Available online at www.sciencedirect.com

SCIENCE DIRECT®

Stud. Hist. Phil. Sci. 36 (2005) 223–240

Studies in History and Philosophy of Science

www.elsevier.com/locate/shpsa

'Spuntar lo scoglio più duro': did Galileo ever think the most beautiful thought experiment in the history of science?

Paolo Palmieri

Department of History and Philosophy of Science, University of Pittsburgh, Pittsburgh, PA 15260, USA

Received 9 August 2004; received in revised form 15 November 2004

Abstract

Still today it remains unclear whether Galileo ever climbed the leaning tower of Pisa in order to drop bodies from its top. Some believe that he established the principle of equal speeds for falling bodies by means of an ingenious thought experiment. However, the reconstruction of that thought experiment circulating in the philosophical literature is no more than a cartoon. In this paper I will tell the story of the thought processes behind the cartoon.

Keywords: Galileo Galilei; Thought experiment; Falling bodies.

1. Introduction

Still today it remains unclear whether Galileo ever climbed the leaning tower of Pisa in order to drop bodies from its top. Thus, James Robert Brown may be right in claiming, in the wake of the great historian Alexandre Koyré, that it was by means of thought experiments that Galileo established the principle that all bodies fall at the same speed in the void. Moreover, I would fully agree with Brown that Galileo's

E-mail address: pap7@pitt.edu (P. Palmieri).

0039-3681/$ - see front matter
doi:10.1016/j.shpsa.2005.03.001

celebrated account in *Two new sciences* (1638) may encapsulate 'the most beautiful thought experiment ever devised'.[1]

Climbing feats aside, however, it is even less clear whether Galileo himself ever 'thought' his most beautiful thought experiment. An important document surviving among Galileo's manuscripts, the so-called *Postils to Rocco* [ca. 1634–1635],[2] which, as far as I know, until now has been ignored, suggests a more complex and, needless to say, more fascinating story. Worse, Galileo's sanitized account of the falling bodies thought experiment in *Two new sciences* may have led historians and philosophers of science astray.[3] Not long ago, Tamar Szabó Gendler recognized that the debate on Galileo's thought experiment had followed the 'somewhat unfortunate practice of considering this thought experiment outside of both its historical and textual contexts'.[4] In this paper I hope to provide some remedy to this distortion.

In what follows, I will discuss the significance of Galileo's autobiographical analysis in the *Postils to Rocco*. Firstly, I will show that the falling bodies thought experiment was originally part of a family of argumentative strategies excogitated by Galileo in the 1590s, in which context it needs to be placed. He always considered it to be conclusive only under the very restrictive condition of bodies with the *same specific gravity*, a condition still specified by Galileo in *Two new sciences*, but which is often forgotten in contemporary discussions.[5] Secondly, and more surprisingly, I will show that in the 1590s the falling bodies thought experiment was believed by Galileo to be perfectly compatible with the theory that bodies of *different specific gravity* fall in the void with speeds proportional to their specific gravities. Thirdly, I will show that almost five decades later, in the *Postils to Rocco*, Galileo, now in possession of the law of accelerated fall, and of a complex theory of fluid resistance, recast his original thought experiment into the punchy presentation that

[1] Brown (2004), p. 24. See Galileo's famous passage of *Two new sciences*, in Galilei (1974), pp. 66 ff. The most recent discussion of Galileo's falling bodies thought experiment is Atkinson & Peijnenburg (2004). As for thought experiment, both in general and in reference to Galileo, I have relied on the following literature: Norton (2004, 1996, 1991), Palmieri (2003), Budden (1998), Gendler (1998, 2000), McAllister (1996), Nersessian (1993), Miščević (1992), Sorensen (1992), Brown (2004, 1991), Prudovsky (1989), Geymonat & Carugo (1981), and Kuhn (1977). Among these contributions, those which touch on Galileo's falling bodies thought experiment systematically ignore its rich historical development. Sometimes one has the impression of being presented a cartoon reconstruction devoid of any historical meaning.

[2] The *Postils to Rocco* is a set of comments and notes that Galileo wrote in response to a book published in 1633 by an Aristotelian philosopher, Antonio Rocco, which attacked the *Dialogue concerning the two chief world systems* (published by Galileo the year before). The *Postils to Rocco* has been published in Galilei (1890–1909), Vol. VII, pp. 569–750, together with Rocco's book.

[3] To be sure, in his famous essay 'Le *De motu gravium* de Galilée: De l'expérience imaginaire et de son abus', Koyré noticed that in *Two new sciences* there are many strands of argument somehow intriguingly connected with the celebrated thought experiment. See Koyré (1973), pp. 224–271. What Koyré failed to notice is that those strands had originally been part of an autobiographical analysis, which Galileo had recounted in the *Postils to Rocco*, but then scattered and rearranged in *Two new sciences*, with no explicit reference to his past intellectual development.

[4] Gendler (1998), p. 402 n.

[5] An exception is Gendler (1998), p. 403, and (2000), p. 39.

was eventually published in *Two new sciences*. Finally, I will show that in *Two new sciences* he omitted to report a most revealing comment on his methodology, which he had made in the *Postils to Rocco*, namely, 'spuntar lo scoglio più duro' ('overcoming the hardest obstacle'). It was with these words that Galileo had qualified the tortuous path towards establishing the equality of speeds in a vacuum for *all* bodies, regardless of their specific gravity.

2. The scenario in the 1590's *De motu*

First of all, we need to understand the context of Galileo's discussion of motion in the 1590s. The document in which Galileo first tackled these issues is the so-called *De motu*, a set of drafts and notes on motion and mechanics which he never published.[6] His starting point is a theory of free fall which he attributes to Aristotle. All bodies of the same kind fall with speeds proportional to their bulk. Thus, a piece of lead twice as large as another piece of lead will fall twice as fast.[7] In addition, Galileo always refers to motions occurring within a fluid medium, such as, for instance, water or air (two of the traditional four elements).

A heavy body *b* moves along line *ce* (Figure 1). Let the line be divided at point *d*. If mobile *b* is divided in the same ratio as that according to which the line is divided by point *d*, then in the same time in which body *b* moves along the whole line, *ce*, its part will move along line *cd*.[8] According to this theory then, all bodies of the same kind, that is, of the same matter, such as wood, or lead, will fall with speed proportional to their magnitudes.

In contrast to Aristotle, Galileo now claims that mobiles of the same kind, although different in volume, will move with the same speed.

