d/dy value
Win. #1	4	10

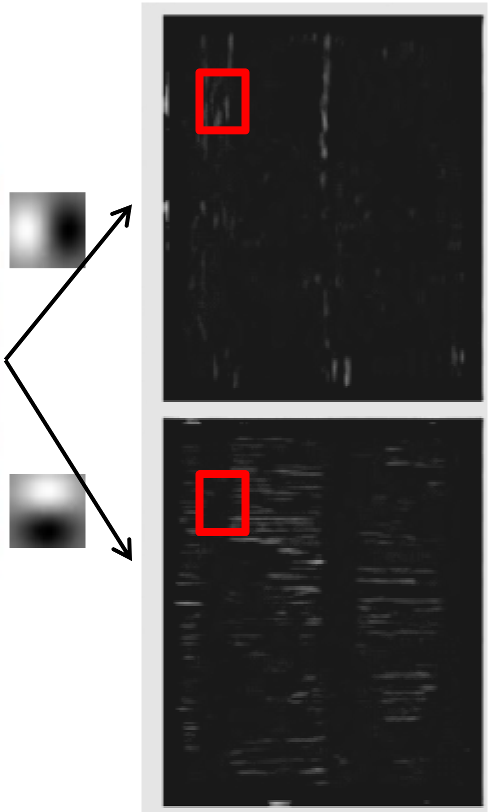
⋮

statistics to summarize patterns in small windows

Kristen Grauman

Texture representation: Example

original image



derivative filter responses, squared

	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2	18	7

⋮

statistics to summarize patterns in small windows

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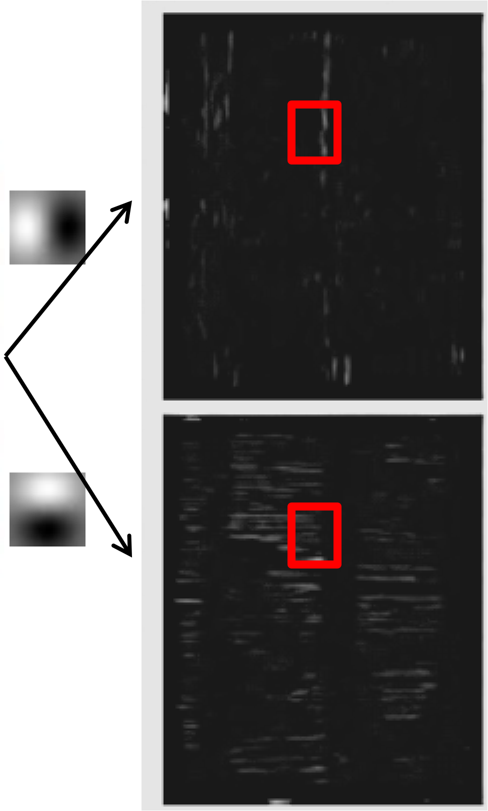
Texture representation: Example

original image



derivative filter responses, squared

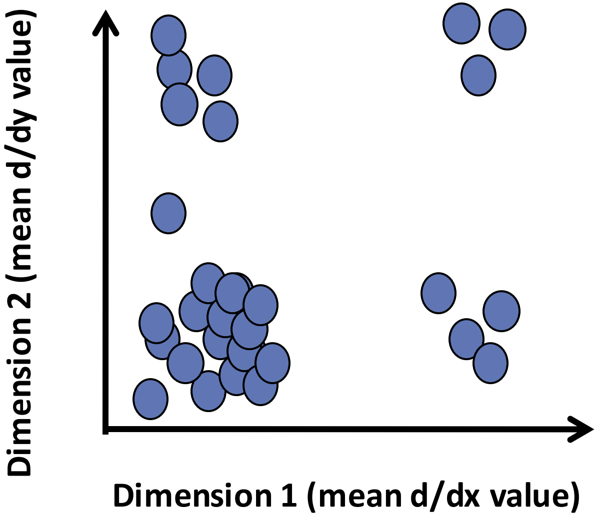
	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2	18	7
⋮		
Win.#9	20	20

⋮

statistics to summarize patterns in small windows

Kristen Grauman

Texture representation: Example



	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2	18	7
⋮		
Win.#9	20	20

⋮

statistics to summarize patterns in small windows

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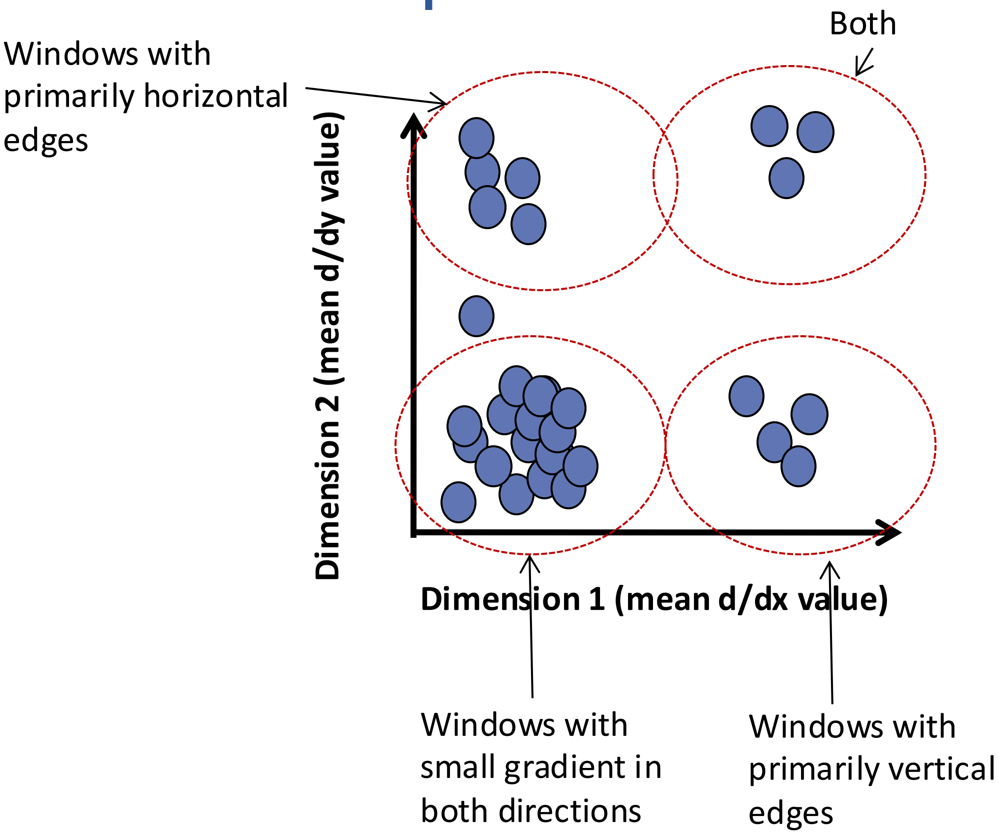
Texture representation: Example



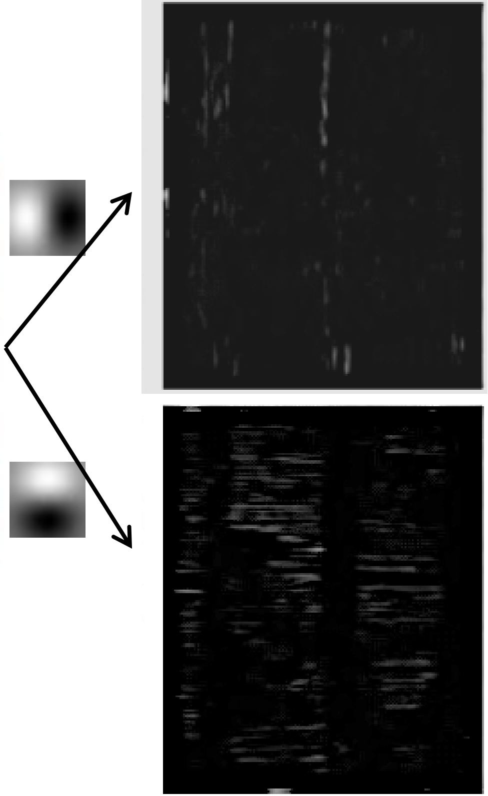
	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2	18	7
⋮		
Win.#9	20	20

⋮

statistics to summarize patterns in small windows

Texture representation: Example

original image



derivative filter responses, squared

visualization of the assignment to texture "types"

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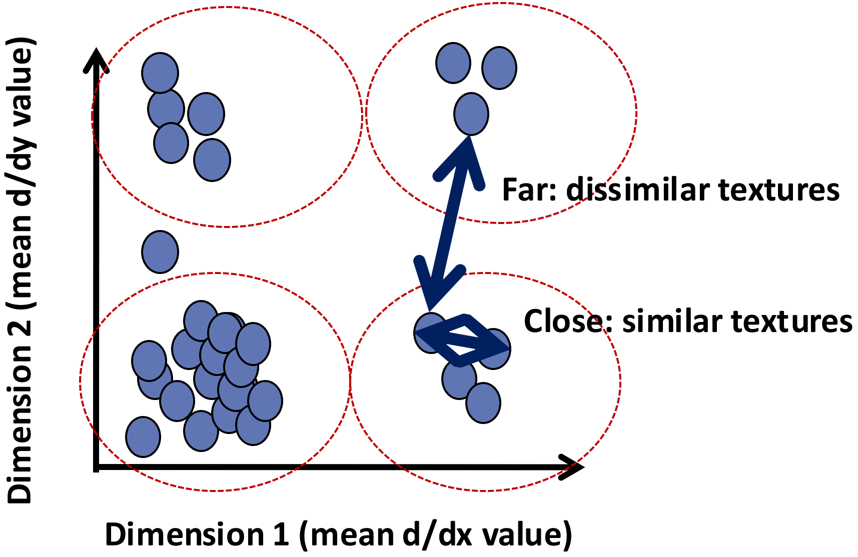
Texture representation: Example



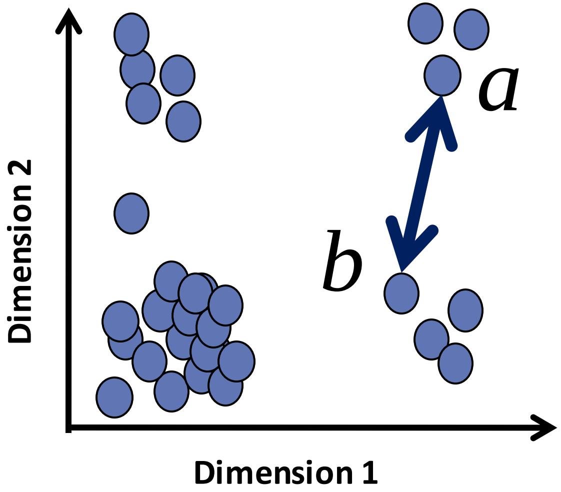
	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2 ⋮	18	7
Win.#9	20	20

