HOMEWOK 6: Due Oct 29

1. Let $A(t+T) = A(t)$ and additionally suppose that $A(-t) = -A(t)$, that is, $A(t)$ is an odd periodic function. Prove that all solutions to $x' = A(t)x$ are periodic.

2. Consider $x'' + a(t)x = 0$ where $a(t+1) = a(t)$, $a(t)$ is continuous, and $\int_0^1 a(t)\, dt > 0$. From Floquet theory we know that there exists a solution, $u(t)$ such that $u(t+1) = \rho u(t)$ for some nonzero ρ. Prove that every such solution must have a zero in $[0, 1]$. (Hint: assume this is not the case and try to obtain a contradiction.)

3. Problem 3.44 Teschl.