> Let us thus say that mobiles of the same kind (things of the same kind are said to be those mobiles which are made of the same matter, such as lead, wood, etc.), though different in volume, will move with the same speed, so that a greater stone will not fall faster than a smaller one.[9]

[6] It is normally dated by scholars around the 1590s. For referencing purposes I have arbitrarily attributed the date of 1590 to the manuscript of *De motu*. See Fredette (1969), Camerota (1992), and Giusti (1998) for the problems concerning the date of *De motu*.

[7] 'De illis mobilibus quae sunt eiusdem speciei dixit Aristoteles, illud velocius moveri quod maius est' (Galilei, 1890–1909, Vol. I, pp. 262–263).

[8] Ibid., p. 263. Cf. Aristotle's *De caelo*, at 301b. The passage is rather difficult. See, for example, a Renaissance edition in Latin, commented by Simplicius (1563), pp. 206–207, and the modern English translations by W. K. C. Guthrie, in Aristotle (1986), p. 277 (with the Greek original text on the facing page), and by J. Barnes, in Aristotle (1984), I, p. 494. Cf. also Palmieri (2003), pp. 258 ff., for a discussion of an interesting interpretation of this passage by an Aristotelian professor of Greek at Pisa University, who was a contemporary of Galileo.

[9] Galilei (1890–1909), Vol. I, p. 263.

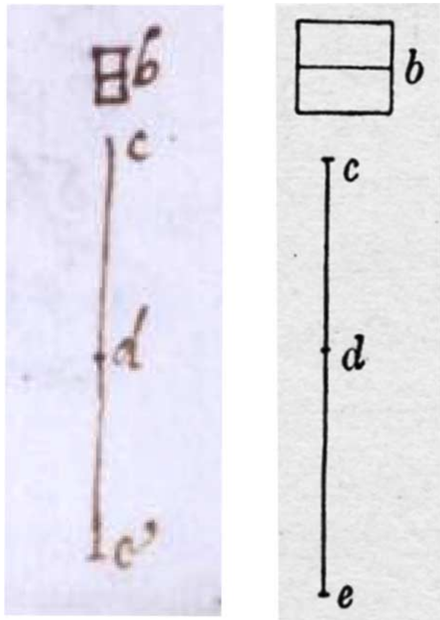

Fig. 1. Aristotle's view, according to Galileo: bodies of the same matter fall with speed proportional to their magnitudes (Galilei, 1590, p. 139 (left); Galilei, 1890–1909, Vol. I, p. 263 (right)).

In my view, the realization that lay at the root of Galileo's line of reasoning in the 1590s is nested in the following startling analogy. The reason why mobiles of the same kind, although different in volume, fall with the same speed is the same as why both a chip of wood and a large wooden beam float.[10]

After the passage where Galileo makes this astonishing claim, we find the beginning of the train of thought leading up to the first appearance of the famous thought experiment in Galileo's writings. It is a sequence in which four distinct stages can be discerned.

In the first two, Galileo begins by saying ' … *si mente concipieremus*', that is, if we conceive *with mind*, or *in mind*. Then, in the third, he proposes an analogy (but does not use this term). Finally, the fourth he calls an 'argument'. I shall discuss them in their original order.

First stage. Let us consider a wooden beam and a chip of the same wood floating on water. Then, imagine the specific weight of water decreasing so that at one point water will become specifically lighter than wood. Who could claim, Galileo asks, that the beam will begin to descend before or faster than the chip? The reason why the beam's behaviour will be the same as the chip's is that while

[10] It is worth reporting the original passage of this startling analogy. 'Qua conclusione [i.e., that all mobiles of the same kind fall at the same speed] qui mirantur, mirabuntur etiam, tam maximam trabem quam parvum lignum aquae supernatare posse: eadem enim est ratio' (ibid., pp. 263–264).

descending both will have to raise an amount of water equal to their volumes. Thus the volumes of water raised will have the same ratio as that of the chip's and the beam's volumes. In consequence, the ratio of the weight of the beam to the weight of its displaced volume of water is the same as that of the weight of the chip to the weight of its displaced volume of water. With the same easiness both the chip and the beam will overcome the resistance of the water which has to be displaced.[11]

Second stage. Consider a volume of wax floating on water (wax is specifically lighter than water). Now imagine mixing with the wax a modicum of material specifically heavier than water, such as sand, so that the mixed body will start descending in water, but very, very slowly. There is no reason, Galileo claims, why a chip of the mixed body will descend slower than the whole mixed body itself, or even not descend at all.[12] Here Galileo has, so to say, reversed the strategy of the first stage. Instead of imagining the specific weight of the medium varying he imagines the specific weight of the body varying.

Third stage. 'And the same one can experience in the balance', exclaims Galileo. In his words:

> ... for if very large, equal weights are placed on each side, and then to one of them something heavy, but only modestly so, is added, the heavier will then go down, but not any more swiftly than if the weights had been small. And the same reasoning holds in water: for the beam corresponds to one of the weights of the balance, while the other weight is represented by an amount of water as great in size as the size of the beam: if this amount of water weighs the same as the beam, then the beam will not go down; if the beam is made slightly heavier in such a way that it goes down, it will not go down more swiftly than a small piece of the same wood, which weighed the same as an [equally] small part of the water, and then was made slightly heavier.[13]

Fourth stage. At this point Galileo claims that the entire reasoning can be confirmed by the following 'argument'. He assumes that if one of two mobiles moves faster than the other the composite of both will move faster than the slower body yet slower than the faster one. Then he goes on to state that mobiles of the same kind and different volumes will move with the same speed (Figure 2). The strategy is as follows. Let *a*, *b* be the two bodies, and *a* be larger than *b*. If possible, let *a* move faster than *b*. The composite will move faster than *b* yet slower than *a*. But the composite is larger than *a*, therefore a larger body will move slower than a smaller body, which is awkward [*quod quidem est inconveniens*].[14]

[11] Ibid., p. 264. Galileo also specifies that it must be proven that volumes of the same matter have the ratio of their weights. He proves this claim further on in *De motu*, cf. ibid., pp. 348-350.

[12] Ibid., p. 264.

[13] Galilei (2000), p. 17. Cf. the original, in Galilei (1890–1909), Vol. I, pp. 264.

[14] Galilei (1890–1909), Vol. I, p. 264.