⋮

statistics to summarize patterns in small windows

Kristen Grauman

Computing Distance using Texture

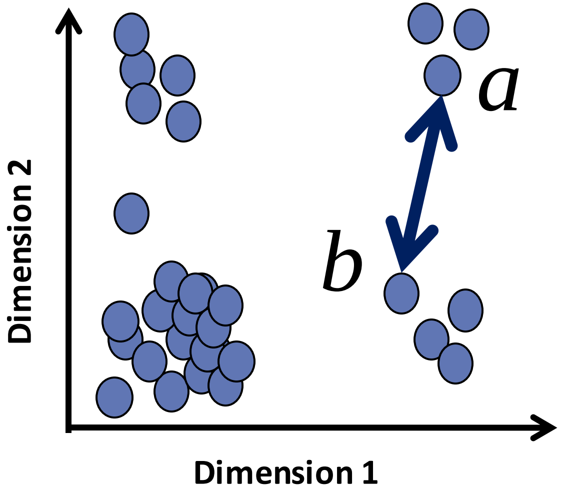


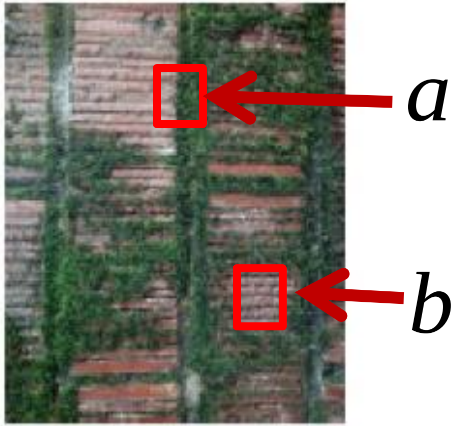
$$D(a,b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$

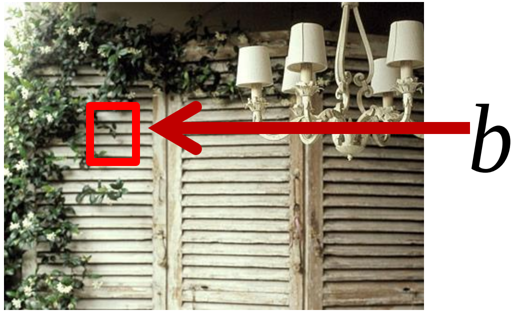
$$= \sqrt{\sum_{i=1}^{d} (a_i - b_i)^2}$$

Euclidean distance (L_2)

Texture Representation: Example







Distance reveals how dissimilar texture from window a is from texture in window b.

Kristen Grauman

Filter banks

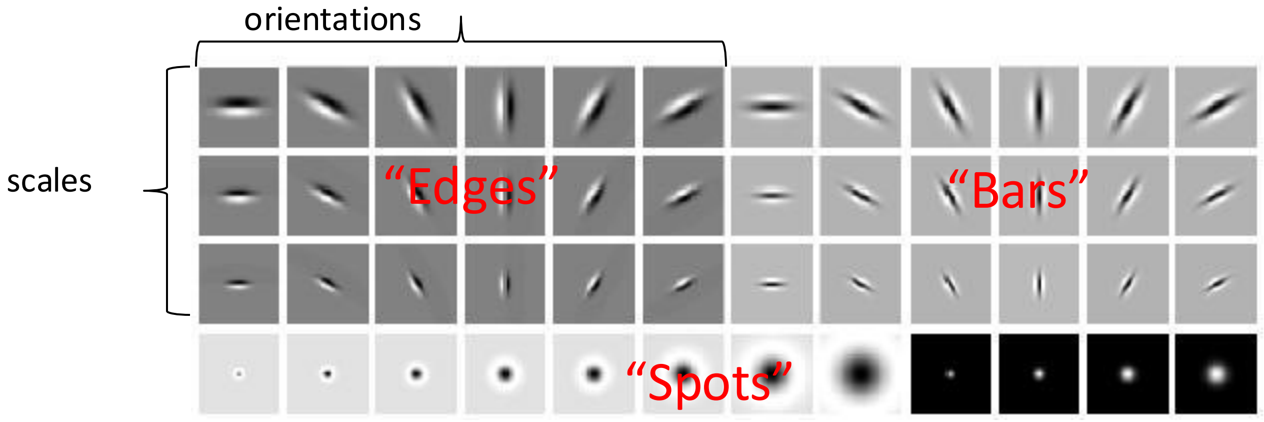
- Our previous example used two filters, and resulted in a 2-dimensional feature vector to describe texture in a window.
  - x and y derivatives revealed something about local structure.

- We can generalize to apply a collection of multiple (*d*) filters: a "filter bank".

- Then our feature vectors will be *d*-dimensional.

Adapted from Kristen Grauman

Filter banks

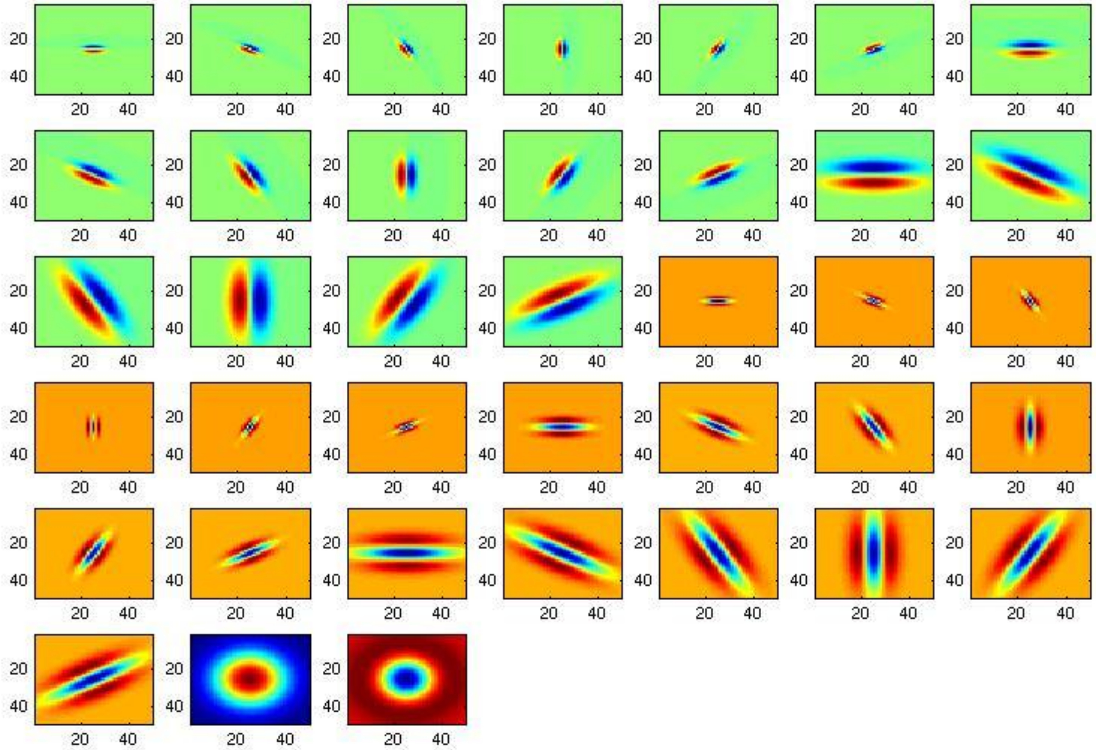


- What filters to put in the bank?
  - Typically we want a combination of scales and orientations, different types of patterns.

Which filters would you use to distinguish buildings from animals? Cheetahs from tigers? Ladybugs from dalmatians?

Adapted from Kristen Grauman, Matlab code: http://www.robots.ox.ac.uk/~vgg/research/texclass/filters.html

