HW 9

1. Find all the Nash equilibria for the following payoff matrix

$$\begin{pmatrix} (5,5) & (5,10) & (8,6) \\ (6,8) & (4,4) & (1,3) \\ (3,1) & (3,1) & (7,7) \end{pmatrix}$$

2. Consider the asymmetric rock,paper,scissors game with the following matrix (where a, b positive):

$$\begin{pmatrix} 0 & -a & b \\ b & 0 & -a \\ -a & b & 0 \end{pmatrix}$$

Let x, y, z denote the fraction of players using rock, papers, scissors respectively. Since $z = 1 - x - y$, I want you to explore the dynamics of this for different choices of a, b in the x, y phaseplane using the replicator dynamics described in class. In particular, I want you to find all the equilibria and assess their stability (numerically, by probing with initial conditions near the equilibria). Consider the following choices of $(a, b) = (1, 1), (.5, 1), (2, 1)$. I have included code for this in matlab and xpp (rps) (note that you will get the wrong answer in matlab for $a = b = 1$ since it uses Euler and this is not accurate enough)

3. Make a table for the following scenario; an extension of Hawk-Dove. Let's call it a Mixed strategy. In this case the player picks Hawk with probability p and dove with probability $1 - p$. So we have H,D, M. Consider H versus M. When M plays H, the payoff for both is $(G - C)/2$ and when M plays D, the payoff for H is G and for M is 0. So this means that when H plays M, the average payoff for H is $p(G - C)/2 + G(1 - p)$. The payoff for M is $p(G - C)/2 + (1 - p)0$. Based on this idea, fill in the rest of the table or payoffs. When D plays M, it is also easy. The hard case is when M plays M. There are 4 different scenarios, $(H, H), (H, D), (D, H), (D, D)$ with probabilities, $p^2, p(1 - p), (1 - p)p, (1 - p)^2$ respectively. So this will yield the payoff for M playing M. Now, suppose that $G = 1$ and $C = 2$. As we saw in class, for just H,D, there is no pure strategy that is a Nash equilibrium. Can you find a p so that the M strategy is the Nash equilibrium? Note that this strategy will not be a strict Nash equilibrium. Instead, it is a strategy in which the payoff does not increase if you switch; it stays the same. More generally, show that for $C > G$, the value of the probability is $p = G/C$.