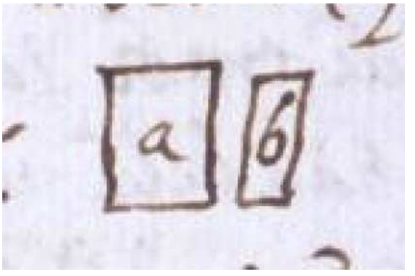

Fig. 2. Fourth stage: the *De motu* version of the famous thought experiment (Galilei, 1590, p. 143).

It is a reductio argument, as the language employed by Galileo clearly suggests (*inconveniens* is almost another way of saying 'absurd').[15]

In sum, the first version of the famous thought experiment is no more than a reductio argument, crafted by Galileo at the end of a complex argumentative strategy. The first three stages seem to have been more of an exploration of the astonishing analogy, according to which the reason why mobiles of the same kind yet different in volume fall with the same speed is the same as why a chip of wood and a large beam float.

3. Different speeds in the void

So far Galileo has restricted his reasoning to bodies of the same kind, that is, of the same specific weight. He now wishes to extend his theory to bodies of different specific weight. He considers equal volumes of different matter moving in a fluid medium. His polemical target is once again Aristotle, against whose opinion he wants to prove that the motions of bodies of different kinds do not have the same ratio as that of their weights. Galileo's new strategy is inspired by Archimedes's theory of buoyancy, expounded in *On floating bodies*.[16]

Let us consider Figure 3, on the left. As far as motion upwards is concerned, Galileo proves that solid magnitudes specifically lighter than water, when forcibly submerged are thrusted upward by a force equal to the difference of the weight of a volume of water equal to the submerged magnitude and the weight of the magnitude. Let us consider Figure 3, on the right. As far as motion downwards is concerned,

[15] This can be gathered by looking at the Latin versions of commentaries on Aristotle circulating in the Renaissance. Cf., for example, Simplicius's commentary on *De caelo*, 301b (Simplicius, 1563, p. 207), in which he asserts that Aristotle's demonstration (*ostensio*) is 'by reductio ad absurdum' ('per deductionem ad impossibilem'), and then concludes his paraphrase of Aristotle's text with the same formula used by Galileo, 'quod est incoveniens'.

[16] Only Latin versions of *On floating bodies* circulated in Western Europe until Heiberg's discovery of the Constantinople palimpsest, in the early twentieth century. Archimedes (1543, 1565) are two Latin editions that Galileo would have consulted.

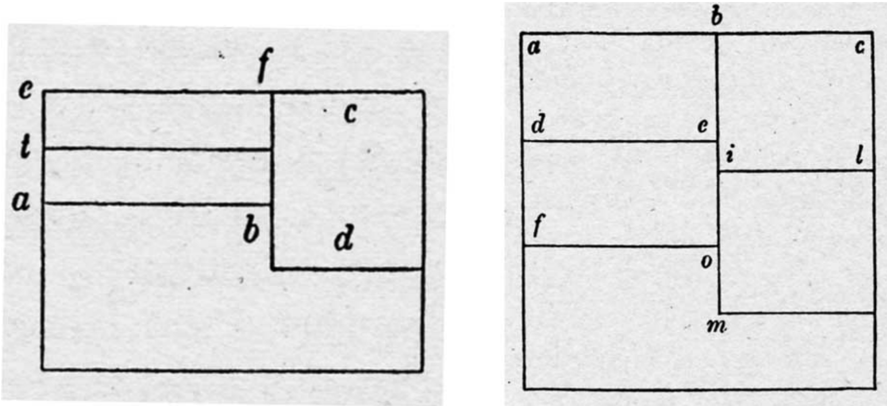

Fig. 3. Galileo's diagrams accompanying the theorems on the force acting on bodies submerging in water. On the left, the case of bodies specifically lighter than water, which float. On the right, the case of bodies specifically heavier than water, which sink. Cf. Galilei (1890–1909), Vol. I, pp. 270, 272.

Galileo proves that solid magnitudes specifically heavier than water sink with a force equal to the difference of the weight of the magnitude and the weight of a volume of water equal to the submerged magnitude. The proofs are not too different. Here is the first, in Galileo's own words (Figure 3, on the left):

> And thus let the first position of the water, before the magnitude is submerged in it, be along surface *ab*; and let the solid magnitude *cd* be forcibly submerged in it; and let the water be raised to the surface *ef*: and since water *eb*, which is raised, has a size equal to the size of the whole submerged magnitude, and the magnitude is assumed to be lighter than water, the heaviness of water *eb* will be greater than the heaviness of *cd*. Then let it be understood that *tb* is that part of the water, whose heaviness is equal to the heaviness of magnitude *cd*: accordingly it must be demonstrated that, magnitude *cd* is carried upward with a force as great as the heaviness of water *tf* (for it is by this heaviness that water *eb* is heavier than the heaviness of water *tb*, that is, than the heaviness of magnitude *cd*). Now since the heaviness of water *tb* is equal to the heaviness of *cd*, water *tb* will exert an upward pressure so as to raise the magnitude with as much force as that with which the magnitude will resist being raised. And thus the heaviness of the part of water exerting pressure, namely *tb*, is equal to the resistance of the solid magnitude: but the heaviness of all the water that exerts pressure, *eb*, surpasses the heaviness of water *tb* by the heaviness of water *tf*: hence the heaviness of all the water *eb* will surpass the resistance of solid *cd* by the heaviness of water *tf*. And so the heaviness of all the water exerting pressure will impel the solid magnitude upward with as much force as the heaviness of part *tf* of the water: which was to be demonstrated.[17]

[17] Galilei (2000), pp. 21–22.

Thus Galileo can conclude that

> the answer to the other question is evident: namely, what ratio the speeds of mobiles equal in size, but unequal in heaviness, observe with one another in the same medium. For the speeds of such mobiles will be to one another as the excesses by which the heavinesses of the mobiles exceed the heaviness of the medium: thus, for example, if two mobiles are equal in size, but unequal in heaviness, the heaviness of one being 8, and of the other 6, but the heaviness of an amount of the medium, equal in size to the size of one of the two mobiles, is 4, then the swiftness of the former mobile will be 4, and that of the latter will be 2. Hence these speeds will observe the ratio of 4 to 2; and not that which is between the heavinesses, namely 8 to 6.[18]

From what has been said, Galileo claims, it should be clear that we do not know the exact heaviness of bodies because we can only weigh them within media. The exact heaviness of bodies would only be given if we could weigh them in a vacuum.[19] Accordingly,

> in a void also a mobile will be moved in the same way as in a plenum. For in a plenum a mobile is moved swiftly according to the excess of its own heaviness over the heaviness of the medium through which it is moved; and thus in a void it will be moved according to the excess of its heaviness over the heaviness of the void: since this is null, the excess of the heaviness of the mobile over the heaviness of the void will be the total heaviness of this same mobile; thus it will be moved swiftly according to its own total heaviness.[20]

Galileo is very clear. In the void speeds are proportional to the gravities since no medium detracts from the weights of bodies. A final example further clarifies.