Filter bank



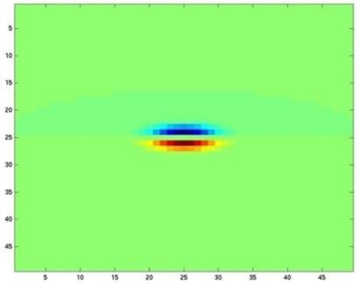
Kristen Grauman

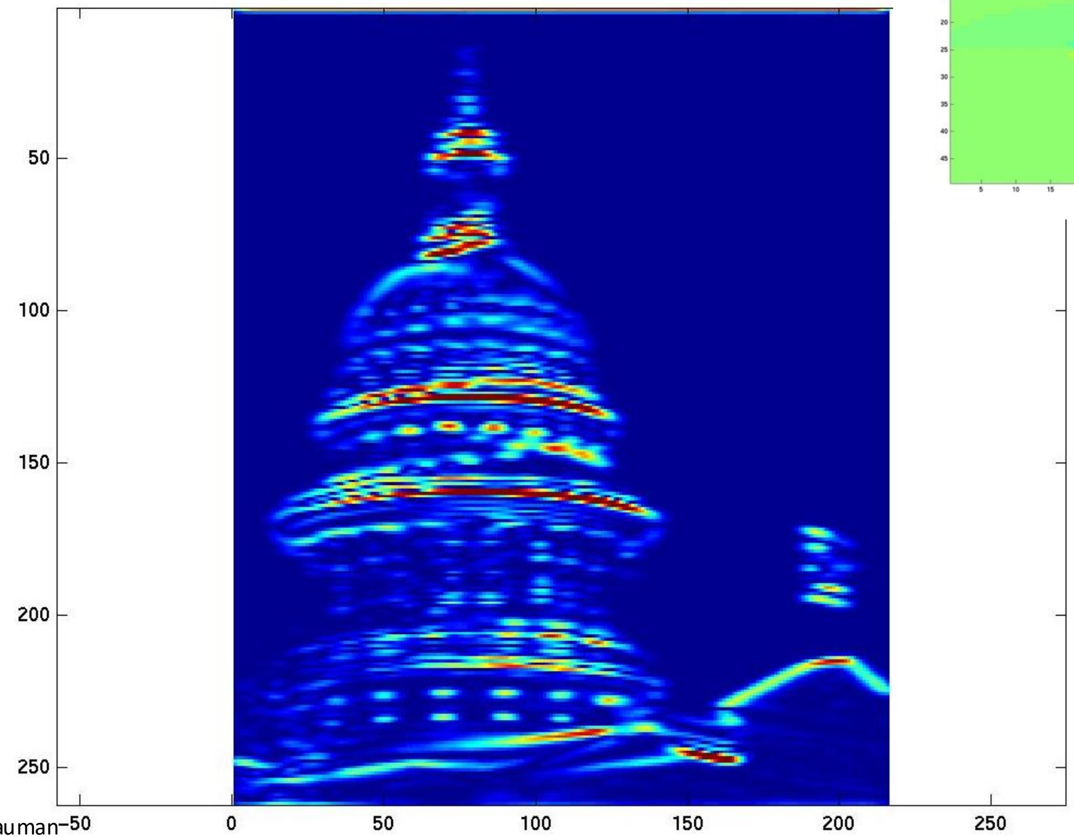
Filter bank: Example

Image from http://www.texasexplorer.com/austincap2.jpg

Kristen Grauman

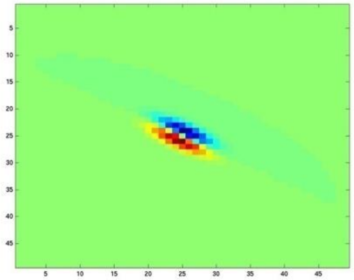
Filter bank: Example

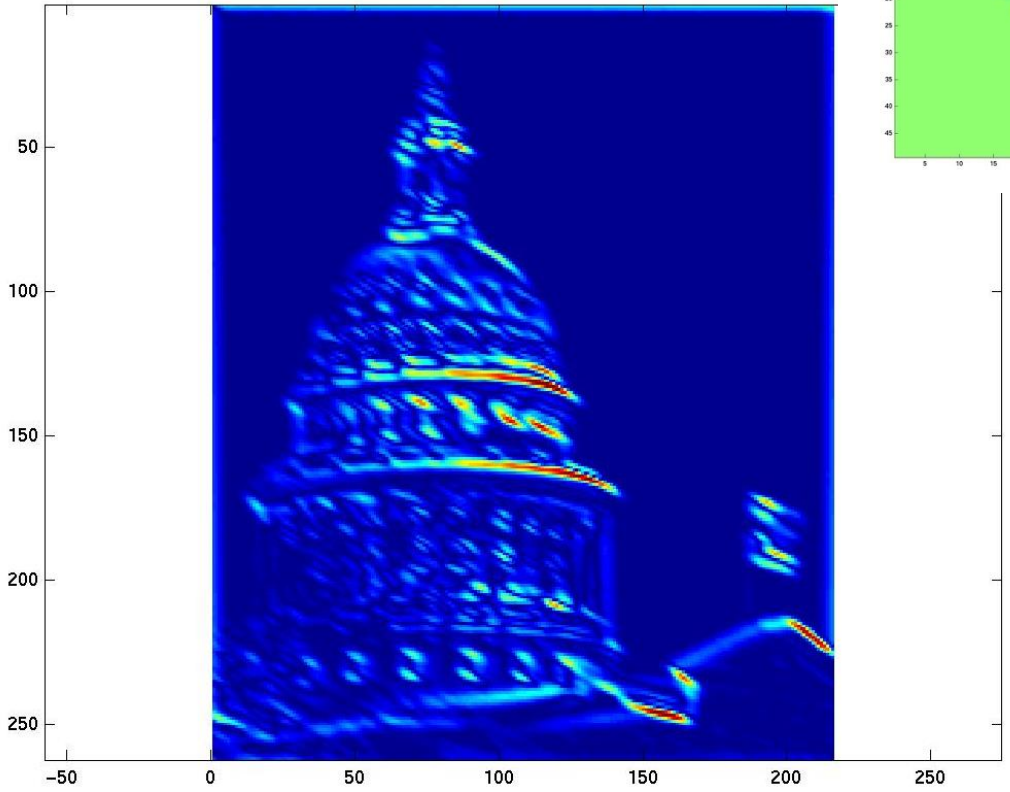




Kristen Grauman

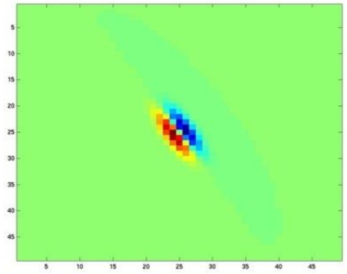
Filter bank: Example

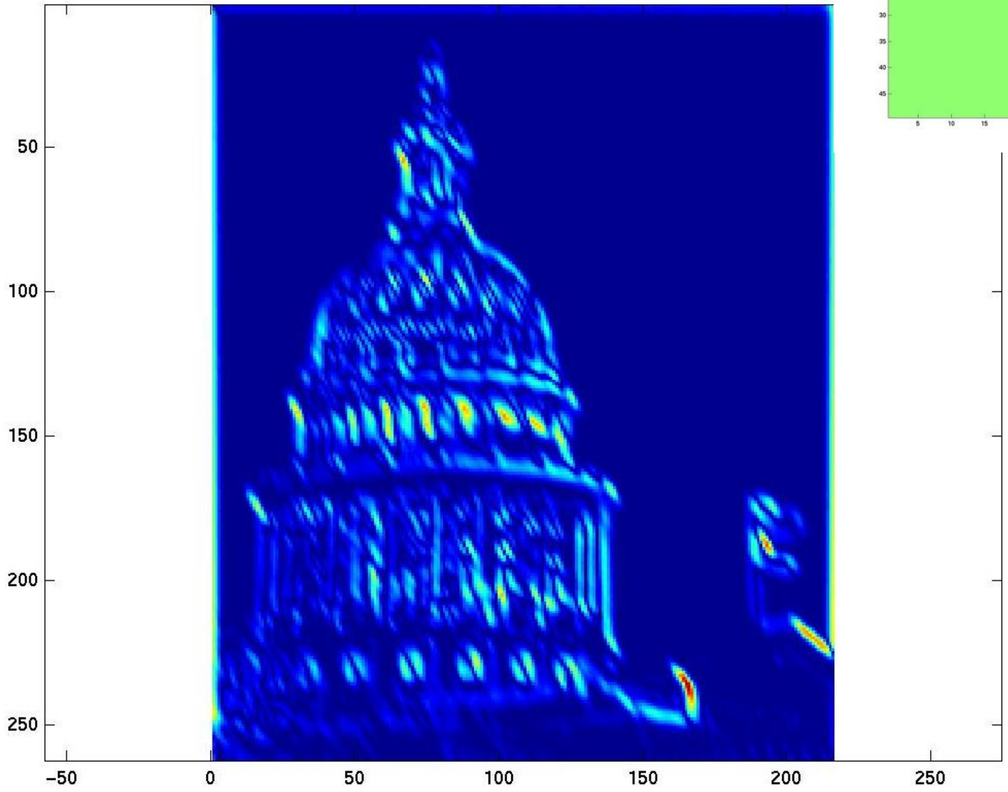




Kristen Grauman

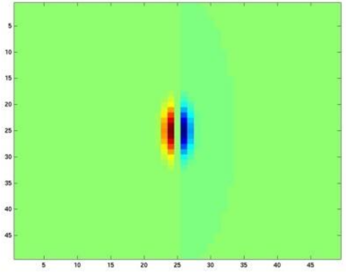
