

HOMEWORK 3  
Due Oct 6 (Not 8th)

1. Problems 2.14 (p 48), 2.17 (p50) 2.20,2.21 (p54) in Teschl
2. Suppose  $\Omega$  is a subset of  $R^n$  and  $f, g : \Omega \rightarrow R^n$  are continuous (and Lipschitz, though that doesn't come into the main point here) and there is a continuous positive function,  $h : \Omega \rightarrow (0, \infty)$  such that  $g(u) = h(u)f(u)$  for all  $u \in \Omega$ . If  $x$  is the unique solution to  $x' = f(x)$  with  $x(0) = a$  defined on the maximal interval of existence and  $y$  is the unique solution of  $y' = g(y)$  with  $y(0) = a$ , also defined on its maximal interval of existence, show there is an increasing function,  $j : \text{dom}(y) \rightarrow \text{dom}(x)$  such that  $y(t) = x(j(t))$  for all  $t \in \text{dom}(y)$ . Note that this means that the phase-portraits of these two ODEs are the same.