

HOMEWORK 5

Due Oct 22

1. Problem 3.25 (p 85) Teschl (only prove second assertion as we proved the first in class); 3.31 (p 86), 3.38 (p 90) 3.39, 3.40 (p96)

2. Let

$$A(t) = \begin{bmatrix} \frac{1}{2} - \cos t & b \\ a & \frac{3}{2} + \sin t \end{bmatrix}$$

Show that there is a solution to $x' = A(t)x$ that becomes unbounded as $t \rightarrow \infty$.

3. Classic example of Markus & Yamabe. Show that if

$$A(t) = \begin{bmatrix} -1 + \frac{3}{2} \cos^2 t & 1 - \frac{3}{2} \sin t \cos t \\ -1 - \frac{3}{2} \sin t \cos t & -1 + \frac{3}{2} \sin^2 t \end{bmatrix}$$

then the eigenvalues of $A(t)$ have negative real part for every t , but

$$x(t) := \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix}$$

which becomes unbounded as $t \rightarrow \infty$ is a solution to $x' = A(t)x$.