Filter bank: Example

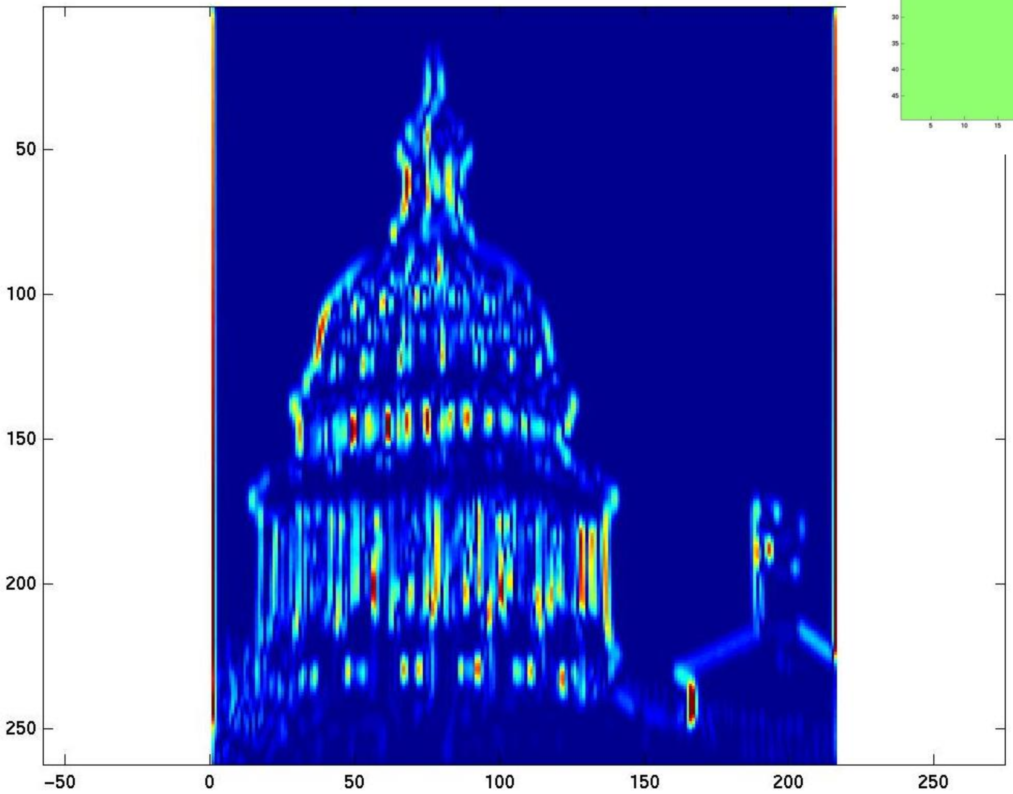




Kristen Grauman

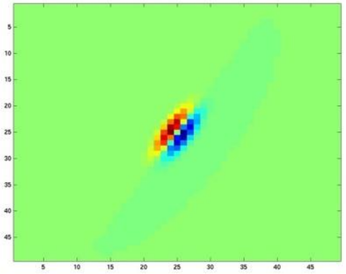
Filter bank: Example

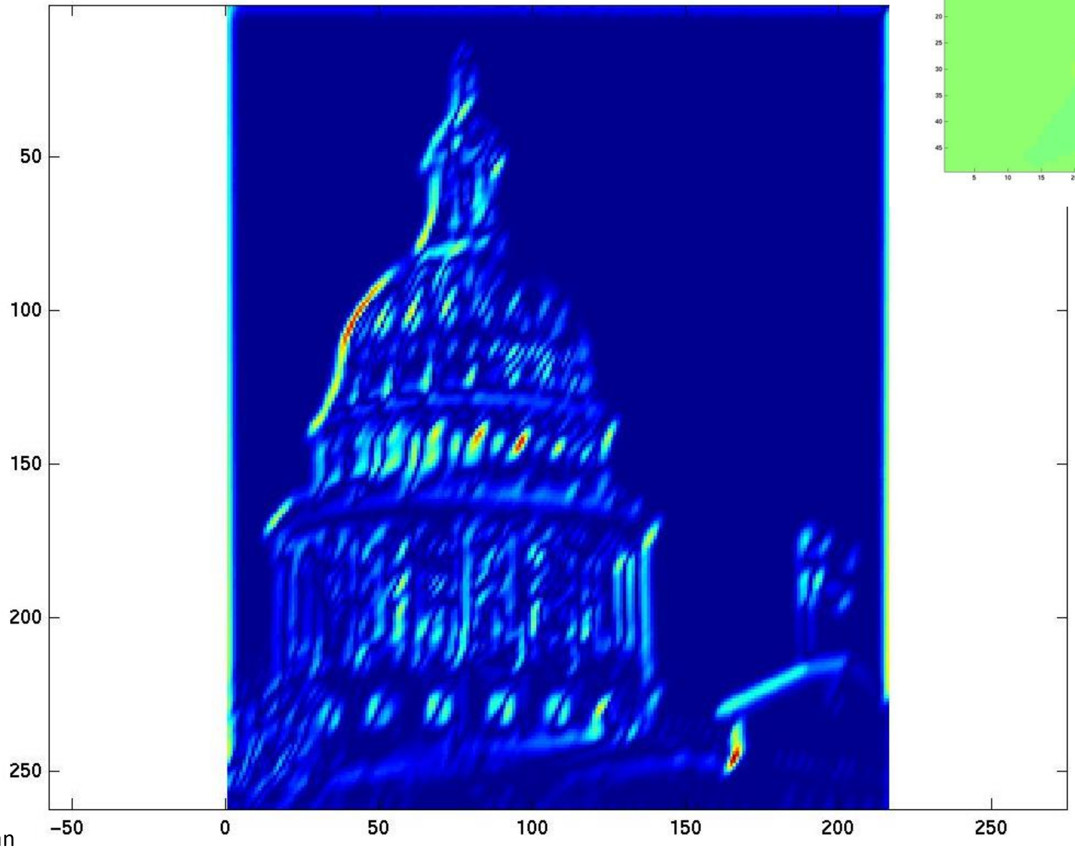




Kristen Grauman

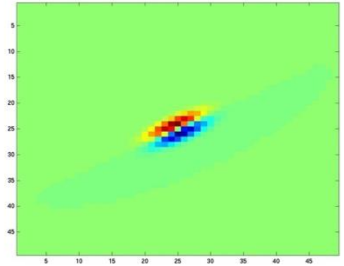
Filter bank: Example

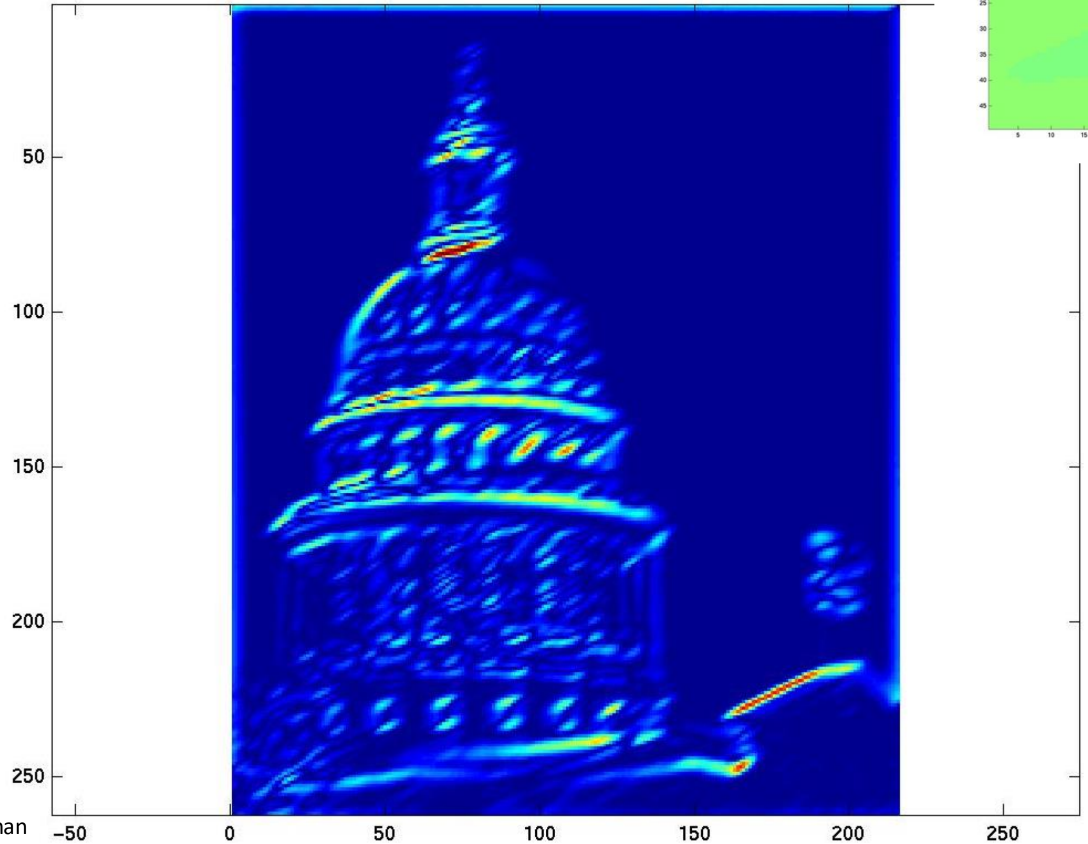




Kristen Grauman

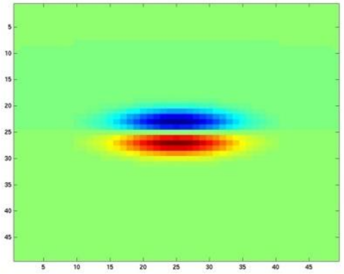
Filter bank: Example

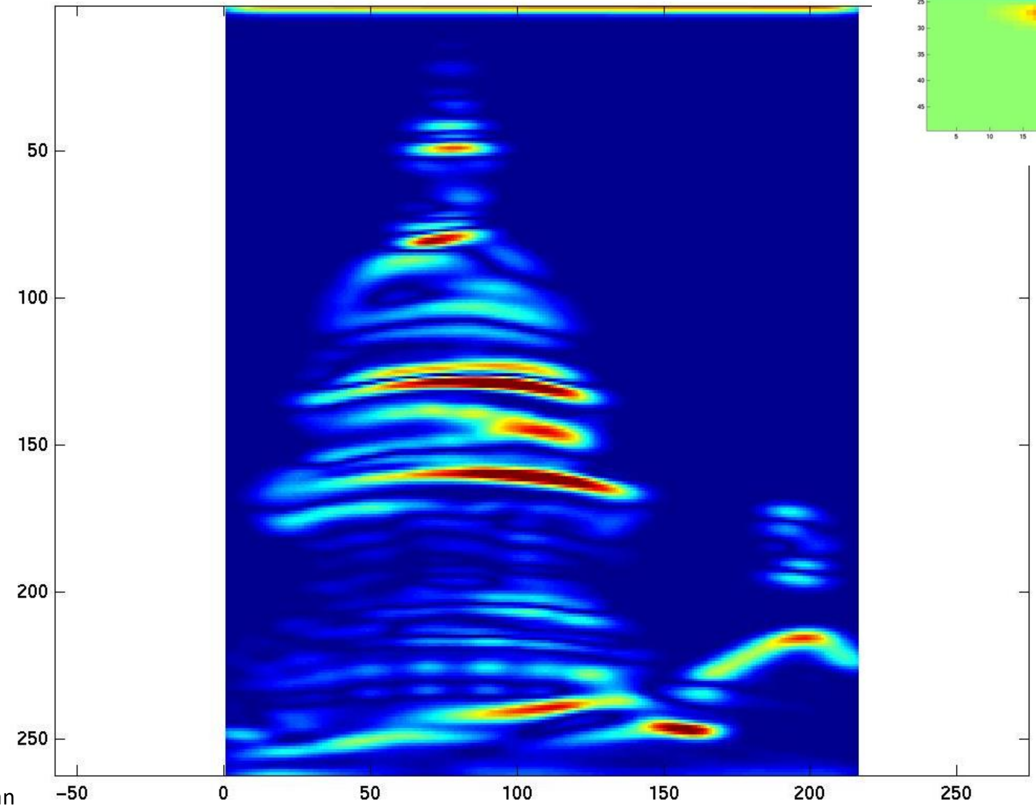




Kristen Grauman

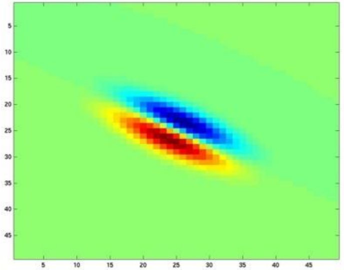
