HW 6 - due Oct 19

1. Consider the pendulum on a spring. Here r is the length of the pendulum and because it is on a spring it can move freely only along the axis (i.e - think if it like a shock absorber on a car with only one degree of freedom) The potential energy, $P.E. = mgy + (k/2)(r - r_0)^2$ where k is the spring constant and r_0 is the rest length of the spring. (a) Express (x, y) in terms of (r, θ) (b) $K.E. = (m/2)(\dot{x}^2 + \dot{y}^2)$. Express K.E. in terms of $r, \theta, \dot{r}, \dot{\theta}$. (c) Express P.E in terms of (r, θ). (d) The lagrangian $L = K.E. - P.E.$ Compute $\partial L/\partial r$, $\partial L, \partial \dot{r}$, $\partial L/\partial \theta$, $\partial L, \partial \dot{\theta}$. (e) Write the equations of motion:

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) &= \frac{\partial L}{\partial r} \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) &= \frac{\partial L}{\partial \theta} \end{aligned}$$

You do not have to solve them!! (e) Show that if $\theta(0) = 0, \theta(\dot{0}) = 0$, then you get:

$$m\ddot{R} = mg - k(r - r_0)$$

the simple linear spring!

2. Consider the "guitar string" illustrated in the figure. The rest state of each spring is R when the mass is centered at the origin. (a) compute d_1, d_2 as a function of (x, y). We have

$$\begin{aligned} P.E. &= (k/2)[(d_1 - R)^2 + (d_2 - R^2)] \\ K.E. &= (m/2)(\dot{x}^2 + \dot{y}^2) \end{aligned}$$

(b) The Lagrangian is $L = K.E. - P.E$ Compute the equations of motion for (x, y) (c) Show that if $y(0) = 0, \dot{y}(0) = 0$, you recover the linear spring on a line!

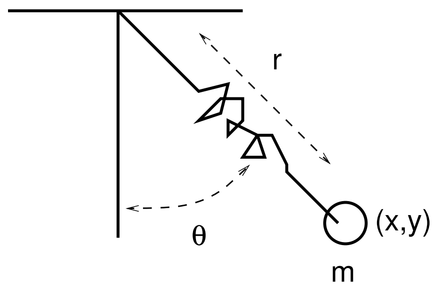
3. Roller coaster tycoon. Okay, lets make a roller coaster that lies in the (x, y) plane. We will keep it simple and suppose that it can be described by a simple function, $y = f(x)$. Let's assume that f is at least twice differentiable. Clearly, $P.E. = mgy$ and $K.E. = (m/2)(\dot{x}^2 + \dot{y}^2)$. Using the chain rule a lot, compute the Lagrangian $L = K.E. - P.E$ and the equations of motion in terms of x (as y is a constrained to lie on the track) :

$$(d/dt)(\partial L/\partial \dot{x}) = \partial L/\partial x$$

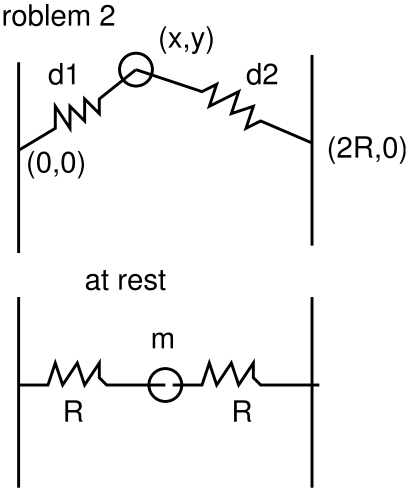
Compute the total energy, $P.E. + K.E.$ For example, here is a fun roller coaster, $f(x) = x^4 - 3x^2$. Derive the equations for this model and simulate it with initial conditions $x(0) = -2, \dot{x}(0) = 0$. Plot $x(t), y(t) = f(x(t))$.

4. Sketch the phase plane for the systems $\ddot{x} = -dF/dx$ with (a)$F(x) = x^4/4 - x^2/2$, (b) $F(x) = \cos(x)$, (c,d) $F(x)$ as drawn in the figures

Problem 1



Problem 2



Problem 4