HOMEWORK 2
Due Sept 24

1. In Teschl: 2.1,2.4,2.6

2. Consider the ODE
$$\dot{x} = x^2, \quad x(0) = a$$
In the existence theorem, solutions exist over an interval $[-T_0, T_0]$ where $T_0 = \min\{T, \delta/M\}$ where T, T_0, M are defined in the book. For this problem, choose a radius, δ for your ball so as to maximize, T_0 for given $a > 0$. Solve the ODE and compare your estimated interval of existence to the actual one.

3. Let $f(x) = Ax$ where A is a constant matrix. Show that each component of the n^{th} Picard iteration to any solution is a polynomial of degree at most n. Can you guess a formula for $x_n(t)$ and from this guess a formula for $x(t)$