

## HOMEWORK 4

Due Oct 15

1. Suppose that for the square matrices of the same dimension  $A, B$  that the origin is asymptotically stable for both  $x' = Ax$  and  $x' = Bx$ . Is it true that the origin for the system  $x' = (A + B)x$  is asymptotically stable? If not, find a counterexample. Suppose  $AB = BA$ . Does your answer change?
2. Find the general solutions to the linear equation  $tx' = Ax$ . (Hint see problem 1.19 in Teschl)
3. Problems 3.9, 3.12, 3.14, 3.18, 3.20 Teschl
4. Let  $\lambda$  be an eigenvalue of  $A$  where  $a_{ij}$  are the entries of matrix  $A$  and let  $v$  be the eigenvector. Suppose  $v_i$  is the component of  $v$  with biggest magnitude. Prove

$$|\lambda - a_{ii}| \leq \sum_{j \neq i} |a_{ij}|.$$

Use this to conclude that all eigenvalues lie in the union of the disks,  $D_k$  in the complex plane defined by

$$D_k = \{z \mid |z - a_{kk}| < \sum_{j \neq k} |a_{kj}|\}$$