Filter bank: Example

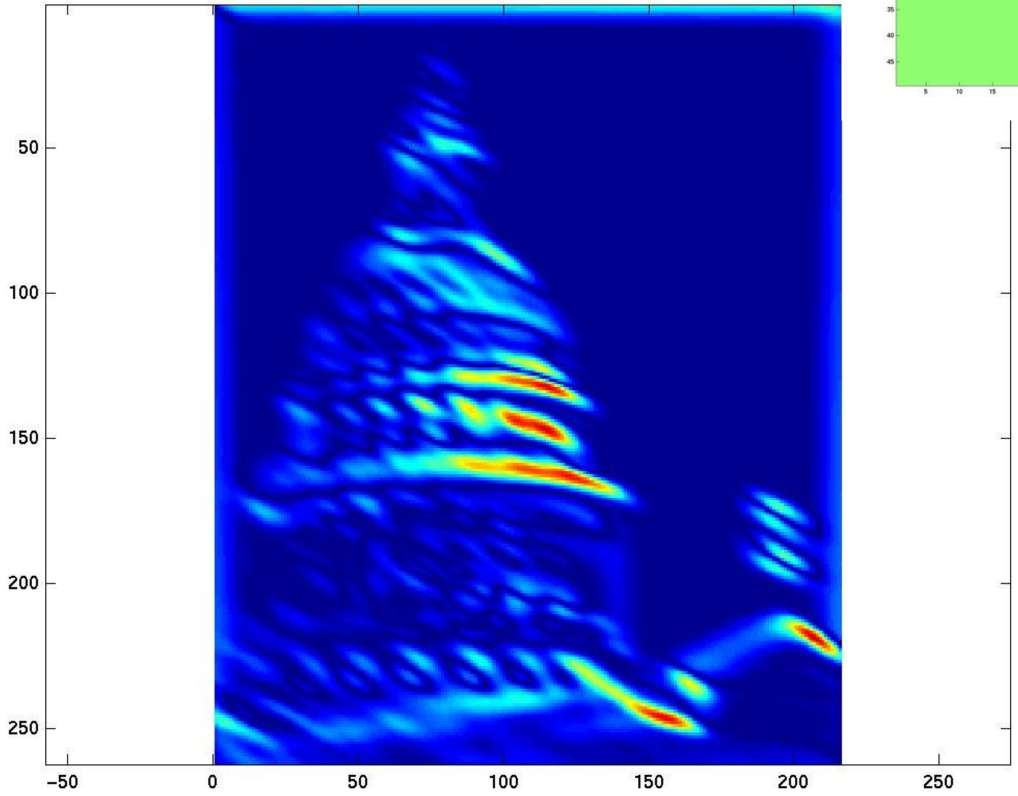




Kristen Grauman

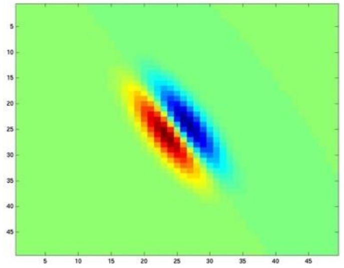
Filter bank: Example

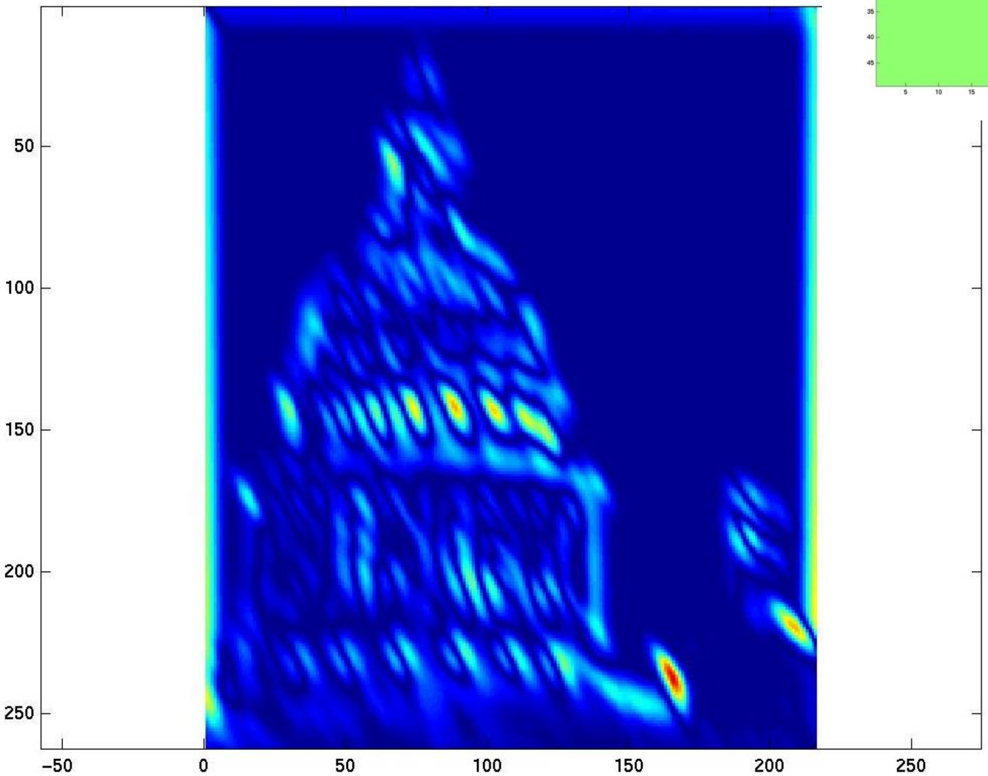




Kristen Grauman

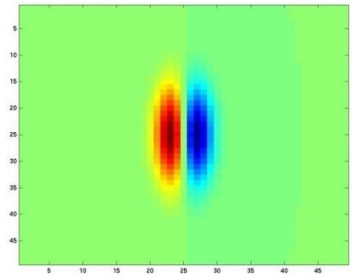
Filter bank: Example

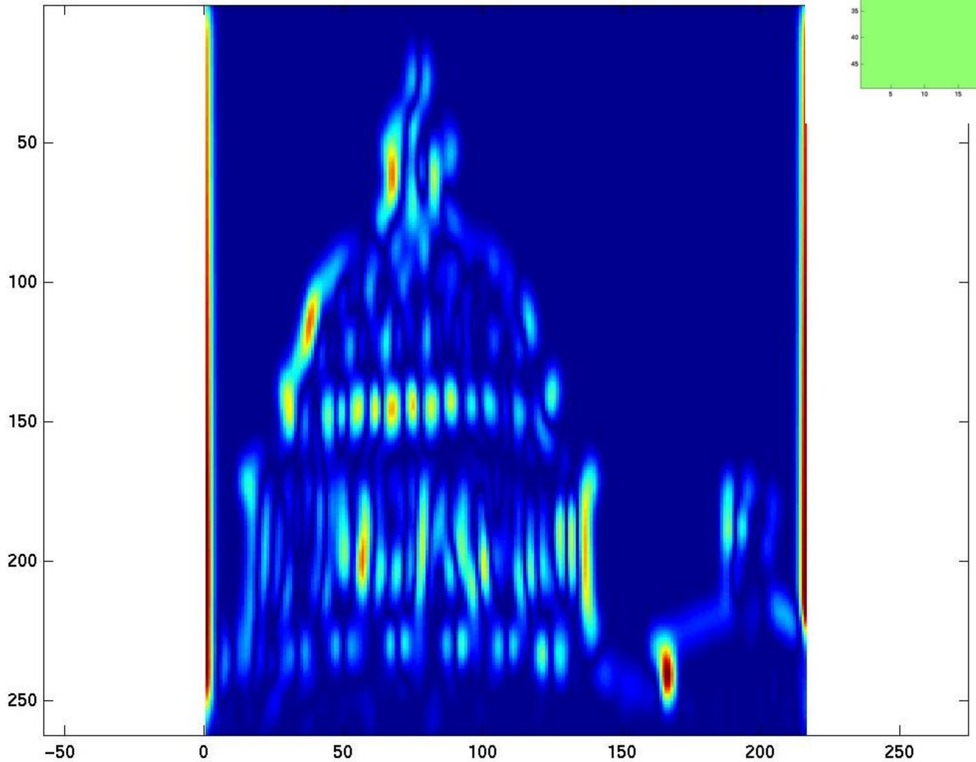




Kristen Grauman

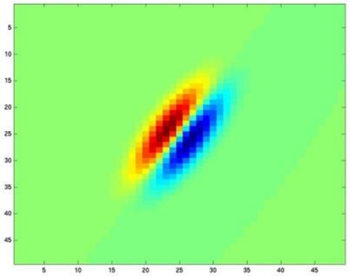
Filter bank: Example

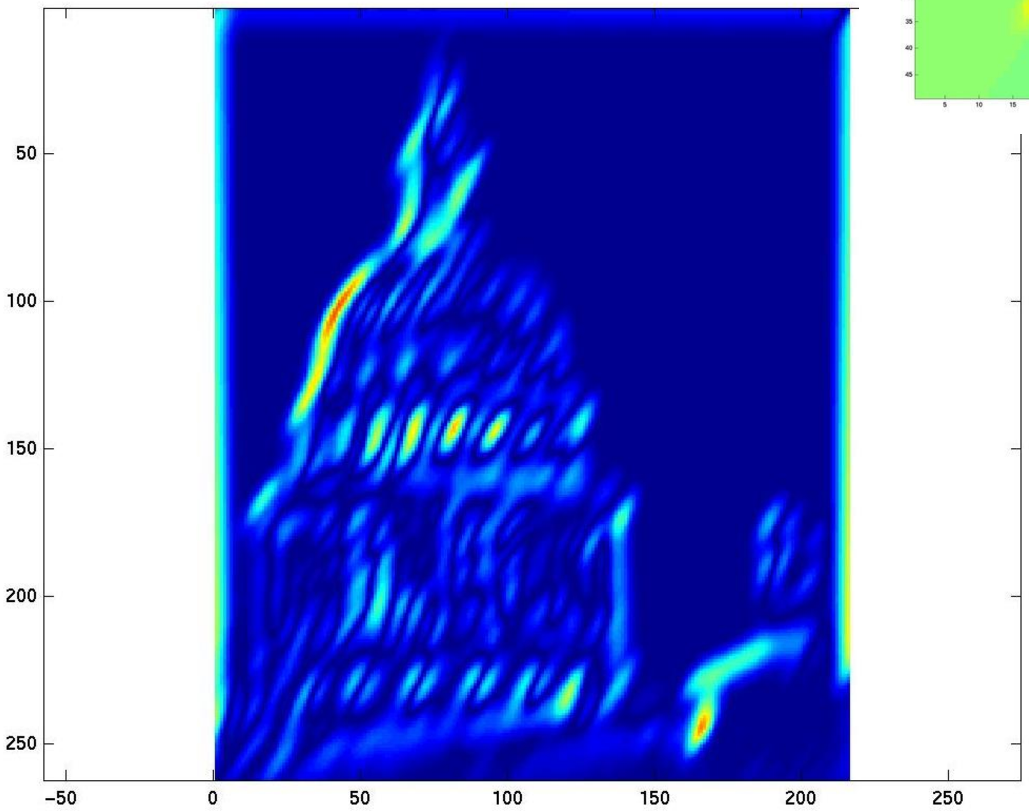




Kristen Grauman