> Thus, to put it briefly, my whole intent is the following: if there is a heavy thing *a*, whose proper and natural heaviness is 1000, in any plenum whatever its heaviness will be less than a thousand, and, consequently, the swiftness of its motion in any plenum whatever will be less than a thousand. And if we take a medium, such that the heaviness of a size of it equal to the size of *a* is only 1, the heaviness of *a* in this medium will be 999; thus its swiftness

[18] Ibid., p. 24.

[19] 'And, from what has been said, it can be manifest to anyone, that we have of no thing its own proper heaviness: for if, for example, two weights are weighed in water, who will say that the heavinesses that we will see then are the true heavinesses of these weights, whose heavinesses, when the weights are weighed in air, will show themselves to be different from these, and will observe with one another another ratio? If again they could be weighed in another medium, for example, fire, the heavinesses would again be different, having to one another another ratio: and in this way they will always vary, according to the difference in the media. And if they could be weighed in the void, in this case surely, where no heaviness of the medium would diminish the heaviness of the weights, we would perceive their exact heavinesses' (ibid., p. 27).

[20] Ibid., p. 32.

> also will be 999: and the swiftness of this same *a* will only be a thousand in the medium where its heaviness is one thousand; and that will be nowhere but in the void.[21]

There is a final, almost bizarre twist to the story of Galileo's 1590's theory of motion. As is well known, Aristotle proposes various arguments against the possibility of the void. Among others that Galileo attributes to the Greek philosopher, the following one stands out. If in the void motion occurred in time, then both heavy and light bodies would move with the same speed, since the resistance of the void to their motions would be nil, which is awkward [*inconveniens*].[22] Aristotle's reason, according to Galileo, is that speedy bodies cleave the medium more strongly, but since the void does not exert resistance all motions must occur at the same speed. Now, Galileo claims that Aristotle sins in that he does not show why the conclusion that in the void heavy and light bodies would move with the same speed is absurd.[23] Perhaps doubt was beginning to creep in Galileo's mind, after all!

4. 'Spuntar lo scoglio più duro': a new turn

Almost five decades later, in Galileo's response to Rocco, we find a fascinating autobiographical analysis concerning the famous thought experiment. Not surprisingly, Galileo's *ex post facto* reconstruction tells a richer story than the one we recounted above on the basis of the 1590's *De motu*.

First of all, Galileo claims that it was reason, not experience, which initially persuaded him that all bodies of the same specific gravity fall at the same speed.

> ... I formed an axiom such that nobody could ever object to. I hypothesized that any heavy body whatever which descends has in its motion degrees of speed so limited by nature and fixed, that it would be impossible to alter them, by increasing or decreasing its speed, without using violence in order to speed up, or slow down, its natural course. I then figured in my mind two bodies equal in volume and weight, such as, for instance, two bricks, which depart from the same height at the same time. These will doubtless descend with the same speed, assigned to them by nature. If this speed has to be increased by another mobile, it is necessary that this mobile move more swiftly. Yet if one imagines the two bricks joining together while descending, which one would be that by adding impetus to the other will double the latter's speed, given that speed cannot be increased by an arriving mobile if it does not move more swiftly?[24]

[21] Ibid., p. 33.
[22] Galilei (1890–1909), Vol. I, p. 401.
[23] Galilei (2000), p. 34.
[24] Galilei (1890–1909), Vol. VII, p. 731.

Then Galileo asserts that 'from this first discourse I moved on to another, more convincing proof', which is more or less the same argument, already given in *De motu*, containing the thought experiment restricted to bodies of the same specific gravity. The only difference (highly significant, however) is that in this document Galileo keeps referring to 'degrees of speed', a clear indication that he now knows very well that speed of fall increases according to a certain progression while a body descends. In 1634 Galileo has of course long been in possession of the times-squared law.[25]

In addition, Galileo has by now reached the conclusion that gravity cannot be regarded as the cause of speed, and that the Aristotelian notion of the proportionality between speed and weight relies on a hidden (and fallacious) assumption, exemplified by the claim that the joining together of the descending bricks increases their weight. In Galileo's view, it is one thing to talk of weight in relation to the effects on a balance, so that a brick put on top of another equal brick will double the latter's weight, but it is quite another to generalize this model to free-falling bricks. Weight, Galileo claims, is but to feel burdened.[26] Put your hand below a 100-pound cannon ball hanging by a rope, you will not feel burdened. But if the rope is severed then you will feel burdened when trying to keep the ball from descending. However, when you make your hand swiftly recede with the same speed below the free-falling ball, you will hardly feel burdened, because no resistance is opposed to the ball. Thus when two equal free-falling bricks join together upon each other, since they fall with the same speed none of them will increase the weight of the other.[27]

Note that by 1634 Galileo has moved away from the *De motu* image of two free-falling bricks *adjacent* to each other and joining together (cf. Figure 2). His thought focuses now on the image of two free-falling bricks *upon* each other, and is clearly motivated by the analogy of the hand and the cannon ball. In other words, Galileo has realized that weight is by no means the cause of speed, and draws the conclusion that a body's speed of fall can only be increased by an arriving body already possessed of a greater speed.

What about bodies of different matters, such as wood, lead, cork, and metals? This became for Galileo the crux of the whole matter. 'Spuntar lo scoglio più duro' ('overcoming the hardest obstacle'), Galileo tells us. What, in his words, ultimately convinced him that all bodies in a vacuum fall with the same speed (always increasing as the square of time, however) are no more than 'conjectures', since, he candidly admits, experience might in this case turn out to be impossible.[28]

[25] Galluzzi (1979), Giusti (1990), and Drake (1995).

[26] In *De motu*, Galileo had already given such a 'psychological' definition of weight, but had not recognized its relevance for the case of free falling bodies. 'We say that we feel burdened [*gravari*] when some weight is placed upon us which tends downwards because of its gravity, in which case we have to oppose a force so that the weight no longer descends; that opposing is what we call to feel burdened' (Galilei, 1890–1909, Vol. I, pp. 288, 388).

