CS 441: Rules of Inference

PhD. Nils Murrugarra-Llerena
nem177@pitt.edu

University of Pittsburgh

Today's topics

- Rules of inference
  - Logical equivalences allowed us to rewrite and simplify single logical statements.
  - How do we deduce new information by combining information from (perhaps multiple) known truths?

What have we learned? Where are we going?

Predicate logic
(refined representation)

Propositional logic
(representation)

Quantifiers
(generalization)

Inference and proof (deriving new knowledge!)

Writing valid proofs is a subtle art

This is called "research"

Step 1: Discover and formalize the property that you wish to prove

Step 2: Formalize the ground truths (axioms) that you will use to prove this property

Subtle, but not terribly difficult

Step 3: Show that the property in question follows from the truth of your axioms

This is the hard part…

What is science without jargon?

A conjecture is a statement that is thought to be true.

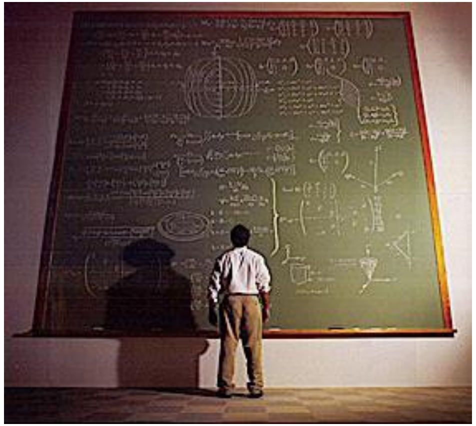
A proof is a valid argument that establishes the truth of a given statement (i.e., a conjecture)

After a proof has been found for a given conjecture, it becomes a theorem

A tale of two proof techniques

In a formal proof, each step of the proof clearly follows from the postulates and axioms assumed in the conjecture.

Statements that are assumed to be true



In an informal proof, one step in the proof may consist of multiple derivations, portions of the proof may be skipped or assumed correct, and axioms may not be explicitly stated.

How can we formalize an argument?

Consider the following argument:

"If you have an account, you can access the network"
"You have an account"
Therefore,
"You can access the network"

This argument *seems* valid, but how can we demonstrate this formally??

Let's analyze the *form* of our argument

p *q*

"If you have an account, then you can access the network"

"You have an account"

Therefore,

"You can access the network"

This is called a "rule of inference"

$p \rightarrow q$

p

$\therefore q$

Rules of inference are logically valid ways to draw conclusions when constructing a formal proof

The previous rule is called modus ponens

- Rule of inference: $p \rightarrow q$
  p
  ―――――
  $\therefore q$

- Informally: Given an implication $p \rightarrow q$, if we know that p is true, then q is also true

But why can we trust modus ponens?

- Tautology: $((p \rightarrow q) \wedge p) \rightarrow q$
- Truth table:

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Any time that p→q and p are both true, q is also true!

There are lots of other rules of inference that we can use!

Addition

- *Tautology*: $p \rightarrow (p \vee q)$
- *Rule of inference*:

- *Example*: "It is raining now, therefore it is raining now or it is snowing now."

Simplification

- *Tautology*: $p \wedge q \rightarrow p$
- *Rule of inference*:

- *Example*: "It is cold outside and it is snowing. Therefore, it is cold outside."

There are lots of other rules of inference that we can use!

Modus tollens

- *Tautology*: $[\neg q \land (p \rightarrow q)] \rightarrow \neg p$
- *Rule of inference*:

.

- *Example*: "If I am hungry, then I will eat. I am not eating. Therefore, I am not hungry."

Hypothetical syllogism

- *Tautology*: $[(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r)$
- *Rule of inference*:

.

- *Example*: "If I eat a big meal, then I feel full. If I feel full, then I am happy. Therefore, if I eat a big meal, then I am happy."

There are lots of other rules of inference that we can use!

Disjunctive syllogism

- *Tautology*: $[\neg p \wedge (p \vee q)] \rightarrow q$
- *Rule of inference*:

- *Example*: "Either the heat is broken, or I have a fever. The heat is not broken, therefore I have a fever."

Conjunction

- *Tautology*: $[(p) \wedge (q)] \rightarrow (p \wedge q)$
- *Rule of inference*:

- *Example*: "Jack is tall. Jack is skinny. Therefore, Jack is tall and skinny."

There are lots of other rules of inference that we can use!

Resolution

- *Tautology*: $[(p \lor q) \land (\neg p \lor r)] \rightarrow (q \lor r)$
- *Rule of inference*:

- *Example*: “If it is not raining, I will ride my bike. If it is raining, I will lift weights. Therefore, I will ride my bike or lift weights”

Special cases:

1. If r = q, we get

.

2. If r = F, we get

In-class exercises

See Top Hat

We can use rules of inference to build valid arguments

If it is raining, I will stay inside. If I am inside, Lisa will stay home. If Lisa stays home and it is a Saturday, then we will play video games. Today is Saturday. It is raining.

Let:

- r ≡ It is raining
- i ≡ I am inside
- l ≡ Lisa will stay home
- p ≡ we will play video games
- s ≡ it is Saturday

We can use rules of inference to build valid arguments

Let:

- r ≡ It is raining
- i ≡ I am inside
- l ≡ Lisa will stay home
- p ≡ we will play video games
- s ≡ it is Saturday

Hypotheses:

Step:

1.
2.
3.
4.
5.
6.
7.
8.
9.

We also have rules of inference for statements with quantifiers

Universal Instantiation

- Intuition: If we know that P(x) is true for all x, then P(c) is true for a particular c
- Rule of inference:

Universal Generalization

- Intuition: If we can show that P(c) is true for an arbitrary c, then we can conclude that P(x) is true for any specific x
- Rule of inference

Note that "arbitrary" does not mean "randomly chosen." It means that we cannot make <u>any</u> assumptions about c other than the fact that it comes from the appropriate domain.

We also have rules of inference for statements with quantifiers

Existential Instantiation

- Intuition: If we know that ∃x P(x) is true, then we know that P(c) is true for some c
- Rule of inference:

Again, we cannot make assumptions about c other than the fact that it exists and is from the appropriate domain.

Existential Generalization

- Intuition: If we can show that P(c) is true for a particular c, then we can conclude that ∃x P(x) is true
- Rule of inference:

Hungry dogs redux

Given: All of my dogs like peanut butter

M(x) *P(x)*

Given: Kody is one of my dogs

M(Kody)

1.
2.
3.
4.

Reasoning about our class

Show that the premises “A student in this class has not read the book” and “everyone in this class turned in HW1” imply the conclusion “Someone who turned in HW1 has not read the book.”

Let:

-
-
-

Premises:

-
-

Reasoning about our class

Let:

- $C(x) \equiv$ x is in this class
- $B(x) \equiv$ x has read the book
- $T(x) \equiv$ x turned in HW1

Premises:

- $\exists x\ [C(x) \land \neg B(x)]$
- $\forall x\ [C(x) \rightarrow T(x)]$

Steps:

1.
2.
3.
4.
5.
6.
7.
8.
9.

In-class exercises

Problem 2: Show that the premises "Everyone in this discrete math class has taken a course in computer science" and "Chike is a student in this discrete math class" lead to the conclusion "Chike has taken a course in computer science."

Others on Top Hat

Final Thoughts

- Until today, we had look at representing different types of logical statements
- Rules of inference allow us to derive new results by reasoning about known truths
- Next time:
  - Proof techniques
  - Please read section 1.8