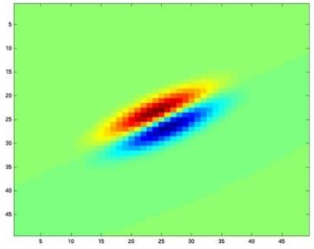
Filter bank: Example

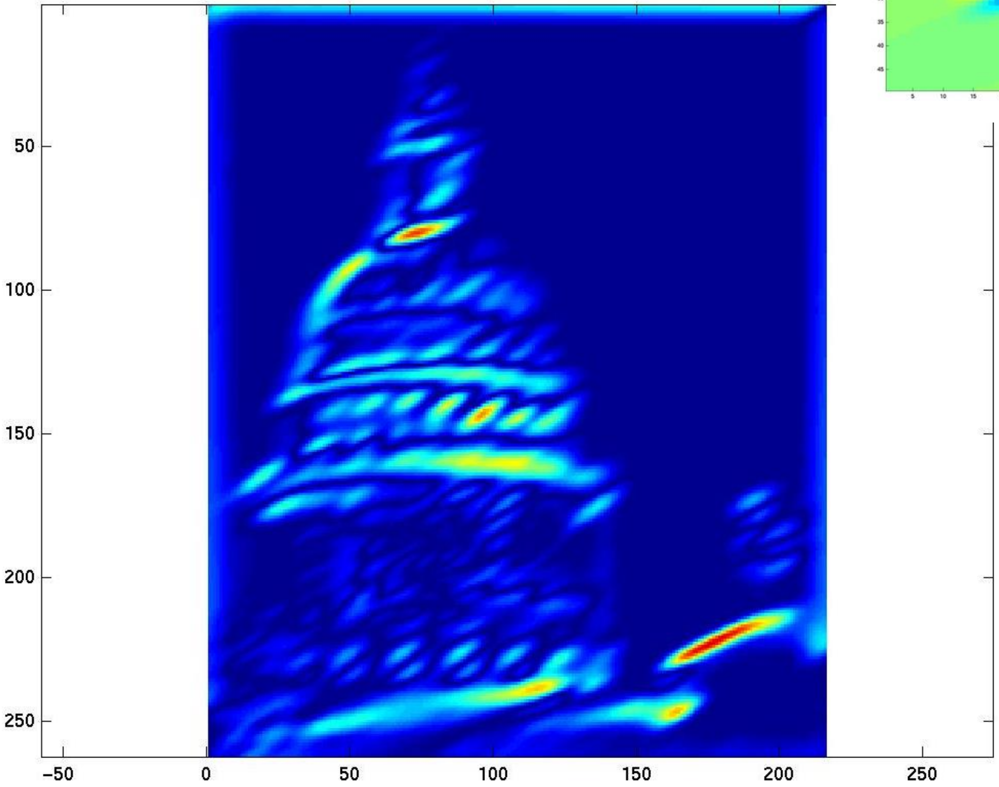




Kristen Grauman

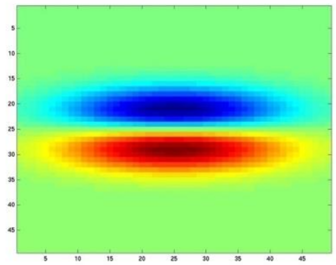
Filter bank: Example

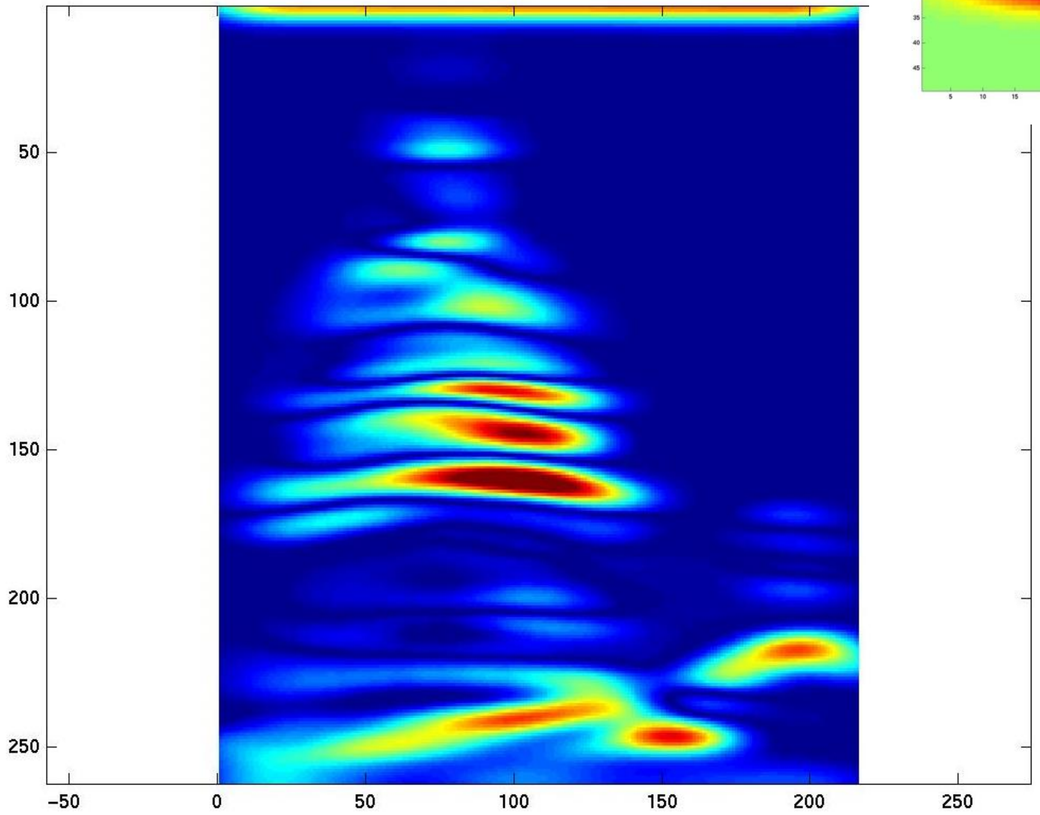




Kristen Grauman

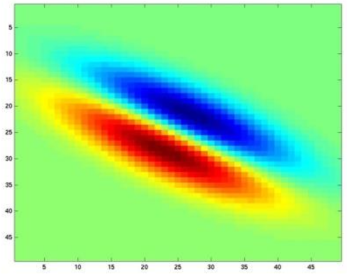
Filter bank: Example

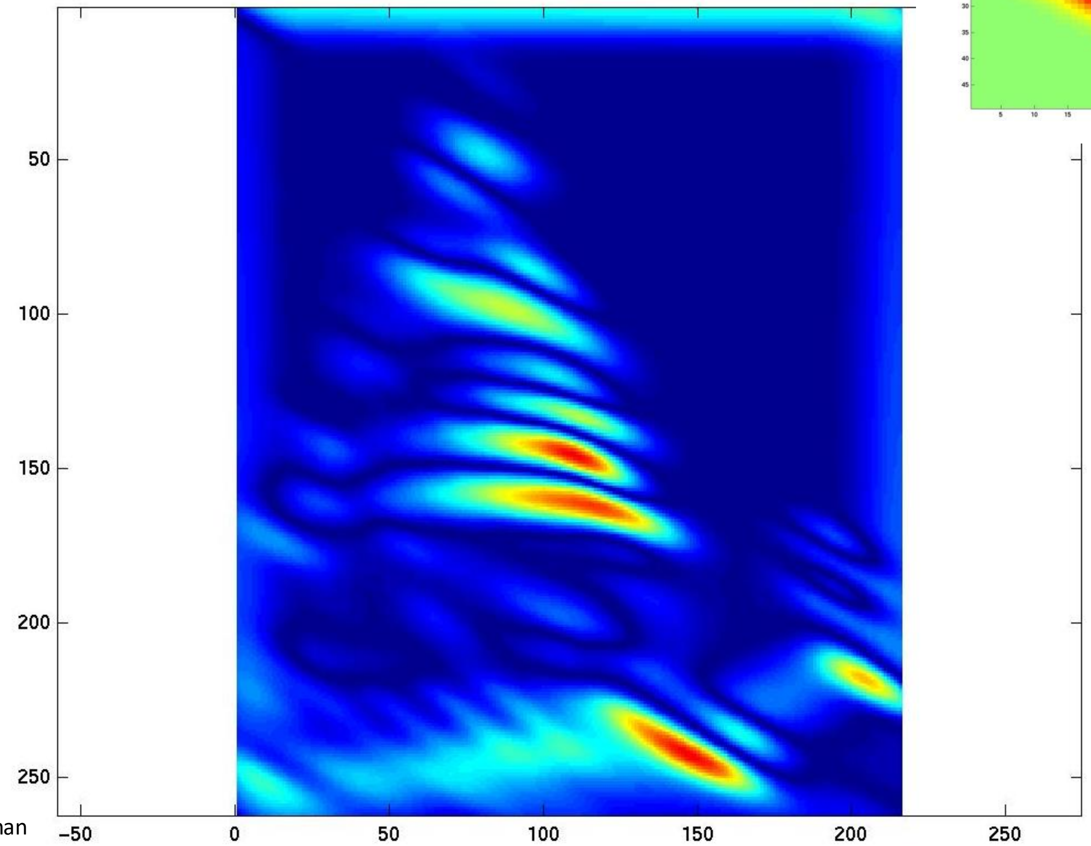




Kristen Grauman

Filter bank: Example





Kristen Grauman

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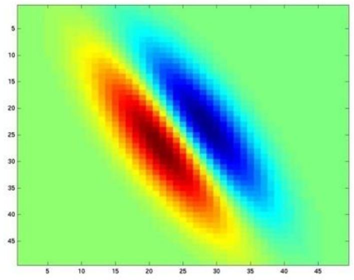

