Homework #2

1. Atmospheric drag is proportional to Sv^2 where S is surface area and v is velocity. Kinetic energy is proportioanl to mv^2 where m is mass. Terminal velocity is that velocity where the drag force balances the force due to gravity (mg). (a) Show that the terminal velocity, v scales like $m^{1/6}$ for similarly shaped objects. (b) Show that the kinetic energy per unit area scales as $m^{2/3}$. (c) Use this to comment on what happens to small vs large animals if they fall down a great height. (d) Show that the height at which terminal velocity is reached scales like $m^{1/3}$ (Hint recall from HS physics that potential energy is mgh.)

2. How long should a turkey be cooked?

   (a) One of the simple rules of thumb for an unstuffed turkey is 15 minutes per pound. So this says that the time to cook scales linearly with mass. However, the following table is taken from a cookbook that provides the following times:

W	time	log10(W)	log10(T)
8	2.75	0.90309	0.43933269
12	3	1.0791812	0.47712126
14	3.75	1.1461281	0.57403129
18	4.25	1.2552725	0.62838894
20	4.5	1.30103	0.65321249
24	5	1.3802112	0.69897002

   where I have also taken the logs for you. (Note that is you want to try different data, the Butterball web page offers different guidelines: http://www.butterball.com/how-tos/roast-a-turkey) Use Least squares to fit the log data to a line, to get the scaling law, $T = cW^p$. The rule of thumb says $p = 1$. Pretty far away from 1, huh. (note some online programs also give you the correlation coefficient for your least squares – close to 1 is a good fit, close to 0 is a bad fit)

   (b) Let's try to do better. The Heat equation (which we will study later determines how heat diffuses (from the oven) through the turkey, $u_t = \kappa \nabla u$ where κ, is a specific parameter with dimensions of L^2/T. We usually have, L, T, M as the independent dimensions. To make life easy, we will add K, temperature as another independent dimension. The main parameters we have are u temperature in the turkey, u_O, temperature in the oven, m mass of the turkey, ρ density of the turkey (dimensions ML^{-3}), time, t and lastly, diffusivity κ. Write:

   $$u^a u_O^b m^c \rho^d t^e \kappa^f$$

   There are 6 unknowns and 4 dimensions, so there will be at least a 2-dimensional nullspace. Use this to find two dimensionaless quantities, π_1, π_2 so that the physical law can be written in terms of these.

(c) Using these quantities and the Buckingham Pi theorem to conclude that assuming a fixed oven temperature and a internal turkey temperature, that $t \propto m^{2/3}$ Does this fit the data better?

3. How does the speed of a water wave depend on various parameters? Let v be the velocity of the waves, g be gravitational acceleration, λ be the wavelength, y be the depth of the water. It can be shown that if the height of the wave is small, we can ignore it. As in the turkey exercise, consider

$$v^a g^b \lambda^c y^d$$

How many dimensionless quantities are there? Find them and show

$$v = \sqrt{\lambda g} F(d/\lambda)$$

for some function F.

4. Do problems 1,7 in section 1.4 of Meerschaert