[27] Ibid., Vol. VII, p. 733.

[28] Clearly Galileo believed that to make experiments in the void might turn out to be impossible (ibid., pp. 743 ff.).

> My conjecture was founded on a certain effect that can be observed concerning speeds of mobiles of different gravity in fluid media. Speeds become more and more different according as the media become heavier and heavier. Only gold, the heaviest matter known to us, descends within quicksilver, in which all other metals float. It is clear that a mixed body of gold and silver can be made so that it will descend within mercury very slowly. Thus gold would sink one braccio within mercury in, say, one pulse beat, whereas the mixed body would take no less than 50 or 100 beats. If we let the bodies fall to the bottom of four braccia of water, pure gold will not precede the mixed body one tenth of the time required of the latter. But in air, from a height of 100 braccia, no difference in time of fall would be discernible.[29]

Other examples by Galileo include a sphere made of beeswax mixed with lead, a marble sphere, a cork sphere, and a gold sphere. The line of reasoning is always the same in all cases. The medium is merely responsible for the divergence in the speeds of fall. There was, however, a serious difficulty in this argument. Galileo had known since the 1590s that the divergence of speeds could also be predicted by his earlier theory of speed proportional to the difference between a body's gravity and the gravity of an equal volume of medium, on the assumption that *ratios* of speeds are taken 'geometrically', that is, according as *ratio* is treated in Euclid's theory of proportions (cf. Table 1, for an example).[30]

In the early *De motu*, in order not to face the complications of forming a non-Archimedean ratio between the specific weight of a medium and that of the vacuum (equal to zero), Galileo had recourse to the theory of 'arithmetical ratio'.[31] In this case, as Galileo explains, ratios are not defined directly, but a criterion of 'sameness' for arithmetical ratios is given, according to which two ratios are the same ratio when the differences between the quantities forming the ratios are the same.[32] On the assumption that ratios are taken arithmetically, divergence of speed does not follow in the early theory (cf. Table 1, where the differences are in fact always equal to 20). The trick worked in *De motu*, but we know from early preparatory material on mechanics and motion, as well as the later *Two new sciences*, that Galileo must soon have realized that toying with arithmetical ratios was a blind alley. Indeed he abandoned the notion of arithmetical ratio for good, as its complete disappearance from his subsequent writings makes clear.

[29] Ibid., p. 743.

[30] Euclid (1956), II, pp. 112–186. For Galileo's mathematics, cf. Armjio (2001), Drake (1973, 1974, 1987), Frajese (1964), Giusti (1981, 1986, 1990, 1992, 1993, 1994, 1995), Maracchia (2001), Palladino (1991), Palmieri (2001, 2002). For a general treatment of the various aspects of the Euclidean theory of proportions I have relied on: Grattan-Guinness (1996), Sasaki (1985), Saito (1986, 1993). Rose (1975) is an extensive, immensely erudite survey of Renaissance mathematics in Italy from a non-technical point of view. Cf. also Sylla (1984), pp. 11–43.

[31] Galilei (1890–1909), Vol. I, pp. 276 ff.

[32] Thus, for example, the arithmetical ratio of 5 to 8 is the same as that of 20 to 23, since $8 - 5 = 3$, which is equal to $23 - 20$ (ibid., pp. 278 ff.).

Table 1. Two bodies of equal volume, A, B, weighing 10 and 30, have speeds as 9, 8, 6, 2, and 29, 28, 26, 22, when falling in media (M) of increasing specific weights, such as 1, 2, 4, 8. Speeds diverge as the specific weight of the medium increases. According to Euclid's theory of proportions, the ratio of 22 to 2 is much greater than the ratio of 29 to 9, but if ratios are considered arithmetically, the ratio of 22 to 2 is the same as the ratio of 29 to 9 (the differences between their quantities being equal).

	Speeds of Body $A = 10$	Speeds of Body $B = 30$
M = 1	9 (= 10 – 1)	29 (= 30 – 1)
M = 2	8 (= 10 – 2)	28 (= 30 – 2)
M = 4	6 (= 10 – 4)	26 (= 30 – 4)
M = 8	2 (= 10 – 8)	22 (= 30 – 8)

Two theories of motion in the void must at one point have competed in Galileo's mind. A 'restricted' theory predicted equality of speeds in a vacuum only for bodies of the same matter. A 'general' theory predicted equality of speeds in a vacuum for bodies of all types of matter. Both theories predicted the divergence of speeds in media of increasing specific weight, on the assumption that ratios are considered geometrically, according to Euclid's *Elements*.

So, how did Galileo go about discriminating between the two theories, now that he had long since and for good converted to geometrical ratios? The question, in other words, still remained open as to whether the divergence of speeds in fluid media could be interpreted *inversely*, as either leading to exact equality of speeds in a vacuum for bodies of all types of matter, or simply leading to a convergence of speeds in a vacuum, still to be considered different and proportional to the specific gravities of the bodies. The diagrams in Figure 4 should clarify what I mean by *inverse* interpretation of the phenomenon of divergence.

Here, on the basis of Galileo's summary account in the *Postils to Rocco*, I will sketch the solution that Galileo found to his dilemma. However, it is a very complex

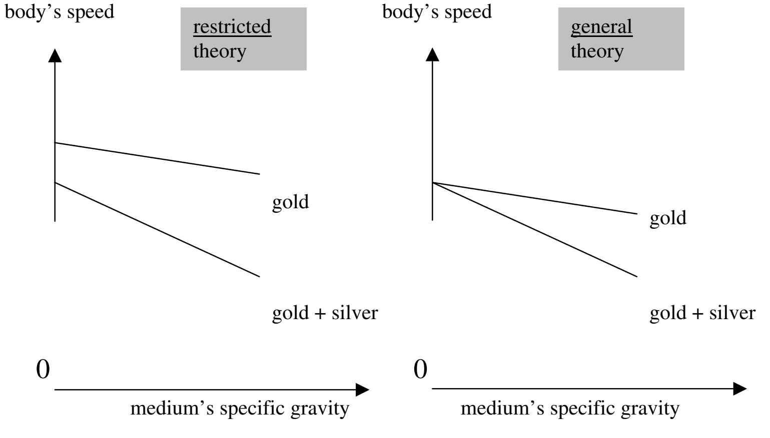

Fig. 4. Two possible inverse interpretations of the phenomenon of the divergence of speeds. In both cases speeds converge as the medium's specific gravity tends to zero (left areas of the diagrams), but in the restricted theory speeds remain different, while in the general theory they become equal.

story that involves all the fundamental discoveries in mechanics and hydrostatics that Galileo had made in the four decades spanning the period from the 1590s to the 1630s. For this reason Galileo's *ex post facto* account, with all its intriguing ambiguities, should be taken with great caution.