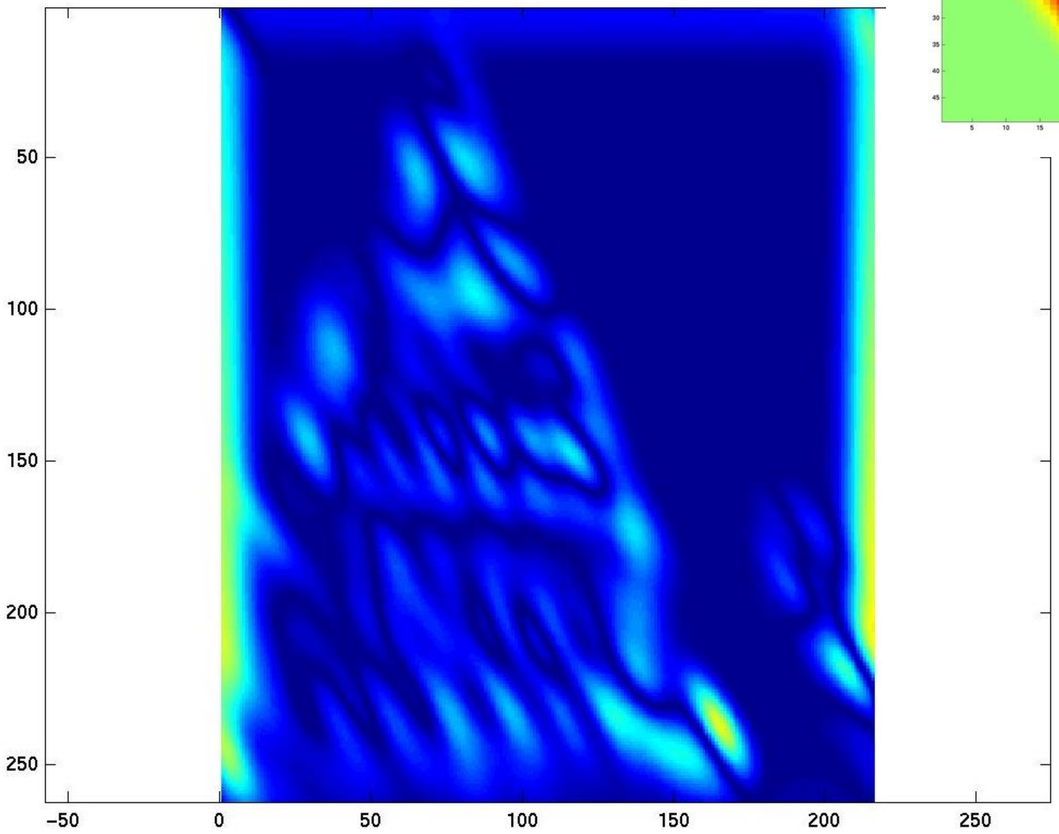

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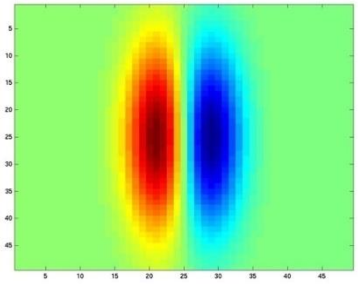
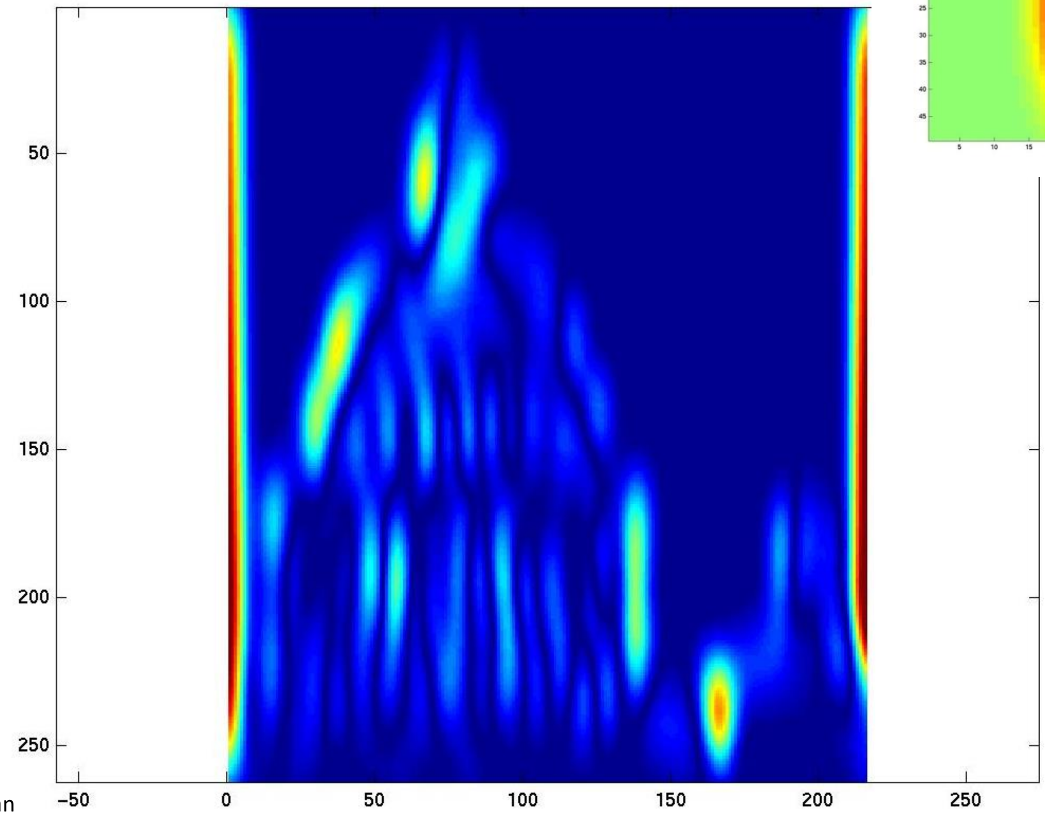
Filter bank: Example





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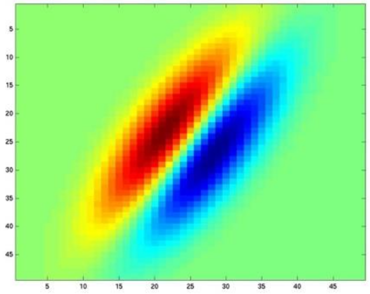
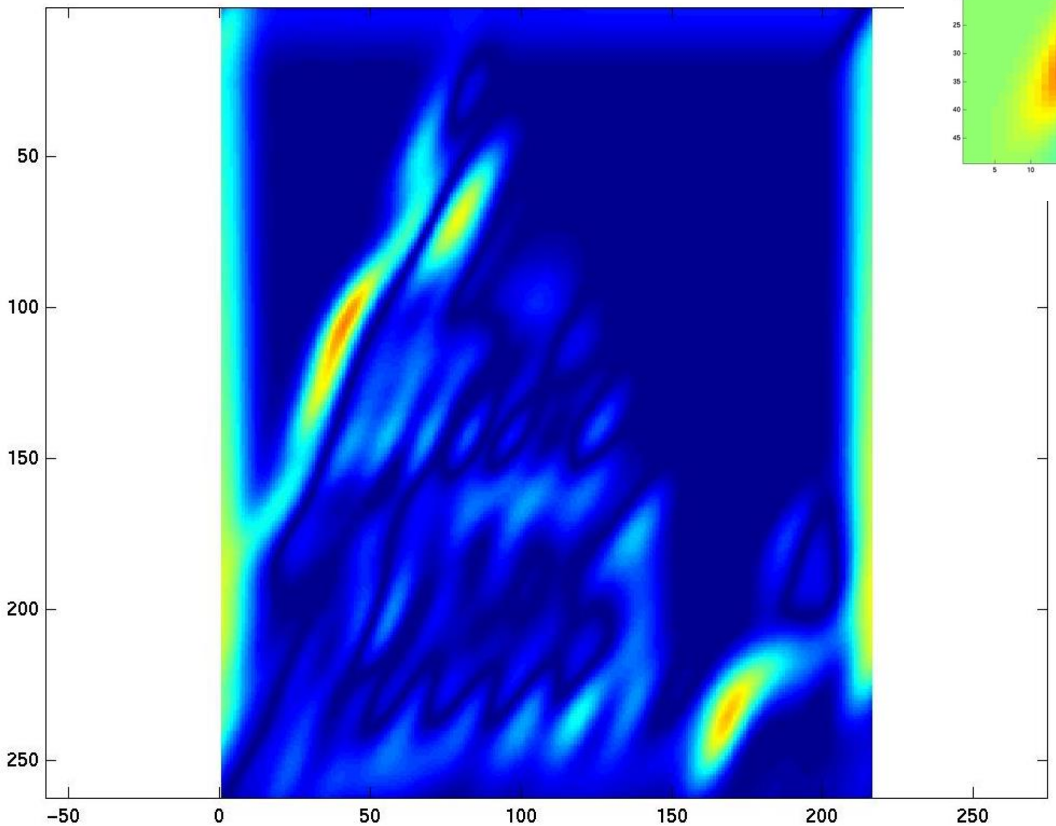
Filter bank: Example





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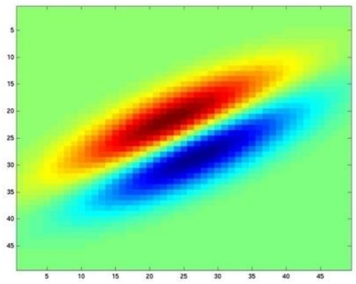
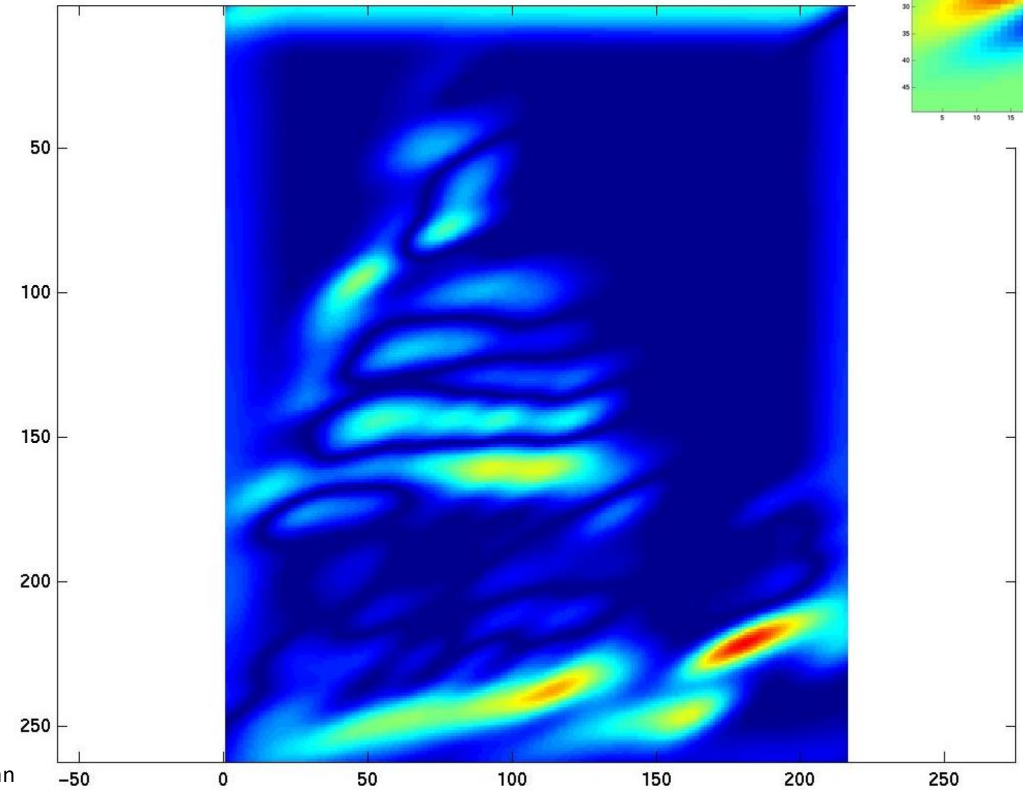
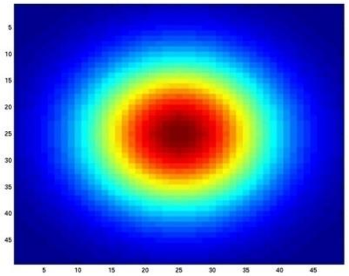
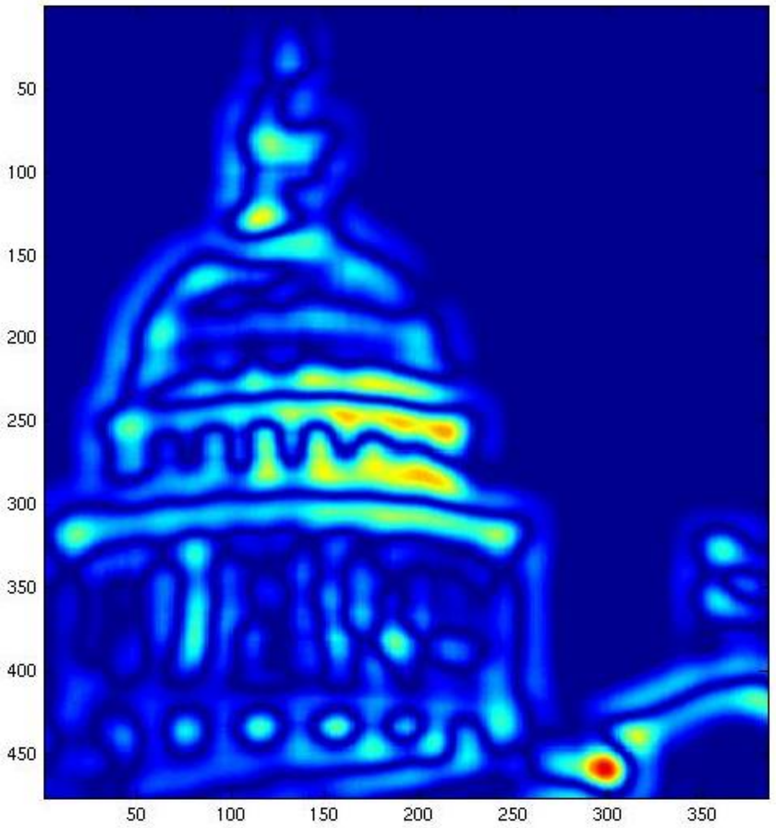




