1. A patient starts taking a drug once a day at an amount b milligrams. From pharmacological studies, it is known that the drugs halflife is h hours. (That is the amount of drug decays exponentially such that after h hours, it is half its original amount.) Let a_n be the amount of drug right after the n^{th} dose. Show that this satisfies:

$$a_{n+1} = ka_n + b$$

Figure out what k should be based on the pharmacological data. Solve the difference equation assuming that $a_0 = 0$ by guessing that $a_n = c + d\lambda^n$. What is the steady state (equilibrium) value of the drug, that is the value as $n \to \infty$?

2. Fireflies from Southeast Asia (and also in the Great Smokey Mtns and in Allegheny National Forest) are known to congregate in large groups and flash synchronously. For the species, *Pteroptyx malaccae*, experimentalists have studied their behavior in the field and have estimated how the timing a single flash of light can shift the timing of their own flash. When left on his own (the animals that flash syncheonously are males), the firefly will flash with a period of P milliseconds. Let T be the period of the strobe light that is being flashed. The shift in the fireflies time to flash is a sinusoidal function of the time that the stimulus arrives. So, let ϕ_n now be the phase (normalized time modulo the period, so that it lies between 0 and 1) that the animal flashes right before the n^{th} stimulus. The above assumptions mean that:

$$\phi_{n+1} = \phi_n + \frac{T}{P} - \frac{a}{2\pi}\sin(2\pi\phi)$$

We define 1:1 locking to mean that the phase advances by exactly 1 between two flashes. That is $\phi_{n+1} = \phi_n + 1$. (a) Find conditions on a and T (the strobe period) such that there will be 1:1 locking and find the values $\bar{\phi}$ such that locking occurs when it can occur (b) determine the stability of the locked solutions; (c) Pick $a = .5$ and $T/P = \{.96, 1.2, 1.9\}$ describe the behavior.

3. Birth death and thresholds. Suppose that a species needs enough of its own around or it will go extinct, and it will stop growing if it gets too crowded. This suggests the following model:

$$x_{n+1} = x_n + bx_n(x_n - r)(1 - x_n), \quad 0 < r < 1$$

(a) Give an interpretation of the parameters and terms in this model, (b) Find all the equilibria and their stability. (c) Sketch the graphical (cobweb) solution for $r = 0.25$ and $b = 0.5, 1.5, 2.5, 3.5$ and several different initial conditions for x. Are these results consistent with your stability analysis?

4. Consider the model for competition between two species:

$$\begin{aligned} A_{n+1} &= \mu_1 A_n - \mu_3 A_n B_n \\ B_{n+1} &= \mu_2 A_n - \mu_4 A_n B_n \end{aligned}$$

where μ_j are all positive constants. (a) Find all the non-negative equilibria (b) Determinetheir stability for the specific case, $\mu_1 = 1.2, \mu_2 = 1.3, \mu_3 = 0.001, \mu_4 = 0.002$

5. A model for a host (H)/parasite (P) system takes the following form:

$$\begin{aligned} H_{n+1} &= H_n e^{1-H_n/K} e^{-aP_n} \\ P_{n+1} &= cH_n P_n \end{aligned}$$

(a) Give an interpretation of the terms in the model. For example, first consider the host equation without the parasite ($P = 0$). What is it's behavior? What does the parasite do to the growth of the host? What us the effect of the host on the growth of the parasite? (b) Let $h_n = H_n/K$ and $p_n = aP_n$ be dimensionless variables. Show that the equations are then:

$$\begin{aligned} h_{n+1} &= h_n e^{1-h_n} e^{-p_n} \\ p_{n+1} &= d h_n p_n \end{aligned}$$

and determine the value of d in terms of c, K, a. (c) Find all the equilibria and determine their stability. (Hint, note that $1 = e^{1-h}e^{-p}$ can be easily solved by taking logs of both sides!). (d) Simulate the model for 200 iterations for with $h_0 = 1, p_0 = .05$ $d = 0.9$, $d = 1.9$, $d = 2.2$. Are your results compatible with your stability analysis? For example, for $d = 2.2$ are there any stable equilibria? If not how did they lose stability in the stability triangle?

Please note that there are a number of applets out there for cobwebbing.

`https://mathinsight.org/applet/function_iteration_cobweb_combined`
`https://www.geogebra.org/m/QJ79IWCL`
`http://math.colgate.edu/math312/Spring1999/iterate.html`