We know, Galileo asserts, that if we let two spheres of gold and cork (having the same volume) fall from a height of 100 braccia in air, the golden sphere will precede the cork one by, say, two or three braccia. However, this is due to resistance caused by the medium. For when the spheres are let fall from a height of one or two braccia the difference will disappear altogether. Thus if we could remove all the impediments caused by the medium the difference would disappear even when the spheres fall from a great height. That the difference must stem from the impediments of the medium, Galileo suggests, is indicated by the following considerations. If gold were faster solely in virtue of gravity then it would be reasonable to expect that once all the impediments due to the medium have been removed the golden sphere's speed would surpass the cork sphere's speed with the same ratio as that of their gravities, even when the spheres fall from very small heights. Therefore, when these experiments are conducted from very small heights, so that all the impediments due to the medium are kept to a minimum, if we observe that the speeds of the two spheres tend to become equal in media of decreasing heaviness, so that even in a very light medium, such as air, differences of speeds all but disappear, then, Galileo claims, we are entitled to hypothesize that in a vacuum their speeds would be identical.[33]

The whole argumentative strategy hinges on a theory of the resistance to motion caused by the fluid medium, which Galileo has expounded in the discussion leading up to this final point.[34] Briefly, he has argued that four forms of resistance are exerted by fluid media. First, the Archimedean thrust, which simply makes the body lighter. Second, the resistance due to the viscous property of the medium. Third, the resistance due to the body's speed, so that the faster the body moves the greater the resistance opposed by the medium. Fourth, the resistance due to friction between a body's surface and the fluid. The latter is caused by the sticking of the medium particles to the asperities of the body's surface.

It should thus be clear why the experimental conditions proposed by Galileo reduce resistance to a minimum. Since he considers equal spheres, both the Archimedean thrust and surface friction are the same. Since speeds need be compared within the same medium the viscosity factor is canceled. Finally, and most importantly, from very small heights bodies do not acquire great speeds, so that resistance due to speed is almost negligible. However, as Galileo is well aware, this resistance cannot be eliminated altogether.

[33] Ibid., Vol. VII, pp. 743–744.
[34] Cf. ibid., pp. 734–742.

This, if we accept his autobiographical account, is how Galileo eventually arrived at the theory of the equality of speeds for all bodies in a vacuum. Did Galileo really conduct the decisive experiments from very small heights that he mentions in his account? As is well known, it has been the business of most Galileo scholarship in the last few decades to decide whether he ever performed the experiments he reports in his writings. In our case, too, we can only add one more tantalizing question to the vexed historiographical problem.

5. An omission in *Two new sciences*

We are now well prepared to consider the 'sanitized' version of the famous thought experiment given by Galileo in *Two new sciences*. Galileo's spokesman, Salviati, addresses his Aristotelian interlocutor, Simplicio, as follows.

> *Salviati*. ... without other experiences, by a short and conclusive demonstration, we can prove clearly that it is not true that a heavier movable is moved more swiftly than another, less heavy, *these being of the same material*,[35] and in a word, those of which Aristotle speaks. Tell me, Simplicio, whether you assume that for every heavy falling body there is a speed determined by nature such that this cannot be increased or diminished except by using force or opposing some impediment to it.[36]

Simplicio's answer is highly significant in the light of our previous discussion. It emphasizes that change in speed can only be caused by the addition of new speed, or impetus.

> *Simplicio*. There can be no doubt that a given movable in a given medium has an established speed determined by nature, which cannot be increased except by conferring on it some new impetus, nor diminished save by some impediment that retards it.

Now Salviati needs Simplicio to agree to the second hypothesis that will make the entire exercise work.

> *Salviati*. Then if we had two movables whose natural speeds were unequal, it is evident that were we to connect the slower to the faster, the latter would be

[35] Emphasis mine. Here Galileo explicitly restricts the argument to bodies of the same matter, that is, as we have seen, of the same specific weight. This is confirmed in a subsequent passage, where Galileo has Sagredo, the third interlocutor, say 'You have clearly demonstrated that it is not at all true that unequally heavy bodies, moved in the same medium, have speeds proportional to their weights [*gravità*], but rather have equal [speeds]. You assumed bodies of the same material (or rather, of the same specific gravity), and not (or so I think) of different density, because I do not believe you mean us to conclude that a cork ball moves with the same speed as one of lead'. To this comments Galileo replies by explaining how he gradually moved away from the restriction of bodies of equal specific weights (Galilei, 1974, pp. 71–72).

[36] Ibid., p. 66.

> partly retarded by the slower, and this would be partly speeded up by the faster. Do you not agree with me in this opinion?
> *Simplicio*. It seems to me that this would undoubtedly follow.

Thus the conclusions may easily be drawn.

> *Salviati*. But if this is so, and if it is also true that a large stone is moved with eight degrees of speed, for example, and a smaller one with four [degrees], then joining both together, their composite will be moved with a speed less than eight degrees. But the two stones joined together make a larger stone than that first one which was moved with eight degrees of speed; therefore this greater stone is moved less swiftly than the lesser one. But this is contrary to your assumption. So you see how, from the supposition that the heavier body is moved more swiftly than the less heavy, I conclude that the heavier moves less swiftly.[37]

A little further on, Galileo has Simplicio raise the objection that the smaller stone adds weight to the greater stone. The solution to Simplicio's objection is of course the same that Galileo related in the *Postils to Rocco*. Simplicio is very doubtful about the extraordinary claim that a stone will not add weight to another stone when both fall at the same rate of speed. Yet the *Postils to Rocco* reveals that Simplicio's astonishment was originally Galileo's. In the following passage from the *Postils*, Galileo addresses Rocco, *imagining* the latter's astonishment if the Aristotelian learned of Galileo's conclusion about the weightless condition of falling bodies (cf. Figure 1, repeated by Galileo in the *Postils*):

> how unheard of, it will be to you, if I showed you that neither the adding of B, nor of one thousand Bs, adds a smidgen of weight to A, and that in consequence since B's weight does not increase, its speed, too, will not increase ...[38]

Finally, we may note that Galileo decided not to present the second part of the argument, which establishes the general validity of the principle that all bodies fall in a vacuum with the same speed, immediately following the famous thought experiment. He disarticulated, so to say, the whole story he had recounted in the *Postils*. He even dropped the revealing phrase '*spuntar lo scoglio più duro*', thus unwittingly leading historians and philosophers interested in his methodology astray. He only discussed this second part after many digressions, further on in *Two new sciences*.