Kristen Grauman

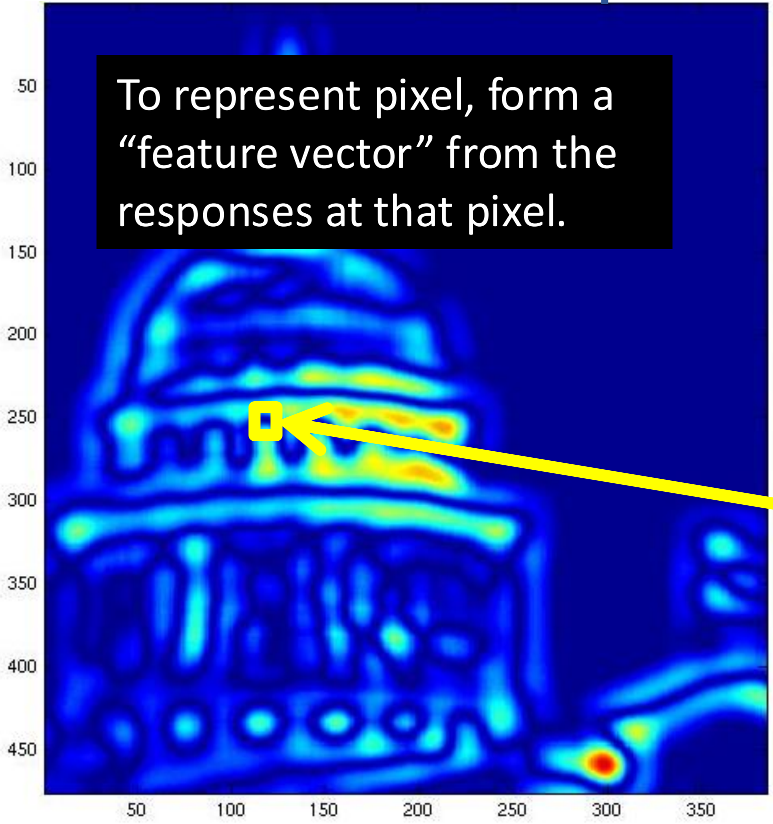
Filter bank: Example



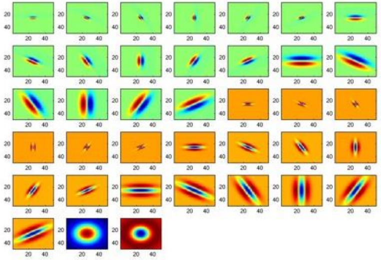


Kristen Grauman

Vectors of texture responses

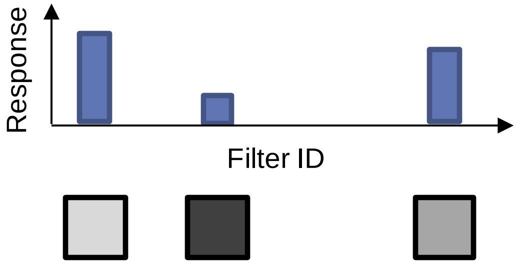


38 filters



1x38 vector representation feature

[r1, r2, …, r38]



Adapted from Kristen Grauman

Vectors of texture responses

To represent pixel, form a "feature vector" from the responses at that pixel.

To represent *image*, compute statistics over all pixel feature vectors, e.g. their mean.

$$[r_{(1,1)}^{1},\ r_{(1,1)}^{2},\ \ldots,\ r_{(1,1)}^{38}]$$

$$[r_{(1,2)}^{1},\ r_{(1,2)}^{2},\ \ldots,\ r_{(1,2)}^{38}]$$

Pixel location (row, column)

Filter ID

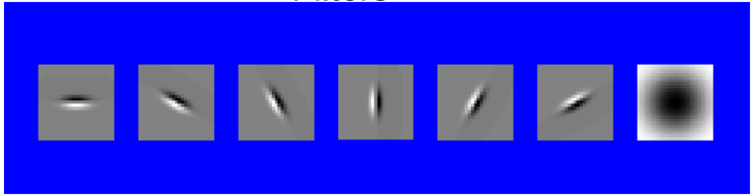
...

$$[r_{(W,H)}^{1},\ r_{(W,H)}^{2},\ \ldots,\ r_{(W,H)}^{38}]$$

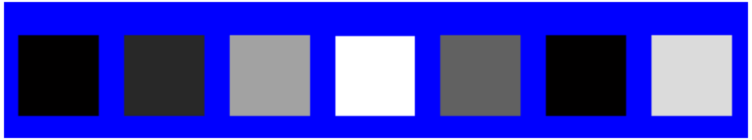
$$[\text{mean}(r_{(:)}^{1}),\ \text{mean}(r_{(:)}^{2}),\ \ldots,\ \text{mean}(r_{(:)}^{38})]$$

You try: Can you match the texture to the response?

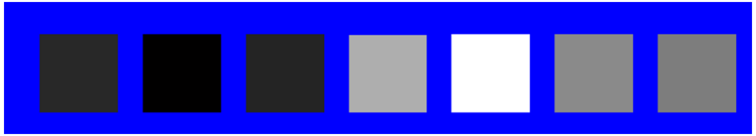
Filters



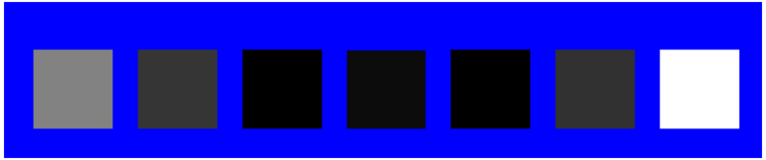
1



2

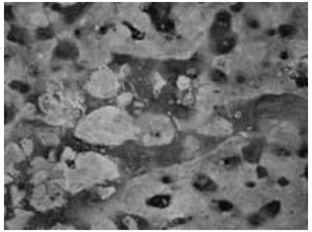


3



Mean abs responses

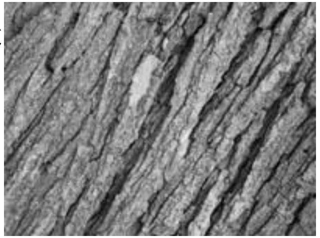
A



B

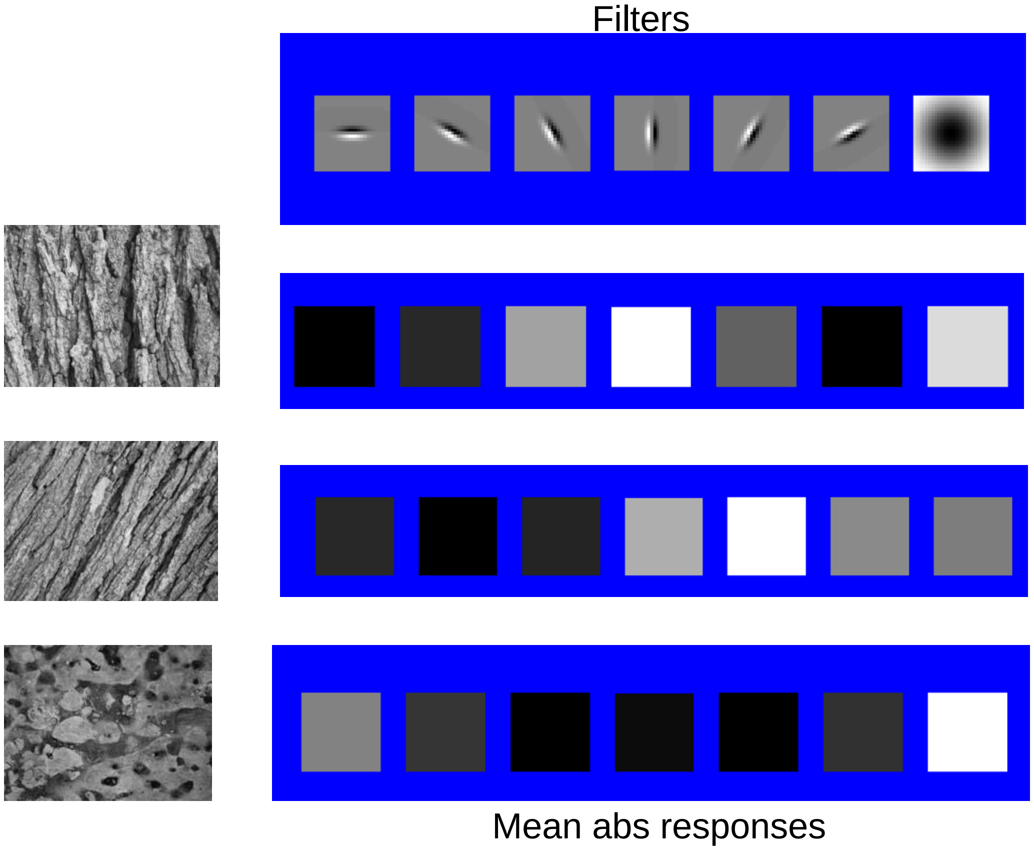
White color means higher response

C



Derek Hoiem

Representing textures by mean absolute response



Derek Hoiem

Classifying materials, “stuff”

Figure by Varma & Zisserman

Summary

- Filters useful for
  - Enhancing images (smoothing, removing noise), e.g.
    - Box filter (linear)
    - Gaussian filter (linear)
    - Median filter
  - Detecting patterns (e.g. gradients)

- Texture is a useful property that is often indicative of materials, appearance cues
  - Texture representations summarize repeating patterns of local structure