HW # 7

Due Thursday November 19

1. Prove that solutions to $x'' = -x/t$ are oscillatory. (Hint: Compare to $x'' = -x/t^2$).

2. Find the eigenvalues and also the Green's function for $Lu = u''$ with $u'(0) = u'(1) = 0$.

3. Find the eigenvalues for the operator:

$$Ku = \int_0^1 e^{-|x-y|} u(y)\ dy.$$

Hint: Write the eigenvalue problem as

$$\frac{1/\lambda}{u} = e^{-x} \int_0^x e^y u(y)\ dy + e^x \int_x^1 e^{-y} u(y)\ dy$$

and try to convert this to a BVP.

4. Given the regular SL system $(P(x)u')' + (\lambda\rho(x) - q(x)) = 0$ along with two sets of boundary conditions $\alpha_j u(a) + \alpha_j' u'(a) = 0$ $(j = 1, 2)$ and $\beta u(b) + \beta' u'(b) = 0$, prove that if $\alpha_2'/\alpha_2 < \alpha_1'/\alpha_1$ then the eigenvalues corresponding to $j = 2$ are smaller than those corresponding to $j = 1$.

5. For a regular SL problem (like in the previous exercise) show that if $\rho_A(x) > \rho_B(x)$ then eigenvalues for case A will be less than those of case B and similarly if $q_A(x) < q_B(x)$

6. Show that if $c(x) < 0$ in $u'' + b(x)u' + c(x)u = 0$, no nontrivial solution of the ODE can have more than one zero. In a related problem, Let $c(x) < 0$ and $u'' + c(x)u = 0$. Show $u(x)u'(x)$ is an increasing function and infer that there can be at most one zero to $u(x)$.

7. Give probelm 5.32 on page 175 in Teschl a try. (If you don't want to do the numerical part, well, you can skip it. If you want to use my free ODE software, here is the file.

```
@ bound=1000000
@ total=200
x'=1
init x=1
y'=cos(y)^2+sin(y^2)/(3*x^2)
init y=0
d
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