[37] Ibid., pp. 66–67.

[38] Galilei (1890–1909), Vol. VII, p. 732. It may be of interest to notice that although Galileo knows that weight is not the cause of speed, the force of traditional language creeps in his comment, 'and that in consequence since B's weight does not increase, its speed, too, will not increase'.

There he also speaks of ingenious experiments with pendula (not mentioned in the *Postils*) that he claims to have conducted in order to obviate the difficulty of directly observing falls from very small heights.[39] But this is really another story.

6. Conclusion

To sum up, what moral can we draw from the tortuous path that Galileo followed from *De motu* to *Two new sciences*? All of us who are interested in the significance of thought experiments in the sciences should be careful when considering their epistemic status and the kind of evidence that they are supposed to afford. All the more so when we examine past thought experiments in a modern frame of mind, in which belief in their efficacy, or doubt as to their conclusiveness, may be affected by our long established assumptions in the natural sciences. Creativity is always surprising. What might today be seen as an instance of a very special form of cognition, which transcends empiricism, and might allow us a glimpse into a Platonic realm of laws, was born in the murky waters of analogical thinking, *ex post facto* cognitive autobiography, and, in all likelihood, real experimentation. Perhaps we need a new approach to the question of thought experiment, capable of integrating results from different disciplinary areas, such as, for instance, the history and philosophy of science and cognitive science.[40] Recent discussions of thought experimentation as mental modeling look promising.[41] In my view, Gendler's perplexity was more than justified. The all too clean baby of today's debate on the most beautiful thought experiment in the history of science should definitely be thrown out, and the bathwater carefully analyzed.

References

Archimedes (1543). *Opera Archimedis Syracusani philosophi et mathematici ingeniosissimi*. Venice: per Venturinum Ruffinellum.

Archimedes (1565). *De iis quae vehuntur in aqua libri duo, a Federico Commandino Urbinate in pristinum nitorem restituti et commentariis illustrati*. Bologna: Ex officina A. Benacii.

Aristotle (1986). *De caelo* (W. K. C. Guthrie, Trans.). Cambridge, MA: Harvard University Press. (First published 1939)

Aristotle (1984). *The complete works of Aristotle* (2 vols.) J. Barnes. (Ed.). Princeton: Princeton University Press.

[39] Galilei (1974), pp. 71 ff.

[40] Here the point might be raised that even in the case of mathematics a lot of trial and error went into the discovery of theorems, such as, for instance, the Pythagorean theorem, but we nevertheless rightly consider them *a priori* knowledge, because we can now justify them without experience. My circumscribed conclusion is that the story recounted in this paper, though perhaps not undermining *a priori* accounts of what transpired, typically to be found in the philosophical literature, suggests that our criteria of what constitutes *a priori* knowledge cannot be defined a-historically. I thank an anonymous referee for pointing out this valuable point.

[41] Miščević (1992), Nersessian (1993), and Palmieri (2003) stress the potential of mental models reasoning as a new framework for understanding thought experiments. With some caveats, mental models are considered as a promising avenue of research also by Norton (2004).

Armjio, C. (2001). Un nuevo rol para las definiciones. In J. Montesinos, & C. Solís (Eds.), *Largo campo di filosofare. Eurosymposium Galileo 2001* (pp. 85–99). La Orotava, Tenerife: Fundación Canaria Orotava de Historia de la Ciencia.

Atkinson, D., & Peijnenburg, J. (2004). Galileo and prior philosophy. *Studies in History and Philosophy of Science, 35A*, 115–136.

Brown, J. R. (1991). Thought experiments: A Platonic account. In T. Horovitz, & G. J. Massey (Eds.), *Thoughts experiments in science and philosophy* (pp. 119–128). Savage, MD: Rowman & Littlefield.

Brown, J. R. (2004). Why thought experiments transcend empiricism. In C. Hitchcock (Ed.), *Contemporary debates in the philosophy of science* (pp. 23–43). Malden, MA: Blackwell.

Budden, T. (1998). Galileo's ship thought experiment and relativity principles. *Endeavour, 22*, 54–56.

Camerota, M. (1992). *Gli scritti 'De motu antiquiora' di Galileo Galilei: Il Ms Gal 71*. Cagliari: Cuec.

Drake, S. (1973). Velocity and Eudoxan proportion theory. *Physis, 15*, 49–64. (Reprinted in Drake, 1999, Vol. II, pp. 265–280)

Drake, S. (1974). Mathematics and discovery in Galileo's physics. *Historia Mathematica, 1*, 129–150. (Reprinted in Drake, 1999, Vol. II, pp. 292–306)

Drake, S. (1987). Euclid's Book V from Eudoxus to Dedekind. *Cahiers d'histoire et de philosophie des sciences, n.s. 21*, 52–64. (Reprinted in Drake, 1999, Vol. III, pp. 61–75)

Drake, S. (1995). *Galileo at work. His scientific biography*. New York: Dover. (First published Chicago: University of Chicago Press, 1978)

Drake, S. (1999). *Essays on Galileo and the history and philosophy of science* (3 vols.) (selected and introduced by N. M. Swerdlow, T. H. Levere). Toronto: University of Toronto Press.

Euclid (1956). *The thirteen books of the Elements* (2nd ed.) (3 vols.) (translated with introduction and commentary by T. Heath). New York: Dover.

Frajese, A. (1964). *Galileo matematico*. Rome: Universale Studium.

Fredette, R. (1969). *Les 'De motu' 'plus anciens' de Galileo Galilei: Prolegomenes*. Ph.D. thesis, University of Montreal. (Available at http://www.mpiwg-berlin.mpg.de/litserv/diss/fred_1/HTML/Page001.htm)

Galilei, G. (1590). *De motu*. Electronic version of the manuscript available at http://xserve02.mpiwg-berlin.mpg.de:18880/echo_nav/echo_pages/content/scientific_revolution/galileo.

Galilei, G. (1890–1909). *Le opere di Galileo Galilei* (edizione nazionale) (20 vols.) (A. Favaro, Ed.). Florence: Barbèra.

Galilei, G. (1974). *Two new sciences. Including centres of gravity and force of percussion* (S. Drake, Trans.). Madison: The University of Wisconsin Press.

Galilei, G. (2000). *De motu* (R. Fredette, Trans.). Electronic version available at http://xserve02.mpiwg-berlin.mpg.de:18880/echo_nav/echo_pages/content/scientific_revolution/galileo.

Galluzzi, P. (1979). *Momento. Studi Galileiani*. Rome: Edizioni dell' Ateneo & Bizzarrim.

Gendler, T. S. (1998). Galileo and the indispensability of scientific thought experiment. *British Journal for the Philsophy of Science, 49*, 397–424.

Gendler, T. S. (2000). *Thought experiment. On the powers and limits of imaginary cases*. New York & London: Garland.

Geymonat, L., & Carugo, A. (1981). I cosiddetti esperimenti mentali nei *Discorsi* Galileiani e i loro legami con la tecnica. In L. Geymonat (Ed.), *Per Galileo* (pp. 81–98). Verona: Bertani Editore.

Giusti, E. (1981). Aspetti matematici della cinematica Galileiana. *Bollettino di Storia delle Scienze Matematiche, 1*, 3–42.

Giusti, E. (1986). Ricerche Galileiane: Il trattato 'De motu equabili' come modello della teoria delle proporzioni. *Bollettino di storia delle scienze matematiche, 6*, 89–108.

Giusti, E. (1990). Galilei e le leggi del moto. In G. Galilei, *Dicorsi e dimostrazioni matematiche intorno a due nuove scienze attinenti alla meccanica ed i movimenti locali* (E. Giusti, Ed.) (pp. ix–lx). Turin: Einaudi.

Giusti, E. (1992). La teoria Galileiana delle proporzioni. In L. Conti (Ed.), *La matematizzazione dell' universo. Momenti della cultura matematica tra '500 e '600* (pp. 207–222). Perugia: Edizioni Porziuncola.

Giusti, E. (1993). *Euclides Reformatus. La Teoria delle proporzioni nella scuola Galileiana*. Turin: Bollati–Boringhieri.

Giusti, E. (1994). Il filosofo geometra. Matematica e filosofia naturale in Galileo. *Nuncius, 9*, 485–498.

Giusti, E. (1995). Il ruolo dela matematica nella meccanica di Galileo. In A. Tenenti et al. (Eds.), *Galileo Galilei e la cultura veneziana* (pp. 321–338). Venice: Istituto Veneto di Scienze, Lettere e Arti.
Giusti, E. (1998). Elements for the relative chronology of Galilei's *De motu antiquiora*. *Nuncius, 13*, 427–460.
Grattan-Guinness, I. (1996). Numbers, magnitudes, ratios, and proportions in Euclid's *Elements*: How did he handle them? *Historia Mathematica, 23*, 355–375.
Koyré, A. (1973). *Études d'histoire de la pensée scientifique*. Paris: Éditions Gallimard. (First published Paris: Presses Universitaires de France, 1966)
Kuhn, T. S. (1977). A function for thought experiments. In idem, *The essential tension* (pp. 240–265). Chicago & London: University of Chicago Press.
McAllister, J. W. (1996). The evidential significance of thought experiment in science. *Studies in History and Philosophy of Science, 27A*, 233–250.
Maracchia, S. (2001). Galileo e Archimede. In J. Montesinos, & C. Solís (Eds.), *Largo campo di filosofare. Eurosymposium Galileo 2001* (pp. 119–130). La Orotava, Tenerife: Fundación Canaria Orotava de Historia de la Ciencia.
Miščević, N. (1992). Mental models and thought experiments. *International Studies in the Philosophy of Science, 6*, 215–226.
Nersessian, N. (1993). In the theoretician's laboratory: Thought experimenting as mental modeling. In D. Hull, M. Forbes, & K. Okruhlik (Eds.). *PSA 1992* (Vol. 2, pp. 291–301). East Lansing, MI: Philosophy of Science Association.
Norton, J. (1991). Thought experiments in Einstein's work. In T. Horovitz, & G. J. Massey (Eds.), *Thoughts experiments in science and philosophy* (pp. 129–148). Savage, MD: Rowman & Littlefield.
Norton, J. (1996). Are thought experiments just what you thought. *Canadian Journal of Philosophy, 26*, 333–366.
Norton, J. (2004). Why thought experiments do not transcend empiricism. In C. Hitchcock (Ed.), *Contemporary debates in the philosophy of science* (pp. 44–66). Malden, MA: Blackwell.
Palladino, F. (1991). La teoria delle proporzioni nel Seicento. *Nuncius, 6*, 33–81.
Palmieri, P. (2001). The obscurity of the equimultiples. Clavius' and Galileo's foundational studies of Euclid's theory of proportions. *Archive for History of Exact Sciences, 55*, 555–597.
Palmieri, P. (2002). *Galileo's mathematical natural philosophy*. Ph.D. thesis, University of London.
Palmieri, P. (2003). Mental models in Galileo's early mathemtization of nature. *Studies in History and Philosophy of Science, 34A*, 229–264.
Prudovsky, G. (1989). The confirmation of the superposition principle: On the role of a constructive thought experiment in Galileo's *Discorsi*. *Studies in History and Philosophy of Science, 20*, 453–468.
Rose, P. L. (1975). *The Italian Renaissance of mathematics. Studies on humanists and mathematicians from Petrarch to Galileo*. Genève: Librarie Droz.
Saito, K. (1986). Compounded ratio in Euclid and Apollonius. *Historia Scientiarum, 31*, 25–59.
Saito, K. (1993). Duplicate ratio in book VI of Euclid's *Elements*. *Historia Scientiarum, 50*, 115–135.
Sasaki, C. (1985). The acceptance of the theory of proportions in the sixteenth and seventeenth centuries. *Historia Scientarum, 29*, 83–116.
Simplicius (1563). *In quatuor libros De coelo Aristotelis*. Venice: apud Hieronymum Scotum.
Sorensen, R. A. (1992). *Thought experiments*. New York & Oxford: Oxford University Press.
Sylla, E. D. (1984). Compounding ratios. Bradwardine, Oresme, and the first edition of Newton's *Principia*. In E. Mendelsohn (Ed.), *Transformation and tradition in the sciences. Essays in honor of I. Bernard Cohen* (pp. 11–43). Cambridge: Cambridge University Press.